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RESEARCH ARTICLE/**ARAŞTIRMA MAKALESİ**

LATENT PROCESS IN A POISSON REGRESSION MODEL

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ABSTRACT

In this study, time series of Poisson count model was investigated. In real situations, mean-variance equality, which is the basic property of Poisson data, cannot be provided. Generally, in such data variance exceeds mean, this is called overdispersion. When the overdisperison is detected, then there may be autocorrelation in latent process for Poisson regression model.

Correlation is assumed to result from a latent process which is added to the linear predictor in a Poisson regression model. A quasi-likelihood approach is used as a parameter estimation technique. Tests for the presence of the latent process and autocorrelation of the latent process are examined. Asymptotic properties of the regression coefficients are investigated by using a simulation study.

As an illustration, monthly number of deathes who were infected by pulmonary tuberculosis for the years 1996 to 2002 in Izmir are investigated as a parameter-driven model and the asymptotic properties of the regression coefficients are investigated, then a suitable model is constructed for forecasting.

Keywords: Quasi-Likelihood method, Latent process, Poisson regression, Overdispersion, Autocorrelation.

POISSIN REGRESYON MODELİNDE GİZLİ SÜREÇ

ÖZ

Bu çalışmada, Poisson sayımlarının zaman serisi modeli incelendi. Poisson dağılan bir verinin temel özelliklerinden olan ortalama-varyans eşitliği uygulamada sağlanamaz. Genellikle, aşırı yayılım olarak adlandırılan varyans değerinin ortalamayı aştığı durum söz konusu olur. Aşırı yayılımın söz konusu olduğu durumda, Poisson regresyon modeli için gizli süreçte otokorelasyonun varlığından söz edilebilir.

Poisson regresyon modelinde doğrusal tahmin ediciye eklenen gizli süreçten kaynaklanan bir korelasyon durumunun olduğu varsayılır. Bu durumda yarı-olabilirlilik yöntemi, parametre tahmin yöntemi olarak kullanılabilir. Gizli süreçin varlığının testi ve gizli sürecin otokorelasyon yapısı bu çalışmada incelenmiştir. Aynı zamanda regresyon katsayılarının asimptotik özellikleri yapılan simulasyon çalışmasıyla araştırılmıştır.

Yapılan uygulamada, 1996'dan 2002'ye kadar İzmir'deki akciğer tüberkulozu ölümleri aylık olarak incelenmiştir. Yapılan çözümleme sonucunda katsayıların asimptotik özellikleri saptanarak uygun bir model oluşturulmaya çalışılmıştır.

Anahtar Kelimeler : Yarı-Olabilirlilik yöntemi, Gizli süreç, Poisson regresyon, Aşırı yayılım, Otokorelasyon

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1. INTRODUCTION

In the last recent years, there has been a great interest in the analysis of time series of counts. In order to get satisfactory modeling in the integer-valued characteristic of the data, time series of count analysis methods are used. The Poisson Regression Model is the basic member in such count data models. Although Poisson Regression Model has the charecteristic property that the expected values and the variances are equal, generally the variance exceeds the mean in count data in real situations. This is overdispersion in the Poisson Model, so with some empirical support, there may be autocorrelation in such data.

There are many techniques in order to identify a suitable model for the correlation structure in the 'latent' process. To decide including a latent process in the specification of the mean of the Poisson counts, there is a need for diagnostic techniques in linear regression with correlated errors. It is needed to know modeling of time series of counts.

The model

$$y_t = x_t^T \beta + \varepsilon_t \quad \textbf{(1.1)}$$

is a linear model with time series errors while Y_t's are continuous variable. While working with such series, first of all autocovariance structure of the time series errors ε_t is determined. $(\beta_1,...,\beta_n)$ parameters are estimated by regressing the data vector $(Y_1,...,Y_n)'$ onto the $(x_1,...,x_n)$ using Ordinary Least Squares (OLS) Method. These estimates ignore the dependence structure of the ε_t. In this condition, these OLS estimate has the same asymptotic efficiency as the Maximum Likelihood estimate. The asymptotic covariance matrix of the OLS and ML estimate depends on the covariance structure of the ε_t. If β is the consistent estimator, then Autocovariance Function (ACVF) of ε_t can be consistently estimated from the sample ACVF of the residuals which is defined as $\hat{\varepsilon}_t = y_t - x_t'\hat{\beta}$.

2. TIME SERIES OF COUNT DATA REGRESSION MODEL

It is a little different from the time series of count data model. Let the outcomes $\{Y_t : t = 1,...,n\}$ are time series of counts. Log-linear models can be used to describe $\mu_t = E(Y_t) = e^{X_t'\beta}$ as a function of a p x 1 vector of covarites x_t with independent observations. Mean function is specified by a linear predictor modified by a 'latent' process.

If Y_t is Poisson, likelihood methods can be used to estimate β in the case $\mathrm{var}(Y_t) = \mu_t$. Quasi-likelihood methods which allow a variety of variance-mean relations are appropriate in the case of $\mathrm{var}(Y_t) > \mu_t$. The two common assumptions are

i) $\mathrm{var}(Y_t) = \mu_t \varphi$

ii) $\mathrm{var}(Y_t) = \mu_t + \mu_t^2 \sigma^2$

where φ and σ^2 are unknown scale parameters. (Zeger, 1988)

If there is no overdispersion, then there is no serial correlation and there will be no need for correcting the covariance matrix estimator of the Poisson. (Brannas ve Johansson, 1994) In time series data, neighbouring observations are dependent. Two classes of models time-dependent data are characterized by Cox (1981):

i) Observation-driven : The conditional distribution of Y_t is specified as a function of past observations, $Y_{t-1},..., Y_1$. As an example autoregressive models for Markov chains for discrete data. Assume that $Y_t | \mu_t$ is Poisson, then the model

$$\log \mu_t = x_t'\beta + \alpha_t , \quad \textbf{(2.1)}$$

where α_t is a function of past observations Y_s, $s < t$ i.e. $\alpha_t = \gamma_1 Y_{t-1} + ... + \gamma_p Y_{t-p}$

ii) Parameter-driven : A latent process is thought as generating autocorrelation in these kind of models. Let $\theta_t = \log \mu_t$ be the canonical parameter for the log-linear model. Here, θ_t is assumed to depend on an unobservable noise process (ε_t), so $\theta_t = \theta(\varepsilon_t, Y_{t-1},..., Y_1)$. If Y_t given ε_t is Poisson, then it is

$$E(Y_t | \varepsilon_t) = \exp(x_t'\beta)\varepsilon_t \quad \textbf{(2.2)}$$

The latent process, ε_t, introduces both overdispersion and autocorrelation in y_t. (Zeger, 1988) The *parameter-driven models* has the advantage of incorparating both overdisperison and autocorrelation and the model specifies an unobserved latent process. It is only assumed that ε_t is a non-negative strictly stationary time series with mean 1 and autocovariance function (ACVF)

$$\gamma_\varepsilon(k) = E[(\varepsilon_{t+k} - 1)(\varepsilon_t - 1)] \quad \textbf{(2.3)}$$

Assume that $Y_t | \mu_t$ is Poisson, then the model

$$\log \mu_t = x_t'\beta + \alpha_t ,$$

where α_t is assumed as a stationary Gaussian AR(1) latent process, i.e. $\alpha_t = \rho\alpha_{t-1} + e_t$ where $|\rho| < 1$, e_t are i.i.d. $N(0, \sigma^2)$. As a property of this kind of model,

$E(Y_t) = \exp(x_t'\beta)E(\exp(\alpha_t)) = \exp(x_t'\beta)$, if $E(\exp(\alpha_t)) = 1$

3. THE MODEL

Let the Poisson probability density function is defined as

$$\Pr(y_t) = \frac{\mu_t^{y_t} e^{\mu_t}}{y_t!} \qquad (t=1,...,n) \qquad \textbf{(3.1)}$$

where y_t is the count or frequency variable at time *t*.

Non-negative time series has the form

$$y_t | \varepsilon_t, x_t \sim P\left(e^{X_t'\beta}\varepsilon_t\right)$$

where $\mu_t = e^{X_t^T\beta}$ is the mean function and $\beta = (\beta_1, K, \beta_p)^T$ is p x 1 parameter vector, x_t is explanotory variable vector.

Conditional on ε_t is assumed as stationary latent process. The marginal moments of y_t is determined as a function of the log-linear coefficients and the parameters of ε_t since y_t follows a log-linear model. (Zeger,1988)

To introduce both overdispersion and autocorrelation, conditional on a latent process ε_t, y_t is a sequence of independent counts with properties

$$E(\mu_t|\varepsilon_t) = \varepsilon_t\mu_t = \exp(x_t'\beta)\varepsilon_t$$
$$\mathrm{var}(Y_t|\varepsilon_t) = \varepsilon_t\mu_t = \exp(x_t'\beta)\varepsilon_t \qquad \textbf{(3.2)}$$

Suppose that the ε_t is a stationary process with $E(\varepsilon_t) = 1$ and $\mathrm{Cov}(\varepsilon_t, \varepsilon_{t+k}) = \sigma^2\rho_\varepsilon(k)$ where σ^2 is the variance and $\rho_\varepsilon(k)$ the autocorrelation function at lag k of the ε_t process. (Brannas and Johansson, 1994) Then,

$$E(Y_t) = \mu_t = \exp(x_t'\beta), \mathrm{Var}(Y_t) = \mu_t + \sigma^2\mu_t^2 \qquad \textbf{(3.3)}$$

and the autocorrelation of y_t is given by

$$\rho_y(t,k) = \mathrm{corr}(Y_t, Y_{t+k}) = \frac{\rho_\varepsilon(k)}{\left[\{1+(\sigma^2\mu_t)^{1}\}\{1+(\sigma^2\mu_{t+k})^{1}\}\right]^{1/2}}$$
$$k \neq 0 \qquad \textbf{(3.4)}$$

Autocorrelation function $\rho_y(t,k)$ varies with t and k. The autocorrelation in y_t must be less than or equal to ε_t. The degree of autocorrelation in y_t relative to ε_t decreases as μ_t and σ^2 decrease.

A consistent estimation procedure for the regression coefficient vector is needed since the existence of latent process should be tested and its correlation structure should be identified. An easy way to compute this type of estimates is obtained from performing the generalized linear model (GLM) analysis Quasi-Likelihood (QL) analysis.

4. ESTIMATION

4.1 Estimation of β

There are at least two options for the estimation of the regression parameters. The Poisson Maximum Likelihood estimator is consistent even if the autocorrelation is not accounted for. In such cases Quasi-Likelihood estimator is used. The conventional covariance matrix of ML estimator is inconsistent. (Brannas and Johansson, 1994)

Zeger (1988) used the quasi-likelihood method in estimating such equation which is a time series of independent counts. If the likelihood function is in complex form, with the help of mean-variance relations, quasi-likelihood approach that is based on only the first and the second moments of the distributions is suggested. The only assumptions on the distribution of the data are first and second moments and some additional regularity conditions relating to the regression equation $E(Y) = \mu = \mu(\beta)$.

Log quasi-likelihood considered as a function of mean μ_t and its variance Var(μ_t) and y, the vector of random variables:

$$Q(\mu_t; y_t) = du\int_{y_t}^{\mu_t} \frac{y_t - u_t}{Var(u_t)}_t + f(y_t) \qquad \textbf{(4.1)}$$

where since each μ_t is a known function of β_j parameters, Var(μ_t)is also a known function.

The sum of this function gives the log quasi-likelihood function such as

$$Q(\mu; y) = \sum_{t=1}^{n} Q(\mu_t; y_t) \qquad \textbf{(4.2)}$$

Log-likelihood function and log quasi-likelihood function have the similar properties and β parameters are asymptotically normal. (McCullagh, 1983). For single-parameter exponential family distributions, Log-likelihood function and log quasi-likelihood function are the same, so it is valid for single-parameter distribution Poisson. But in real data, variance-mean equality assumption is violated, generally variance exceeds the mean, so it causes overdispersion. Although this excess variation has little effect on parameter estimates, standard errors, tests and confidence intervals may be wrong unless it is appropriately taken into account. (Dean,1992) The variance in this case is considered as

$$\mathrm{Var}(Y_t) = \varphi \mathrm{Var}(\mu_t) = \varphi\mu_t \quad \textbf{(4.3)}$$

where φ is an unknown dispersion parameter

$$\phi = \frac{1}{N - p}\sum_{t=1}^{n}\frac{(y_t - \hat{\mu}_t)^2}{Var(\hat{\mu}_t)} \quad \textbf{(4.4)}$$

In the quasi-likelihood approach, variance ($\phi\mu_t$) of the Poisson Regression model is equal to the variance of the Generalized Poisson Regression Model, so the log quasi-likelihood functions of the Poisson Regression and the Generalized Poisson Regression are the same, then the quasi-likelihood estimates for both function will be the same. However, the log-likelihood functions of the Poisson Regression and the Generalized Poisson Regression are different, so the maximum likelihood estimates are also different.

Quasi-likelihood estimates of the parameters can be obtained by taking first derivative of the log quasi-likelihood function according to μ_i and then equalizing to 0. Since μ_t's depend on regression parameters β_j,

$$q_j = \frac{\partial Q(\mu,y)}{\partial \beta_j} = \sum_{t=1}^{n}\left(\frac{\partial \mu_t}{\partial \mu_j}\right)\frac{y_t - \mu_t}{Var(Y_t)} = 0 \quad j=1,..,p \quad \textbf{(4.5)}$$

5. ABOUT THE LATENT PROCESS

Before finding again the parameter estimates, it is needed to test for the existence of a latent process. Once a latent process is detected, then autocorrelation should be tested. While covariance estimates are often biased, the standard estimates of correlation proposed by Zeger are in use.

5.1 Test for a Latent Process

There are tests that are used to detect overdispersion in Poisson distribution. Brannas and Johansson (1994) used the following statistic,

$$S = \frac{\sum_{i=1}^{n}[(y_t - m)^2 - y_t]}{\sqrt{\sum_{t=1}^{n}\hat{m}_t^2}} \quad \textbf{(5.1)}$$

under the hypothesis Lagrange multiplier test of the Poisson distribution against Negative Binomial or more general Katz distribution. Dean and Lawless (1989) improved this test statistic for the small samples

$$S_a = \frac{\sum_{t=1}^{n}\left[(y_t - \hat{\mu}_t)^2 - y_t + \hat{h}_t\hat{\mu}_t\right]}{\sqrt{\left[2\sum_{t=1}^{n}\hat{\mu}_t^2\right]}} \quad \textbf{(5.2)}$$

where h_t is the *t*th diagonal element of the *hat* matrix. "Hat" matrix is $H = \Lambda^{1/2}X(X^T\Lambda X)^{1}X^T\Lambda^{1/2}$, where $\Lambda = \mathrm{diag}(\mu_1,...,\mu_n)$ and $X = (x_1,...,x_n)^T$ is the design matrix. (Fahrmeir &Tutz, 1994) Both of these test statsitic asymptotically distributed as N(0,1) under "there is no latent process" hypothesis.

Davis, et al. (1998) introduced an alternative test specifically designed for overdisperison in the case of latent process in Poisson process. Since this test uses higher moment properties of Poisson observation, it is more powerfull than S_a.
Under the null hypothesis that there is no latent process $(i.e., \varepsilon_t \equiv 1)$ the Pearson residuals,

$$e_t = \frac{y_t - \hat{\mu}_t}{\sqrt{\hat{\mu}_t}} \quad \textbf{(5.3)}$$

have approximately zero mean and unit variance.

$$Q = \frac{\left(\frac{1}{n}\sum_{t=1}^{n}e_t^2 - 1\right)}{\hat{\sigma}_Q} \quad \textbf{(5.4)}$$

where

$$\hat{\sigma}_Q^2 = \frac{1}{n}\left(\frac{1}{n}\sum_{t=1}^{n}\frac{1}{\hat{\mu}_t} + 2\right) \quad \textbf{(5.5)}$$

may be used to test the presence of a latent process. (Davis et al., 1998) Q statistic is distributed as N(0,1) approximately under the hypothesis that the variance of a latent process is zero.

5.2 Estimation of the Autocorrelation Function and Autocovariance Function of the Latent Process

In literature, there are many suggestions on various estimates of the autocovariances. Zeger (1988) obtained the estimation of nuisance paramaters with the help of moments method. In this method, σ^2 is estimated as

$$\hat{\sigma}^2 = \frac{\sum_{t=1}^{n}\left\{(y_t - \hat{\mu}_t)^2 - \hat{\mu}_t\right\}}{\sum_{t=1}^{n}\hat{\mu}_t^2} \quad \textbf{(5.6)}$$

where $\operatorname{var}(Y_t) = \mu_t + \sigma^2 \mu_t^2$.

Similarly, the autocorrelation function of ε_t (latent process) can be estimated by

$$\hat{\rho}_\varepsilon(k) = \hat{\sigma}^{-2} \sum_{t=k+1}^{n} \frac{\{(y_t - \hat{\mu}_t)(y_{t-k} - \mu_{t-k})\}}{\sum_{t=k+1}^{n} \hat{\mu}_t \hat{\mu}_{t-k}} = \frac{\hat{\gamma}_\varepsilon(k)}{\hat{\sigma}^2} \qquad \textbf{(5.7)}$$

where autocovariance function of ε_t (latent process) is given by

$$\hat{\gamma}_\varepsilon(k) = \frac{\sum_{t=\tau+1}^{n} \{(y_t - \hat{\mu}_t)(y_{t-k} - \hat{\mu}_{t-k})\}}{\sum_{t=k+1}^{n} \hat{\mu}_t \hat{\mu}_{t-k}} \quad \text{(Zeger, 1988)} \qquad \textbf{(5.8)}$$

There are several alternative residuals for the Poisson regression model. One of them is Pearson residual given by

$$\varepsilon_{tP} = \frac{(y_t - \hat{\mu})}{\sqrt{\hat{\mu}}} \qquad \textbf{(5.9)}$$

which has an autocorrelation at lag k depends on $\hat{\mu}_t, \hat{\mu}_{t-k}$ as well as on $\hat{\sigma}^2$.

The pattern of the estimated autocorrelations is useful for the identification of basic autoregressive moving average (ARMA) model.

5.3 Testing for Autocorrelation

The aim is to detect the autocorrelation for the analysis; but for this, firstly overdispersion should be determined to examine the autocorrelation. On detecting the overdispersion, we need to see whether this overdispersion is generated by ε_t which has an autocorrelation.

By using any of the estimators, autocorelation coefficients for ε_t can be estimated from the residulas. The test hypothesis is 'There is no autocorrelation (white noise)'. For this Brannas and Johansson (1994) suggested the following test statistics given by

$$Q_{BP} = T \sum_{k=1}^{k} r_k^2 \quad \text{(Box and Pierce (1970))} \qquad \textbf{(5.10)}$$

$$Q_{LB} = T(T+2) \sum_{k=1}^{k} \frac{r_k^2}{T-k} \quad \text{(Ljung and Box (1978))} \qquad \textbf{(5.11)}$$

where r_k is the estimated autocorrelation at lag k.

Davis, Dunsmur and Wang (1998) suggested another test statistic, since there is problem with the correlated Poisson model. This problem is that the variance and covariances have different forms of dependence on the mean function μ_t and there is no single normalization of residuals. These normalization eliminate the dependence from the variance and from the covariance terms required to construct autocorrelations. So, the usual normalization procedure in the Box-Pierce and Ljung-Box portmanteau statistics will be incorrect.

$$H^2 = \sum_{k=1}^{L} \left[\frac{\hat{\gamma}_\varepsilon(k)}{s.e.(\hat{\gamma}_\varepsilon(k))} \right]^2 \qquad \textbf{(5.12)}$$

where L is the maximum lag, is proposed for testing for serial correlation in the mean of the observed count time series. It is analogous to the Box-Jenkin's statitics. Under the hypothesis of independence H^2 will have an approximate χ^2 distribution on L degrees of freedom.

6. ILLUSTRATION

As an illustration, monthly number of deathes who were infected by pulmonary tuberculosis, for the years 1996 to 2002 in İzmir are investigated as a parameter-driven model. These data are reported by Health Directorate of İzmir Administrative Province. Our interest is to determine a long-term decrease in the rate of pulmonary tuberculosis infection.

Since there is seasonality, monthly number of deathes is regressed on a linear trend with sine and cosine pairs at the annual and semi-annual frequencies. Figure1 shows the trend analysis of the response variable.

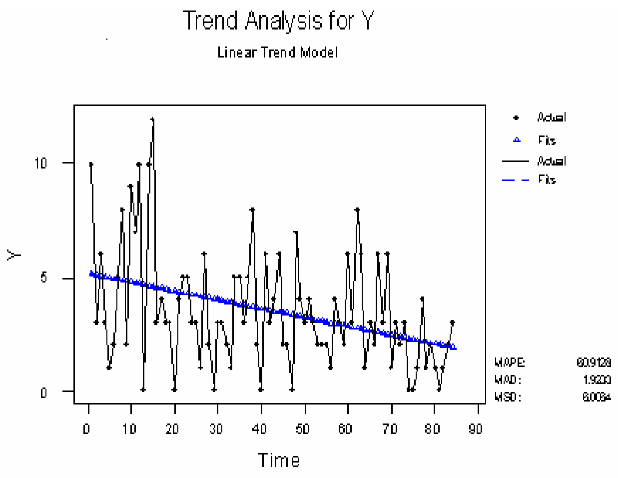

Figure 1 Trend Analysis for Monthly Number of Deathes

Our aim is to identify the β parameters. For the Poisson model

$$f(Y|x,\beta) = \frac{e^{-\mu(x,\beta)} \mu(x,\beta)^y}{y!}$$

where

$$\mu(x,\beta) = E(Y|x,\beta) = \exp(x',\beta)$$

The MLE of the parameter β is obtained by maximizing the log-likelihood function

$$L(\beta) = \sum_{i=1}^{n} y_t \log \mu(x_t,\beta) - \mu(x_t,\beta) - \log(y_t!)$$

On specifiying correct conditional mean function and conditional Poisson distribution of Y, the MLE is consistent, efficient and asymptotically normally distributed, with variance matrix consistently estimated with

$$Var(\hat{\beta}) = \left(\frac{\sum_{i=1}^{n} \frac{\partial \mu(x,\beta)}{\partial \beta} \frac{\partial \mu(x,\beta)}{\partial \beta}}{\mu(x,\beta)} \right)^{-1}$$

In the case of rejection of the mean-variance equality assumption, the model is misspecified. Here the Poisson estimator may also interpreted as a quasi-likelihood estimator (QLE). These QLEs are robust in the sense of producing consistent estimates of the parameters of a correctly specified conditional mean, even whether the distribution is incorrectly specified. For these QL models, only a correct spesification of $\mu(x,\beta)$ is needed for consistency.

Hovewer, the estimated standard errors won't be consistent unless the conditional distribution of Y is correctly specified. But, it is possible to get the robust standard errors in order to make valid inferences even whether the distribution is incorrectly specified by using QL standard errors. But it doesn't posess any efficiency properties.

An intercept term, a linear trend, and harmonics of 6 and 12 months are used as regressors. The design matrix is

$$x_t = \left(1, t'/1000, \cos\left(2\pi t'/12\right), \sin\left(2\pi t'/12\right), \cos\left(2\pi t'/6\right), \sin\left(2\pi t'/6\right)\right)$$

where $t' = t - 37$ is the intercept term at January 1999.

A simulation study was done to investigate these estimates over many trails, and 1000 realizations on the parameter-driven model fitted to the data were generated and this is also repeated 1000 times. On using the correct standard errors for the trend term it is concluded that the trend term is significant. A simulations column of the Table 1 gives the true regression parameters. The latent process in this simulation was assumed to be a LogNormal AR(1) with $\varphi = 0.80$. On comparing the true value of the parameters, it can be concluded that there is no significant bias.

Table1. Coefficients and Standard Errors of Regression Parameters

	QML		**PML**		Asym	**Simulations**	
	Coeff.	Std. Error	Coeff	Std. Error	s.e. (Coeff)	Ave (Coeff)	s.d. (Coeff)
Intercept	1.259704	0.069613	1.259704	0.059526	0.059951	1.258452	0.060632
$Trend x 10^{-3}$	-11.00053	3.050726	-11.00053	2.471986	2.472688	-10.58603	2.513145
$cos(2\pi t/12)$	0.247722	0.096764	0.247722	0.082393	0.082619	0.234196	0.082685
$sin(2\pi t/12)$	-0.087644	0.103449	-0.087644	0.086066	0.086446	-0.074159	0.087014
$cos(2\pi t/6)$	0.082642	0.093502	0.082642	0.083067	0.083324	0.083165	0.086735
$sin(2\pi t/6)$	0.073830	0.10719	0.07383	0.083638	0.083871	0.072462	0.084761

Since φ (Dispersion Parameter = $\frac{Pearson\_\chi^2}{df}$) = 1.525 > 1 then we can conclude that there is overdispersion. When overdipserion is detected then we invsestigate the presence of latent process. With the help of Q statistics we decide that there is latent process.

Since Pearson residuals have the same correlation structure as ε_t (Zeger, 1988), it is assumed that there is lag one autoregressive correlation structure with E(ε_t)=1. The latent process has a lognormal distribution as depicted Table 2.

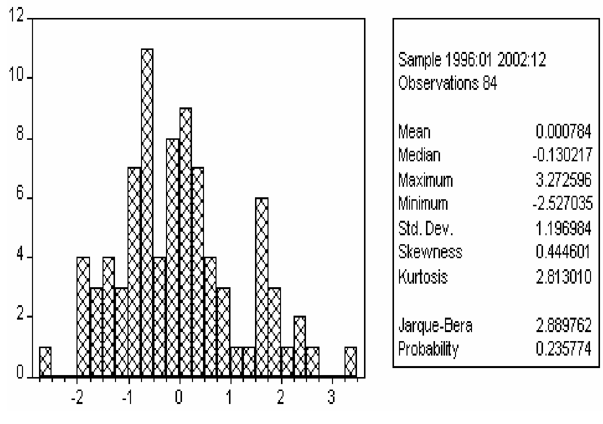

Figure2 Descriptive Statistics of the Pearson Residuals

Table 2 Descriptive Statistics of the Simulation

	β_0	β_1	β_2	β_3	β_4	β_5
Mean	1.258452	-10.58603	0.234196	-0.074159	0.083165	0.072462
Median	1.261703	-10.66275	0.233171	-0.071803	0.086023	0.075050
Maximum	1.448640	-2.913805	0.553063	0.171008	0.351765	0.388515
Minimum	1.026540	-18.69673	-0.024499	-0.350561	-0.219456	-0.201225
Std. Dev.	0.060632	2.513145	0.082685	0.087014	0.086735	0.084761
Skewness	-0.154144	-0.121542	0.017149	-0.136640	-0.102956	-0.049945
Kurtosis	3.132887	2.986629	3.078314	3.053805	3.389450	3.192151
Jarque-Bera	4.695835	2.469523	0.304563	3.232361	8.086281	1.954178
Probability	0.095568	0.290904	0.858747	0.198656	0.017542	0.376405
Observations	1000	1000	1000	1000	1000	1000

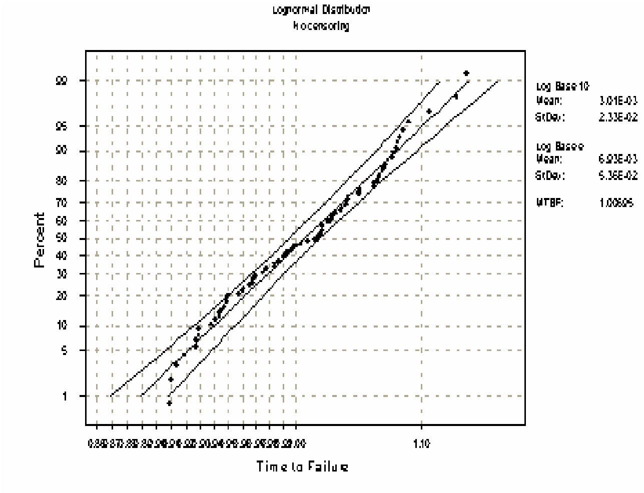

Figure 3 Probability Plot of the Latent Process

The quasi-likelihood estimator is consistent and asymptotically normal. This is a robust approach in making consistent inferences about β only that $E(Y_t) = \mu_t$ whether or not equal to $var(Y_t)$.

Table3 Model with an Intercept Term, a Linear Trend, and Harmonics of 6 and 12 Months

Variable	Coeff.	Std. Error
C	1.259704	0.069613
TREND	-11.00053	3.050726
COS12	0.247722	0.096764
SIN12	-0.087644	0.103449
COS6	0.082642	0.093502
SIN6	0.07383	0.10719
S.E. of regression		2.411857
Log likelihood		-184.4295
LR statistic (5 df)		33.61566
Probability(LR stat)		2.84E-06
Akaike criterion	4.534037	
Schwarz criterion	4.707666	

Table 4 Model with an Intercept Term, a Linear Trend, and Harmonics of 12 Months

Variable	Coeff.	Std. Error
C	1.263258	0.071392
TREND	-11.20847	3.046832
COS12	0.257259	0.102682
SIN12	-0.07705	0.100826
S.E. of regression		2.386169
Log likelihood		-185.3152
LR statistic (3 df)		31.84426
Probability(LR stat)		5.64E-07
Akaike criterion	4.5075	
Schwarz criterion	4.6232	

Table 5 Model with an Intercept Term, a Linear Trend, and Harmonics of 6 Months

Variable	Coeff.	**Std. Error**
C	1.276383	0.074122
TREND	-11.01905	3.184534
COS6	0.100139	0.097493
SIN6	0.069366	0.115191
S.E. of regression		2.493434
Log likelihood		-189.4683
LR statistic (3 df)		23.53815
Probability(LR stat)		3.12E-05
Akaike criterion	4.606388	
Schwarz criterion	4.722141	

Table 6 Model with an intercept term and a linear trend

Variable	Coeff.	Std. Error
C	1.281134	0.075442
TREND	-11.21862	3.206079
S.E. of regression		2.479241
Log likelihood		-190.5601
LR statistic (1 df)		21.35459
Probability(LR stat)		3.82E-06
Akaike criterion	4.5847	
Schwarz criterion	4.6426	

According to the Akaike and Schwarz Criterions, we choose the second model for forecasting. Then the model will be

$$\log Y = 1.263258 - 11.20847\left((t-37)/1000\right) +$$
$$0.257259\left(\cos 2\Pi(t-37)/12\right) -$$
$$0.077050\left(\sin 2\Pi(t-37)/12\right)$$

This model gives very close forecasts as in the first model. The model has the significant trend function.This means that the pulmonary tuberculosis deathes are decreasing by the time.

Even there is latent process with autocorrelation for this Poisson regression model, consistent coefficient estimates and robust variance estimates can be obtained by using Quasi Likelihood method, so valid inferences can be made.

7. CONCLUSION

In order to identify the latent process and its correlation structure, a consistent estimation procedure for the regression coefficient is needed. For consistent estimation procedure, a quasi-likelihood method which is based on only the first and the second moments of the distribution is suggested. In the case of rejection of the mean-variance equality assumption, the model is misspecified. These Quasi maximum likelihood estimates are robust in the sense of producing consistent estimates of the parameters of the correctly specified conditional mean, even whether the distribution is incorrectly specified. For these QML models, only a correct specification of the mean function is needed for consistency.

With simulation study, QMLEs asymptotic behaviour is examined. For this monthly pulmonary tuberculosis data, parameter estimates are consistent, asymptotically normal and asymptotically efficient. And also since the variance estimates of the parameters are robust, we can make valid inferences about this model only that $E(Y_t) = \mu_t$ whether or not equal to $Var(Y_t)$.

REFERENCES

Agresti A. (1984). *Analysis of Ordinal Categorical Data*. Canada, John Wiley & Sons.

Agresti A. (1990). *Categorical Data Analysis.* Canada, John Wiley & Sons.

Bartholomew D. & Knott M. (1999). *Latent Variable Models and Factor Analysis* (2nd ed.). London, Arnold.

Bowerman B. & O'Conell R. (1987). *Time Series Forecasting.* (2nd ed.). Canada, Duxbury Press.

Brannas K. & Johanson P. (1994). Time Series Count Data Regression. *Communication Statistics-Theory and Methods* 23, 2907-2925.

Blais M. & MacGibbon B. & Roy R. (2000). Limit theorems for regression models of time series of counts. *Statistics&Probability Letters 46*, 161-168.

Breslow N.E. (1984). Extra-Poisson Variation in Log-Linear Models. *Appl. Statist.* 33, 38-44.

Breslow N. (1990). Tests of Hypotheses in Overdispersed Poisson Regression and Other Quasi-Likelihood Models. *JASA* 85, 565-571.

Brown R. (1963). *Smoothing Forecasting and Prediction of Discrete Time Series.* USA, Prentice Hall.

Cameron A. C. & Trivedi K. P. (1998). *Regression Analysis of Count Data*. USA, Cambridge University Press.

Campell M.J. (1994). Time Series Regression for Counts: An Investigation into the Relationship between Sudden Infant Death Syndrome and Enviromental Temperature. *JASA* 157, 191-208.

Chan K.S. & Ledolter J. (1995). Monte Carlo EM Estimation for Time Series Models Involving Counts, *JASA* 90, 242-252.

Consul P.C. & Famoye F. (1988). Maximum Likelihood Estimation for the Generalized Poisson distribution hen the sample mean larger than the sample variance. *Commun. Statist.-Theory and Methods* 17*,* 299-309.

Consul P.C. (1988). On some models leading to the generalized Poisson distribution, *Commun. Statist.-Theory and Methods* 17, 423-442.

Consul P.C. & Famoye F. (1992). Generalized Poisson Regression Model. *Commun. Statist.-Theory and Methods* 21, 89-109.

Cox R. (1981). Statistical Analysis of Time Series: Some Recent Developments. *Scandinavian Journal of Statistics* 8, 93-115.

Cox R. (1983). Some Remarks on Overdispersion. *Biometrika 70*, 269-274.

Davis R., Dunsmuir T.M. & Wang Y. (2000). On Autocorrelation In a Poisson Regression Model. *Biometrics 87*, 191-208.

Dean C.B. (1992). Testing for Overdipersion in Poisson and Binomial Regression Models. *JASA* 87, 451-457.

Dobson J. A. (1990). *An Introduction to Generalized Linear Models*. London, Chapman & Hall.

Famoye F. (1993). Restricted Generalized Poisson Model. *Commun. Statist.-Theory and Methods* 22, 1335-1354.

Frome L. E., Kutner M.H. & Beauchamp J.J. (1973). Regression Analysis of Poisson- Distributed Data. *JASA* 68, 935-940 .

Frome E. (1983). The Analysis of Rates Using Poisson Regression Models. *Biometrics* 39

Frome E. & Checkoway H. (1985). Use of Poisson Regression Models in estimating incidence rates and ratios. *American Journal of Epidemiology* 121, 309-323.

Ganio L.M. & Schafer D.W. (1992). Diagnostics for Overdisperion. *JASA* 87, 795-804

Guo J. & Li T. (2002). Poisson Regression Models with errors-in-variables: implication and treatment. *Journal of Statistical Planning and Inference* 104, 391-401.

Haberman J.S. (1978). *Analysis of Qualitative Data* Volume 1. USA: Introductory Topics Academic Press.

Johnson N., Kotz S. & Kemp A. (1992). *Univariate Discrete Distributions* (2nd ed.). Canada, John Wiley & Sons.

Jorgensen B., Christensen S., Song P. & Son L. (1999). A state space model for multivariate longitudinal count data. *Biometrika* 86, 169-181.

Kleinbaum, Kupper, Muller & Nizam. (1998). *Applied Regression Analysis and Other Multivariable Methods* (3th ed.). Canada, Duxbury Press.

Krzanowski J. W. (1998). *An Introduction to Statistical Modelling*, London, Arnold.

McCullagh P. (1983). Quasi-Likelihood Functions. *The Annals of Statistics* 11, 59-67.

Oh M. & Li Y. (2001). Bayesian analysis of Time Series Poisson data. *Journal of Applied Statistics* 28, 259-271.

Özmen İ. (1992). Poisson Regresyon Çözümlemesinde Logaritmik Doğrusal Modellerin Kullanımı ve Uygulaması. *Bilim Uzmanlığı Tezi*, H.Ü.Fen Fakültesi, Ankara.

Özmen İ. (1998). Poisson Regresyon Çözümleme Teknikleri. *Doktora Tezi*, H.Ü. Fen Fakültesi, Ankara.

Ramanathan R. (1998). *Introductory Econometrics with Applications*. USA , The Dryden Press

Tuenter H.J.H. (2000). On the Generalized Poisson distribution. *Statistica Neerlandica* 54, 374-376.

Wang P., Puterman M.L., Cockburn I. & Le N. (1996). Mixed poisson Regression Models with Covariate Dependent Rates. *Biometrics* 52, 381-400.

Wedderburn R.W.M. (1974). Quasi-Likelihood Functions, generalized linear models, and the Gauss-Newton method. *Biometrika* 61, 439-447.

Zeger S.L. (1988). A Regression Model for Time Series of Counts. *Biometrika 75*, 621-629.

Zeger S.L. & Qaqish B. (1988). Markov Regression Models for Time Series: A Quasi-Likelihood approach, *Biometrics* 44, 1019-1031.

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