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ON PHYSICAL AUTHENTICITY OF PLASTICITY PROBLEMS

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ABSTRACT

At present many plasticity problems do not have a reliable solution. This is connected to the fact that in the theoretical calculations, based on the classical plasticity theories, a complexity of the process of loading a body is not taken into consideration. Sometimes this leads to considerable errors. These particularly manifest themselves when there are stress concentrators in the construction body, as well as a multiparametric loading takes place.

The purpose of this note is to draw attention to the issue of authenticity in solving problems and to present possible ways of solving this problem. One of these ways is resulted from the CL-computer method of Ilyushin known as the method of evaluating the physical authenticity of plasticity problems.

Key Words: Plasticity, Complex loading, Postulate of Isotropy, Principle of delay, Physical authenticity, Processes of loading.

PLASTİSİTE PROBLEMLERİNİN FİZİKSEL GÜVENİRLİLİĞİ

ÖZ

Mevcut bir çok plastisite probleminin gerçek bir çözümü yoktur. Bu klasik plastisite teorileri temeline dayanan teorik hesaplamalar ile ilişkilidir. Bir elemanın yüklenmesi işleminin karşılığı göz önüne alınmamıştır. Bazen bu kayda değer hatalara yol açmaktadır. Bu hatalar, özellikle çok parametreli yüklenenin meydana geldiği durumlarda ve yapı elemanlarında gerilme yığılmaları olduğunda ortaya çıkmaktadır.

Bu notun amacı problemlerin çözümünde güvenirliliğin önemine dikkati çekmek ve bu problemin olası çözüm yollarını göstermektir. Bu yollardan biri de plastisite problemlerinin fiziksel güvenirliliği metodu olarak bilinen Ilyushin CL-bilgisayar metoduyla elde edilmiştir.

Anahtar Kelimeler: Plastisite, Karmaşık yükleme, İzotropi kabulü, Gecikme prensibi, Fiziksel, güvenirlik, Yükleme işlemi.

1. INTRODUCTION

The development of modern engineering is connected to the necessity of creating structures, installations and manufactures that correspond not only to their functional specifications but also to the requirements of optimal decisions (in terms of weight, dimensions, safety factor, etc.). This, in turn, requires that theoretical, numerical, and experimental studies of the objects to be carried out , taking into consideration plastic deformations resulted, for instance, from considerable heterogeneity of the stress field (due to concentrators: holes, recesses, cuts, etc.), appearing in the process of rolling, punching and depression. Problems of fastness beyond the elastic limit, elastoplastic waves propagation under combined stress conditions and many other problems may be listed as such examples. An exact mathematical statement of an appropriate boundary problem on the basis of physically authentic correlation between stresses and strains is realised within the framework of the A.A. Ilyushin's theorem on simple loading (Ilyushin,

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1948). However in many cases such a problem statement proves to be unreal and, in order to have it realised, special approximate approaches are required with the use of numerical and CL experiments (CL-complex loading). One of these special approaches is the general mathematical plasticity theory based on the theoretically and experimentally substantiated mechanics of continua postulates and principles (the macroscopic determinability hypothesis, the isotropy postulate, the principle of delay, etc.) and on geometry of the loading (strain) process (Ilyushin, 1963). Based on these approaches the CL–computer method (Babamuratov et al. 1987) is intended to solve the plasticity problems at incomplete theoretical information about constraint equations between stresses and strains. The most important consequence of the CL-computer method is the method of estimation of physical authenticity of the solutions of bound-ary plasticity problems.

This article is devoted to the development of a practical method of analysis and estimation of physical authenticity of problem solutions obtained by any version of the plasticity theory. The method, in essence, is this: ordinary calculations of constructions, carried out on the basis of any plasticity theory, are supplemented with the construction of theoretical (calculated) stress trajectories $\bar{\sigma} = \bar{\sigma}(x_k, \lambda)$ (x_k is point of construction with number k, λ is parameter of process) and strain trajectories $\bar{Э} = \bar{Э}(x_k, \lambda)$ in the most important points of the body, which in some cases are given by the customer of the problem. Based on this data, using a small number of testing experiments (usually 2 or 3) on the CL-test facility, there is estimated a degree of accuracy (physical authen-ticity) of the results obtained.

2. ESTIMATION METHOD OF PHYSICAL AUTHENTICITY OF PLASTICITY PROBLEM'S SOLUTIONS

In an assumption where the body under consideration is plastically incompressible, a hydrostatic pressure can be considered as an exterior parameter. In this case, when the constraint between stresses and strains is investigated, it seems natural to turn to deviators. As among the six components of deviators only five are independent (first invariant of strain and stress deviators are zero), it is reasonable to introduce five-dimensional spaces of stress and strain vectors.

In this work we shall consider a case of plane stress. In the Ilyushin's vector space the latter are corresponded to three-dimensional loading processes. Since the strain paths in many cases have a small second curvature, it is possible to use a procedure of an approximate flattening (Babamuratov, 1989) of the appropriate three-dimensional trajectories. Consequently, the two-dimensional problem can be reduced to consideration of stress-strained state of a thin cylindrical shell at any combination of a pair of three actions: torsion, tension and internal pressure. The Isotropy Postulate gives us an opportunity to reduce a variety of two-dimensional processes to consideration of this problem.

We shall present an algorithm of the method of estimation of physical authentic-ity in conformity with the problem of homogenous stressstrained state of a thin cylindrical tube (Lensky, 1978), an equation of which state being shown in the form of

$$d\bar{\sigma} = N\, d\bar{Э} - (N - P)\frac{\bar{\sigma} d\bar{Э}}{\sigma^2}\bar{\sigma} \qquad (1)$$

$$d\bar{Э} = \frac{1}{N} d\bar{\sigma} - \frac{N - P}{NP} \cdot \frac{\bar{\sigma} d\bar{\sigma}}{\sigma^2}\bar{\sigma}\,, \qquad (2)$$

where N and P are preassigned coefficients which are calculated depending on the chosen plasticity theory version (Babamuratov et al. 1987), $\bar{\sigma}, \bar{Э}$ –stress and strain vectors, at that $|\bar{\sigma}| = \sigma, |\bar{Э}| = Э$

In case of the two-dimensional version, in the experiments under complex load-ing (P-M_k experiments, here P is tensile force, M_k is torque), the stress vector

$$\bar{\sigma} = \bar{\sigma}_1\, \bar{e}_1 + \bar{\sigma}_3\, \bar{e}_3, \sigma = \sqrt{\sigma_3^2 + \sigma_1^2}\,, \qquad (3)$$

and similarly the strain vector:

$$\bar{Э} = \bar{Э}_1\, \bar{e}_1 + \bar{Э}_3\, \bar{e}_3,\ Э = \sqrt{Э_3^2 + Э_1^2}\,, \qquad (4)$$

are expressed in terms of tensile forces and a torque

$$\sigma_{11} = P / F,\ \sigma_{12} = M_k / RF\ (F = 2\,\pi Rh)$$

and respective strains

$$Э_{11} = \Delta l / l,\ 2\varepsilon_{12} = R\varphi / l,$$

where F is a cross-section area of the tube, R is its radius and h – thickness; Δl is exten-sion of the tube, φ is a angle of twist at the section under consideration l.

In the experiments with the use of the CL-set the program of loading includes the strain trajectories (4) (for the machines of kinematics type), or the stress trajectories (3) (for the machines of dynamics type) in the form of time function or in the form of some λ parameter. The components of both stress vector (3) and strain vector (4) are expressed in terms of stresses σ_{11}, σ_{12} and strains $Э_{11}$, $Э_{12}$ by the formulas as:

$$\bar{\sigma}_1 = \sqrt{2/3}\,\bar{\sigma}_{11} = \sqrt{2/3}\,\bar{P} / F;\ \bar{\sigma}_3 = \sqrt{2}\,\bar{\sigma}_{12} = \sqrt{2}\,\bar{M}_k / PF;$$

$$|\bar{\sigma}| = \sqrt{\sigma_1^2 + \sigma_3^2} = \sqrt{2/3}\,\bar{\sigma}_i \qquad (5)$$

$$\bar{\ni}_1 = \sqrt{3/2}\,\bar{\varepsilon}_{11} = \sqrt{3/2}\,\Delta l/l;\ \bar{\ni}_3 = \sqrt{2}\,\bar{\varepsilon}_{12} = R\varphi/1\sqrt{2};$$

$$|\bar{\ni}| = \sqrt{\ni_1^2 + \ni_3^2} = \sqrt{3/2}\,\bar{\varepsilon}_i \qquad \textbf{(6)}$$

Let us, for example, assign the tube loading (Figure 1) law in the form of proportions

$$P(\lambda) = \begin{cases} \neq 0, \text{ increased to } P_0 \text{ value (at } \lambda \leq \lambda_0) \\ = P_0 \quad \text{at} \quad \lambda \geq \lambda_0 \end{cases}$$

$$M_k(\lambda) = \begin{cases} = 0, \quad \text{at} \quad \lambda \leq \lambda_0 \\ 0, \text{ increased at } \lambda > \lambda_0 \text{ to } M_k^0 \text{ value} \end{cases}$$

The stress vector in this case represents the known function of parameter

$$\sigma_1^{(0)} = \begin{cases} \sigma_1^{(0)}\lambda, \text{ at } 0 \leq \lambda \leq \lambda_0 \\ \sigma_1^{(0)}\lambda_0 = \sigma_0, \text{ at } \lambda \geq \lambda_0 \end{cases}, \quad \sigma_3^{(0)} = \begin{cases} 0, \text{ at } 0 \leq \lambda \leq \lambda_0 \\ \sigma_3^{(0)}(\lambda - \lambda_0), \text{ at } \lambda \geq \lambda_0 \end{cases}, \qquad \textbf{(7)}$$

If, for instance, the material is to be brought into a plastic state in the absence of the torsion moment (σ_3=0), then we shall define the initial conditions of the problem as follows:

$$\sigma_{1s}^{(0)} = \sqrt{2/3}\,\sigma_s,\ \ni_{1s}^{(0)} = \sqrt{3/2}\,\ni_s,\ \sigma_{3s}^{(0)} = \ni_{3s}^{(0)} = 0 \qquad \textbf{(8)}$$

Thus, if the tube loading law was assigned, then by solving the equations system (2) at the initial conditions of the (8) and the assigned N, P, we shall be constructing the trajectory $\bar{\ni} = \bar{\ni}(\lambda)$ which is considered as the program of loading the specimen in the experiments where the CL-set is used. According to this program the stress trajec-tory $\bar{\ni}^*(\lambda)$ is determined on the CL-set, which is then compared with the trajectory (7) by the criterion:

$$2\,|\sigma^* - \sigma^0| / |\sigma^* + \sigma^0| \leq \delta_\sigma \qquad \textbf{(9)}$$

In such a way there is identified a deviation degree of the trajectory, calculated by any of the $\bar{\sigma} \sim \bar{\ni}$ versions, from the experimental one. The technique of setting and conducting the experiments on the CL-set is described in detail in (Babamuratov et al. 1987).

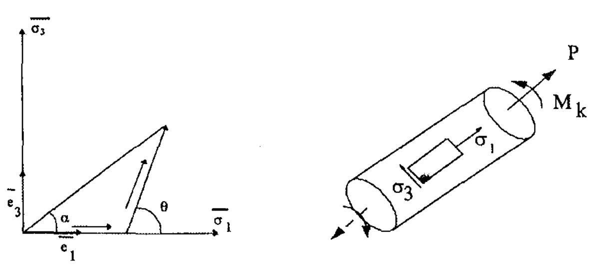


Figure 1. Tube Under Complex Loading.

When a boundary problem is considered, then according to the hypothesis of macroscopic determinability (Ilyushin, 1963) the certain point x_k of the body may be compared with the specimen of finite sizes (the thin cylindrical tube under consideration). While solving the problem the stress $\sigma^0 = \sigma^0(x_k, \lambda)$ and strain trajectories $\bar{\ni} = \bar{\ni}(x_k, \lambda)$ are obtained in every point of the body. Based on the strain trajectories, experiments are organised and the stress trajectories are found. Then the method esti-mation of physical authenticity is essentially reduced to the comparison of $\sigma^0 = \sigma^0(x_k, \lambda)$ and $\bar{\sigma}^* = \bar{\sigma}^*(x_k, \lambda)$ trajectories with each other by the criterion (9). Since the discretization of the body at a computational solving can be great, it is advisable to conduct an ample quantity of experiments. But this can be avoided, if charac-teristic trajectories are selected out of all the variety of processes and the experiments are carried out only with these trajectories.

Sometimes it seems reasonable to carry out strain experiments in place of stress experiments. In this case the CL machine of dynamics type is used to construct the ex-perimental strain trajectory. As for the rest, the algorithm is the same as described above.

Analysis of physical authenticity of the solution of plasticity problems can be carried out in the numerical experiments as well. In that case the CL-set experiments are replaced by solving the equations (1) or (2) type, parameters N and P being specifically chosen. This is the case, when the physical authenticity of these relations against the processes (trajectories) under consideration has been determined beforehand.

In order to identify the area of physical authenticity of the theory, let us consider the constitutive relations for the isothermal quasi static processes in the form of bi-linear streak lines. In the differential form these equations look like (Shevchenko et al. 1982) as

$$d\bar{\sigma} = N\,d\bar{\ni} - (N - P)\,\bar{\ni}_k\,d\bar{\ni}_k / \ni_k^2 \qquad \textbf{(10)}$$

here $N = N_\sigma \sin(\theta - \vartheta)/\sin\theta - |\bar{\sigma}|(\cos(\theta - \vartheta)/\sin\theta)\,N_\vartheta$;

$P = N_\sigma \cos(\theta - \vartheta)/\cos\theta + |\bar{\sigma}|(\sin(\theta - \vartheta)/\cos\theta)\,N_\vartheta$;

$N_\sigma = \partial|\bar{\sigma}|/ds,\ N_\sigma = \partial\vartheta/\partial s,$

$\ni_k$ is vector of strain in bend-point; θ is angle of bend-point, ϑ is approach angle of vectors $\bar{\sigma}$ and $d\bar{\sigma}$, s - length of the strain trajectory.

Based on the experiments of simple loading (pure shearing) and on the experi-ments on the trajectories with the bending angle θ=90^0 (Babamuratov et al. 1988), we shall construct the stress-strain diagrams.

In the first case we shall obtain

$$|\tilde{\sigma}|\,\Phi(\tilde{\text{э}}) = \begin{cases} \tilde{\text{э}}, & \text{at } s \le 1 \\ a\text{э}^2 + b\tilde{\text{э}} + c, & \text{at } 1 \le s \le 3.84, \\ k\tilde{\text{э}} + d, & \text{at } s \ge 3.84 \end{cases} \qquad (11)$$

here a = -0,0052929; k = 0,030286; b = 0,0432235; d = 0,9337; c = 0,9620694.

In the second case we shall obtain

$$|\tilde{\sigma}|\,\Phi(\tilde{\text{э}}) = \begin{cases} \tilde{\text{э}}, & \text{at } s \le 1 \\ a\text{э}^2 + b\tilde{\text{э}} + c, & \text{at } 1 \le s \le 4.6 \\ a_1\tilde{\text{э}} + b_1, & \text{at } 4.6 \le s \le 7.39 \\ a_2\tilde{\text{э}}^2 + b_2\tilde{\text{э}} + c_1, & \text{at } 7.39 \le s \le 11.05 \\ a_3\tilde{\text{э}} + b_3, & \text{at } s \ge 11.05 \end{cases} , \qquad (12)$$

here a = -0,01389; a_1 = 0,3571; a_2 = -0,05368; a_3 = 0,02193; b = 0,1277; b_1 = -0,462; b_2 = 0,6124; b_3 = 0,9684; c = 0,8861; c_1 = -0,6418.

The relations (11) and (12) are given in dimensionless quantities, where $\tilde{\text{э}} = \text{э} / \text{э}_s$ and $\tilde{\sigma} = \tilde{\sigma} / \sigma_s$, and have been constructed on the basis of the experiments with the specimens produced from brass LS-59. The Table 1 shows the data character-ising the relative deviations of the results of the equations' (10) solutions at the initial conditions as $\tilde{\sigma}_1 = 0$, $\tilde{\sigma}_3 = 1$ at $\tilde{\text{э}}_3 = 1$ from the experimental data under complex loading (material: brass LS-59, E=112815 MPa, G=44145 MPa, σ_s = 138 MPa, F= 79E-6 m^2 is the cross-section area of the cylindrical specimen). The complex loading processes were being realised in the form of the bi-linear streak line with the bend-angle $\Theta = 155^0$.

From Table 1 it follows that the calculations performed with the use of (10), (12) equations are adequately describing the process under consideration with the exception of the small neighbourhood of the bend-point (δ_ϑ >20% at s=7,36). At the same time, the use of the curve (11) in the calculations results in the deviations δ_ϑ>15%. As it follows from Table 1 the relations (10), (12) can be used in place of the experiments on the CL-set in the analysis of physical authenticity of the elementary versions of plasticity for bilinear processes when the materials being hardened are concerned.

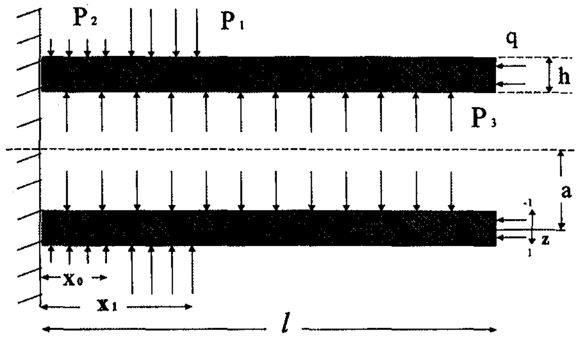
3. ANALYSIS OF AUTHENTICITY OF THE BOUNDARY PLASTICITY PROBLEMS' SOLUTIONS.

The most boundary problems of plasticity are normally solved on the basis of the classical theory of plasticity, which, generally speaking, give acceptable results only for the processes close to a simple loading. However, in case that the body has stress concentrators and a multiparametric loading is applied, the issue of accuracy of the problem solution remains open. This conclusion is confirmed by the results obtained from the problem solutions based on various theories of plasticity.

The assessment of physical authenticity was first used in solving a boundary problem of a combined multiparametric loading of the final cylindrical shell (Babamuratov et al. 1987) rigidly fixed at the left end (Figure 2).

Table 1. Comparison of Experimental and Theoretical Results.

№	The given Data		Experimental facts		Deviations of calculated results by (10) from experimental data (in %)			
					Calculations by (10), (11)		Calculations by (10), (12)	
	$\tilde{s}$	$\tilde{\text{э}}$	$\tilde{\sigma}$	$\cos\vartheta$	δ_σ	δ_ϑ	δ_σ	δ_ϑ
1	7.36	3.9	0.93	0.76	6.9	20.9	5.53	20.2
2	9.66	4.9	1.07	0.95	15.73	1.57	7.68	6.2
3	11.04	5.9	1.10	0.95	15.22	2.21	9.6	1.8
4	12.42	6.9	1.12	0.98	13.95	4.7	9.3	1.1
5	13.8	8.0	1.15	0.99	12.94	5.19	9.2	1.9
6	15.18	9.25	1.17	0.998	11.76	5.51	9.0	2.4
7	16.56	10.6	1.21	0.996	10.86	5.09	9.1	2.4
8	17.48	11.35	1.21	1.00	9.11	5.03	7.9	2.3
9	18.86	12.9	1.22	0.998	7.36	4.49	7.0	2.25
10	20.24	14.0	1.24	0.998	5.46	4.17	6.0	2.13
11	21.62	15.8	1.26	0.998	3.29	3.97	4.75	2.12
12	23.00	17.4	1.27	1.00	1.16	3.81	3.5	2.11



$\tilde{x}_0 = 2.4,\ \tilde{x}_1 = 3.6,\ \tilde{l} = 12$, a=15e-3m, h=1.6e-3m,
$\tilde{P}_1 = 11,\ \tilde{P}_2 = 9.6,\ \tilde{P}_3 = 1.2,\ \tilde{q} = 1.2$

Figure 2. Shell Under External Forces.

The sequence of loading the shell is as follows: at first the external circular pressure P_1 is being applied until a plastic strains occurs; then at fixed P_1, the forces P_2, P_3, q are applied. Depending on the value of the forces applied to shell and the geometric characteristics it, the form of the flexure and the distribution of plastic deformations of the shell are changed.

The problem was solved by the introduction of non-dimensional parameters:

$$\tilde{q} = q / 2\pi a h \sigma_s,\ \tilde{P}_i = 4aP_i / Gh\varepsilon_s\ (i = 1,2,3),\ \tilde{x} = x\sqrt{3} / \sqrt{2ah},$$
$$\tilde{l} = l\sqrt{3} / \sqrt{2ah}$$

It is determined that for this problem the strain processes (trajectories) in the form of bi-linear streak lines are characteristic. The experimental stress trajectories constructed in accordance with the programs $\bar{\ni} = \bar{\ni}\ (x_k, \lambda)$ in two typical points of the shell were compared with the strain trajectories calculated by the Ilyushin's theory, Prager's theory and that of Kadashevich and Novozhilov. The experiments with the LS-59 brass cylindrical specimens were carried out on the CL-set based on the calculated strain trajectories. It is shown that at the point (x=2; z=0) of the shell, for which the bend-angle of the strain trajectory is θ<pπ2, the values of components of the strain vectors, calculated according to the Ilyushin's theory and the Kadashevich and Novozhilov's theory, were proved to be closest to the experimental results. In the point (x=2,2; z=1) (q^{a}p§2), the results close to those of experimental have been obtained by the Kadashevich and Novozhilov's theory. The deviation of the experimental values from the theoretical one constituted as follows: by the Prager's theory – 37 %, the Ilyushin's theory – 47 %, and the Kadashevich and Novozhilov's theory – 22 %.

In the problem of torsion of an elastic-plastic rod having the rectangular section, as shown in the calculations, there are taking place deformation processes in the form of mean curvature trajectories (complex loading), and the difference between the solutions of this problem (Babamuratov et al. 1992), obtained on the basis of the Prager's theory, the perfect plasticity theory and the Ilyushin's theory reaches as much as 200%. The version of the Prager's theory which takes into consideration the strain hardening effect, gives the solution nearest to the experimental results (the deviation of the theoretical values from the experimental ones is not more than 30–40 %).

Similar conclusions (Babamuratov et al. 1998) were made regarding the problem for a rectangular plate with a cut (stress concentrator) which was under tensile force, the plate being rigidly fixed on both sides, and the problem of combined three-parametric loading of the shell of revolution. In solving the problems of this kind, as seen from the calculations, the neglect of the complexity of loading process will bring considerable errors.

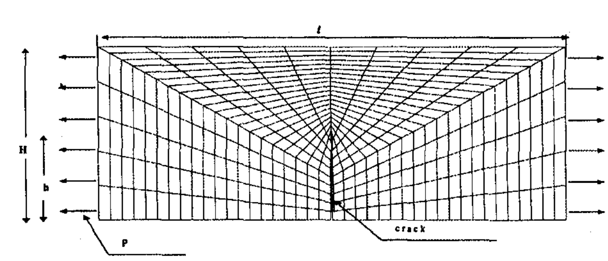
Let us consider the problem of the specimen with one boundary rupture (Babamuratov et al. 1998), being subject to tensiling forces (Figure 3), the load being evenly distributed. The diagram of the material is in the form of:

$$\sigma = \frac{E \cdot \sigma_s\ \varepsilon + C}{\sigma_s + E \cdot C}$$, where C = 0.2333, E = 205800 MPa, ss = 480.2 MPa.

The calculation was made based on the theory of Prandtl and Reiss (incremental theory of plasticity) and on the theory of Ilyushin.

On the bases of the problem solution there are shown isolines of stress intensity (Figure 4, 5) and presented the strain trajectory projections onto different planes (Figure 6).

As seen from the calculations the strain trajectories proved to be complicated (mean curvature), and therefore the solutions derived on the basis of the various



P=13680 N, l=2m, H=0.8m, h=0.4m

Figure 3. Specimen Calculation Scheme.

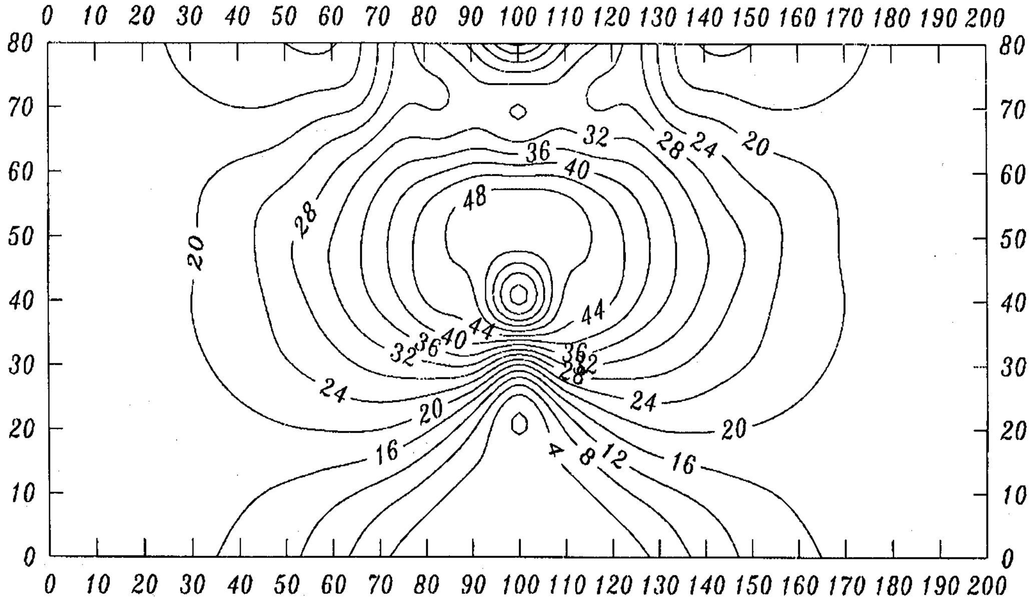


Figure 4. Isolines of Stresses Intensity (Incremental Theory of Plasticity).

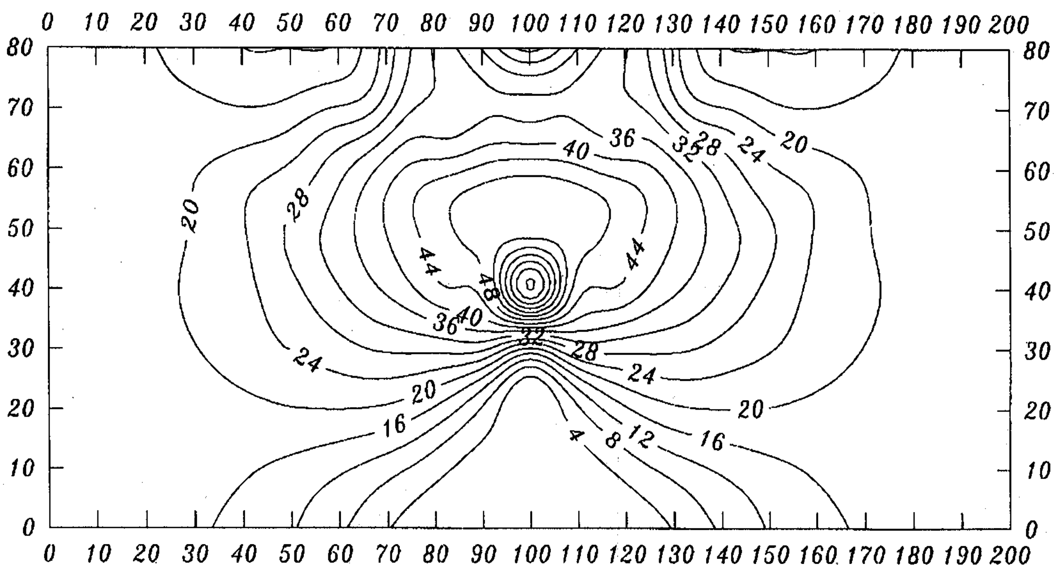
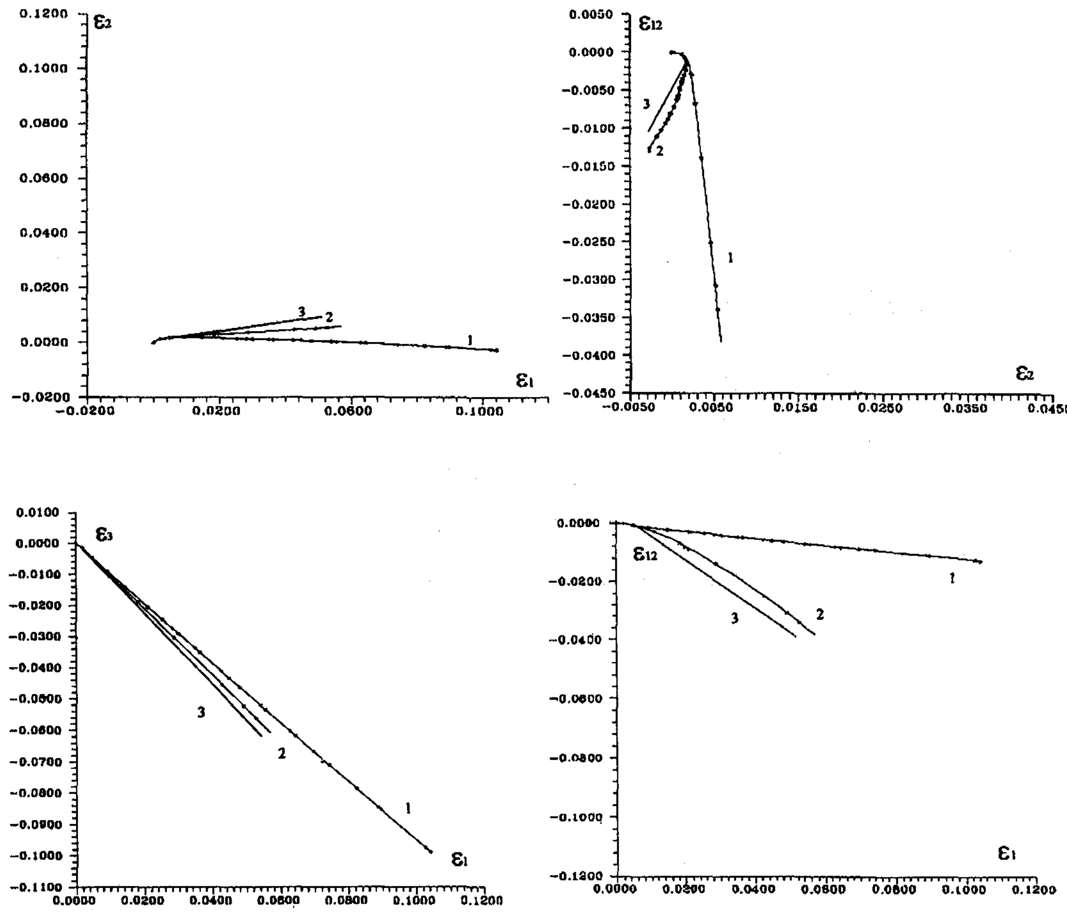


Figure 5. Isolines of Stresses Intensity (Ilyushin's Theory).

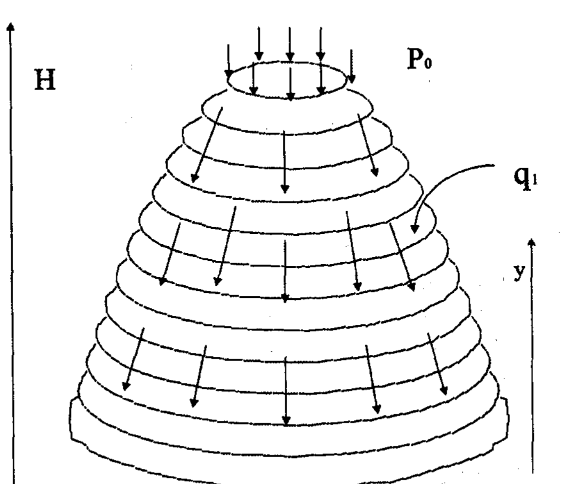
theories require the assessment of physical authenticity their. The stress intensity isolines (Figure 4, 5) show that the qualitative picture of behaviour of the scalar properties is preserved, when calculated by various theories, but for all that the solution based on the strain theory gives excessive strain values throughout the surface of the plate. This is reflected in the strain values calculated by the strain theory, which also have excessive values (up to 30%) compared with the results obtained according to the incremental theory of plasticity. In terms of the vector properties this difference amounts to 60%.

Since the Prager's theory (Babamuratov et al. 1992) describes the mean curvature trajectories better than the other theories do, we shall use the calculations of this theory (i.e. carry out the numerical experiment) to have the obtained results compared with those derived from calculations by the theories of Prandtl and Ilyushin. Referring to Figure 6, there are strain trajectories in various plains and, as one can see, the calcula-tions made on the basis of the Prager's theory are close to the results obtained from the calculations by the Prandtl and Reiss's theory (the deviation is not more than 5%).



1- Ilyushin's theory, 2- theory of Prandtl, 3- theory of Prager

Figure 6. Trajectories of Deformations.

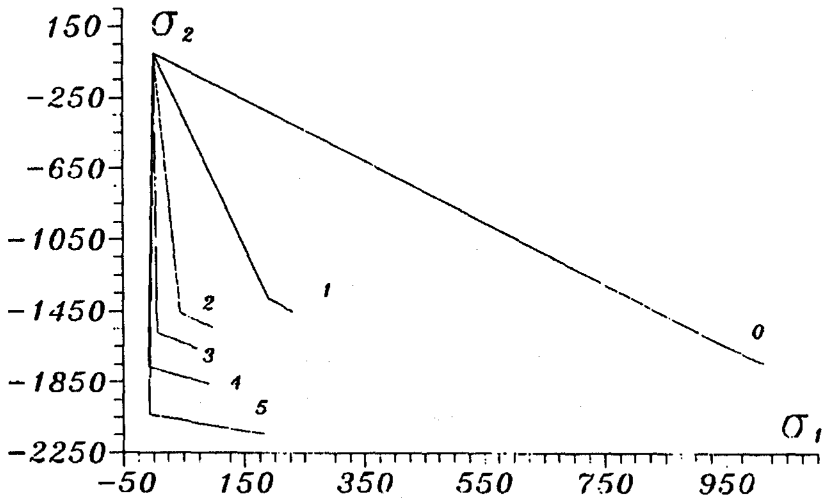


P_0 - external force q_1-distributed surface efforts, q_n-internal pressure
H=0.514 m; P_0=980 N; q_1=1.96 MPa; q_n=4.116 MPa.

Figure 7. Shell of Revolution Under External and Internal Forces.

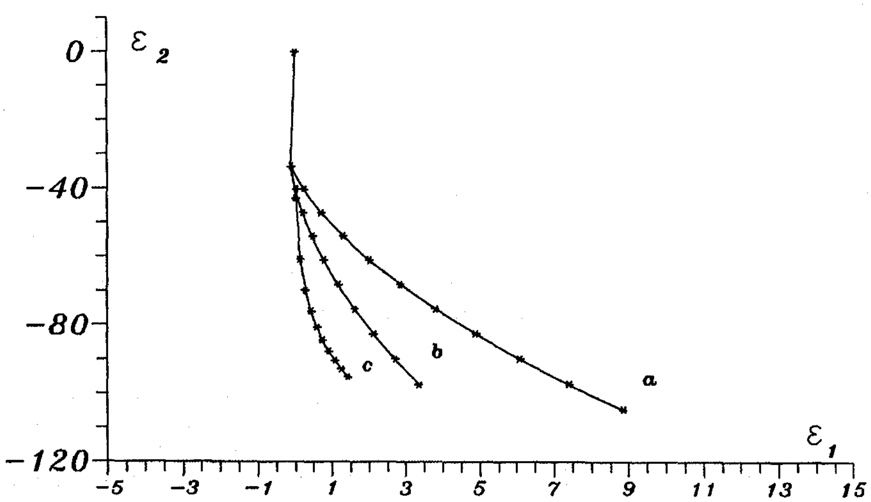
As a next illustration, let us consider the problem of combined loading of the shell of revolution being under the effect of the surface load q_1, the compressive force P_0, and the external normal pressure q_n (Figure 7). We shall be loading the shell by step-by-step force application. First we are applying the external normal pressure q_n until the first plastic deformations occur. Then, while increasing q_n, we are applying P_0 and q_1. When such a loading takes place, at each section of the shell there complex processes of loading occur in the form of bi-linear streak lines (Figure 8). As seen in the diagram, the bend value of the load trajectory trends to increase towards the base.

Then we shall consider the solutions of this problem by using the Prager's and Ilyushin's theories, and, in order to perform an accurate solution, we shall take the relations of the modified theory of bi-linear streak lines (Abirov, 1997), where the coefficients of plasticity are designed taking into account the values of the experiments on specimens under complex loading. The Ilyushin's and Prager's theories give the increased values of deformation (Figure 9), the deviations increasing as the applied load is being increased.



[0]-y=0.514m; [1]-y=0.498m; [2]-y=0.474m; [3]-y=0.354m; [4]-y=0.231m; [5]-y=0m

Figure 8. Trajectories of Stresses In Different Sections of Shell.



a – Ilyushin's theory, b - theory of Prager, c - modified bi-linear streak lines theory

Figure 9. Trajectories of Strains.

4. CONCLUSION

From the above-mentioned examples it is clear that the issue of physical authenticity of the solutions of the plasticity problems remains open. Particularly, when a construction element has stress concentrators and when a multi-parameter loading takes place. The purpose of this note was to draw attention to this problem.

One of the ways of carrying out analyses is the estimation of physical authenticity of the solutions of plasticity problems presented in this work. Whatever the case, in order to carry out such analyses a test unit of CL-set type is required.

And what is more, the CL-set based experiments on the homogeneous cylindrical specimens will allow to construct new and more authentic models of the constraints between stresses and strains. And this in turn will open up new possibilities for solving plasticity problems more authentically.

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