ANADOLU ÜNİVERSİTESİ

SHORTCUTS TO ADIABATICITY FOR ULTRACOLD GASES

Zalihe ÖZÇAKMAKLI

PhD Dissertation

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ABSTRACT

PhD Dissertation

SHORTCUTS TO ADIABATICITY FOR ULTRACOLD GASES

Zalihe ÖZÇAKMAKLI

Anadolu University
Graduate School of Sciences
Physics Program

Supervisor: Assoc. Prof. Dr. Cem YÜCE
2013, 102 pages

In order to manipulate Bose–Einstein condensates for different applications, it is important to study and control their response to time-dependent changes of the confining fields. A natural approach to avoid undesired excitations is to modify the trap adiabatically, i.e. very slowly, so that, if the initial state is in the ground state the final state will be the ground state as well. However, this may require very long times and become impractical. Thus natural objective is to cut down the time to arrive at the same final state, in other words, to find shortcuts to adiabaticity, by designing optimal adiabatic pathways.

In this thesis, fast frictionless expansion for growing Bose–Einstein condensates described by the Gross–Pitaevskii gain equation is investigated. It is shown that fast frictionless expansion can be generalized to a condensate continuously replenished by pumping from the reservoir. Our formalism is applied to the harmonically trapped condensate and an optical lattice.

Key Words: Bose–Einstein condensates, harmonically trapped condensate, an optical lattice, shortcuts to adiabaticity.

ÖZET

Doktora Tezi

ULTRASOĞUK GAZLARDA ADİYABATİKLİK İÇİN KISAYOLLAR

Zalihe ÖZÇAKMAKLI

Anadolu Üniversitesi
Fen Bilimleri Enstitüsü
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Bose-Einstein yoğuşmasının kontrolünde önemli olan yoğuşmanın uygulanan tuzaklama alanlarının zamana bağlı değişimine olan tepkisini kontrol altına almaktır. İstenmeyen uyarılmalardan kaçınmak için tuzağın adiyabatik olarak yani çok yavaş değiştirilmesi doğal bir yaklaşımdır. Böylece eğer sistem başlangıçta taban durumunda ise son durumu da taban durumu olacaktır. Ancak bu durum çok uzun zaman gerektirebilir ve bu süreç kullanışsız hale gelir. Bu nedenle aynı son duruma ulaşmak için zamanı kısaltmak doğal bir amaçtır, bir başka deyişle, amaç en uygun adiyabatik geçiş yollarını tasarlayarak adiyabatikliğin kısa yolunu bulmaktır.

Bu tez de, Gross-Pitaevskii çoğalma denklemi ile tanımlanan içerdiği parçacık sayısı artan Bose-Einstein yoğuşması için hızlı sürtünmesiz genleşme araştırıldı. Hızlı sürtünmesiz genleşmenin, rezervuardan pompalanan parçacıklarla sürekli artması sağlanan yoğuşmaya genelleştirilebileceği gösterildi. Bu formülasyonu sırasıyla harmonik olarak tuzaklanmış yoğuşma için ve optik örgü için uygulandı.

Anahtar Sözcükler: Bose–Einstein yoğuşması, harmonik olarak tuzaklanmış yoğuşma, optik örgü, adiyabatikliğin kısa yolu

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LIST OF SYMBOLS

H : Hamiltonian

N : Number of particles

N_0 : Initial number of particles

V(x) : Trap potential

m : Atomic mass

h : Planck constant

p : Momentum

R_i : Extension of the cloud

ω : Frequency

B : Magnetic field

μ_i : Magnetic moment

U_0 : Interaction potential

E : Energy

μ : Chemical potential

μ_B : Bohr magneton

d : Electric dipole moment operator

ε : Electric field vector

F_{dipole} : Dipole force

g_F : Land g-factor

$I(t)$: Invariants of motion

a : The s-wave scattering length

$I(r,z,t)$: Intensity profile of a Gaussian laser beam

$\psi(x,t)$: Mean-field amplitude

$|\psi(x,t)|^2$: Particle number density

z_R : Rayleigh range

I_{sat} : Saturation intensity

δ : Detuning

Ω : Rabi frequency

$\hat{J}$: Pseudo-angular momentum operator

T : Temperature

$\gamma(t)$: Linear gain term

$L(t)$: Expansion of the condensate

κ : Ratio of final spacing to the initial spacing

k_L : Optical lattice wave number

1. INTRODUCTION

It is desirable to control the quantum systems in quantum mechanics. A standard operation to control the quantum system consists in changing the external parameters of the Hamiltonian. In many cases the optimal transformations from an initial to a final parameter configuration are the ones that do not induce any transitions [1, 2]. The standard solution to this requirement is to perform the changes “adiabatically”. In quantum mechanics, the evolution of a system described by a time-dependent Hamiltonian $H(t)$ is adiabatic when the transition probabilities between the instantaneous eigenstates of H are negligible. This happens when H is time-independent, or when its rate of change is slow compared to the typical time-scales involved [3, 4].

Even adiabatic techniques are typically slow [5], while experimentalist physicists are often constrained by finite lifetimes or coherence times of their samples. This motivated the investigation for fast schemes reproducing or approaching adiabatic transformations. Some methods use minimization techniques to optimize the transition to a target state [6, 7], in contrast other methods provide the exact same state that would have been reached after an adiabatic transformation [8]. The latter technique called as fast frictionless process or shortcut to adiabaticity and many physicists pay attention to this research [9-31]. Fast frictionless expansion of harmonically trapped ultracold ^{87}Rb atoms was experimentally realized [13, 14]. In the experiment [14], a 3D interacting Bose-Einstein condensate (BEC) confined in an anisotropic harmonic trap is decompressed in a time comparable to the final radial trapping period. The technique is based on engineering the time dependent parameters of the Hamiltonian. Recently, nearly perfect fidelity for a two-level quantum system was achieved experimentally [15]. Choi et al. discussed that optimal cooling of an atomic species may be obtained by means of sympathetic cooling with another species whose trapping frequency is dynamically changed to maintain constancy of the Lewis-Riesenfeld adiabatic invariant [16]. Fast frictionless expansion of an optical lattice was investigated in [17].

Shortcut to adiabaticity offer many hopeful research and application ways with practical and elementary implications. Actually, shortcut to adiabaticity open interesting prospects to improve quantum information and technology operations, by implementing new fast and robust transport or expansion approaches, internal state manipulations, and cooling protocols. Nuclear magnetic resonance is another field where developing ideal pulses may take advantege from shortcut to adiabaticity. In this thesis, an overview of recent work on shorcuts to adiabaticity covering a broad span of methods and physical systems will be presented and shortcut to adiabaticity for growing condensate will be studied.

In this thesis, we will investigate the possibility of fast frictionless expansion for a condensate with variable number of particles. The condensate growth by particle injection was experimentally realized with ^{23}Na [32], ^{7}Li [33] and ^{87}Rb [34]. Several methods have been introduced theoretically to account for the growth of Bose-Einstein condensate [35–40]. We will use Gross-Pitaevskii gain equation to study fast frictionless expansion for both harmonically trapped condensate and an optical lattice. We will show that a correct choice of expansion trajectory allows us to get a final adiabatic state in a non-adiabatic way for the growing condensate.

This thesis is divided into four chapters: the first one contains the background needed to address the physics of Bose-Einstein condensates of neutral atoms, both from the theoretical and experimental point of view. Its first section gives a remainder of basic statistical physics. The stress is put on the results providing experimental tools. The basic equations useful to the description of BECs in various geometries are given. The most important techniques used to produce BECs are presented: laser cooling and radio-frequency evaporative cooling in a magnetic trap.

The second chapter details the study of shortcuts to adiabaticity with both a very dilute normal gas, which can be considered ideal, and a dense interacting Bose-Einstein condensate. This chapter contains both theoretical and experimental information of shortcuts to adiabaticity. Summarizing of the investigations up to now on shortcuts to adiabaticity is also given in this chapter. We overview different methods which used by researchers to obtain shortcuts to adibaticity.

And at the end of this chapter, we will detail experimental procedure of the experimental realization of shortcuts to adibaticity.

Chapter three contains our study of shortcuts to adiabaticity. Differently from the literature we will generalize fast frictionless expansion to growing condensates. Firstly, we will obtain the general formalism and then apply it specifically to a harmonically trapped condensate and an optical lattice. The problem of harmonically trapped condensate is investigated in two cases, first one is studied with Thomas-Fermi approximation and in the second one we will look for the conditions for the fast frictionless process of the condensate without omitting the kinetic energy.

The last chapter is concerned with the results of our study. Interpretation of calculations is given in this chapter.

2. THEORETICAL BASICS

There are two main purposes of this chapter. Firstly, it gives the fundamental theoretical information needed to describe and analyze the objects which we are going to work with: trapped cold gases of neutral atoms. Secondly, it gives a general review of the main experimental techniques used to produce such ultracold gases.

2.1. Bose-Einstein Condensation

Quantum statistics is governed by the principle of indistinguishability of identical particles. The indistinguishability of identical particles, which is the fact that the measurable quantities should not change when the positions of any two particles are swapped, together with the laws of quantum mechanics lead to the striking fact that quantum particles can be divided into two main categories: bosons, for which the many-body wave function is unchanged by the exchange of two particles, and fermions, for which the wave function changes sign. Bosons obey Bose-Einstein statistics in which there is no restriction on the occupation number of any single-particle state. Fermions obey Fermi-Dirac statistics in which not more than one particle can occupy any single-particle state.

The spin-statistics theorem [41] emphasizes that particles with half-integer spins are fermions, whereas particles with an integer spin are bosons.

Shortly, two identical neutral atoms whose nuclei have an even number of neutrons (such that they are bosons) can be put at the same position in space and with the same velocity, pointing in the same direction (same state). On the contrary, neutral atoms with odd number of neutrons obey Pauli's exclusion principle: they can not be in the same state.

Bose-Einstein condensation was first proposed as a theoretical concept in the last century. Albert Einstein was impressed by Bose' s work and extended it to a gas of massive, noninteracting particles [42]. Einstein realized that for sufficiently low tempratures a large fraction of particles would "condense" into the lowest energy state and hence behave all in the same manner. The idea of

Bose-Einstein condensation was born and searchs for this phenomenon began. In 1980s, laser and magnetic based cooling techniques were developed that allowed experimentalists to cool dilute gases of neutral atoms down to extremely low temperatures [43, 44].

In 1995 Bose-Einstein condensation in dilute alkali gases was achieved for the first time in a series of experiments using Rubidium in the group of Eric Cornell and Carl Wieman, Lithium in the group of Randall Hulet and Sodium in the group of Ketterle [45-47].

2.2. Theory of the Bose-Einstein Condensates

In the present subsection we will give general information about the structure of the Bose-Einstein condensed state in the presence of interactions. In the following we will firstly discuss the Gross-Pitaevskii equation [48], which describes the zero-temperature properties of the non-uniform Bose gas when the scattering length a is much less than the mean interparticle spacing. Secondly, the Thomas-Fermi approximation will be examined.

2.2.1. The Gross-Pitaevskii Equation

The equation that controls the actions of all the properties of Bose-Einstein condensates of dilute atomic gases is the interacting many-body Schrödinger equation. The many-body Schrödinger equation is very difficult to solve, even for few particles, and approximations are absolutely necessary.

Definitely the most popular of these approximative methods is the famous Gross-Pitaevskii theory which was developed independently by Gross and Pitaevskii in 1961. The Gross-Pitaevskii equation is derived as a special case of a more general method to solve the many-body Schrödinger equation [49]. It is possible to explore inhomogeneous, interacting condensates in arbitrary trap geometries with using Gross-Pitaevskii theory.

Now let us overview the Gross-Pitaevskii theory. In coordinate space the effective interaction between two particles at low energies corresponds to a

contact interaction $U_0\delta(r-r')$, $U_0 = 4\pi\hbar^2 a/m$, where $\boldsymbol{r}$ and $\boldsymbol{r}'$ are the positions of the two particles [50]. In the fully condensed state, all bosons are in the same single-particle state, $\phi(\boldsymbol{r})$, and therefore we may write the wave function of the *N*-particle system as

$$\Psi(r_1, r_2, ..., r_N) = \prod_{i=1}^{N} \phi(r_i). \tag{2.1}$$

The single-particle wave function $\phi(\boldsymbol{r_i})$ is normalized as below,

$$\int dr \left|\phi(r_i)\right|^2 = 1. \tag{2.2}$$

After all the effective Hamiltonian can be written as

$$H = \sum_{i=1}^{N}\left[\frac{p_i^2}{2m} + V(r_i)\right] + U_0 \sum_{i<j} \delta(r_i - r_j), \tag{2.3}$$

where $\boldsymbol{V}(\boldsymbol{r})$ is the external potential. Hence the energy of the state (2.1) is given by

$$E = N\int dr\left[\frac{\hbar^2}{2m}\left|\nabla\phi(r)\right|^2 + V(r)\left|\phi(r)\right|^2 + \frac{(N-1)}{2}U_0\left|\phi(r)\right|^4\right]. \tag{2.4}$$

Let us explain the terms in the parenthesis, the first term is kinetic energy of the condensate, the second is the trap energy and the third one is the interaction energy.

In the following we will consider a uniform Bose gas. In a uniform system the interaction energy of a pair of particles is U_0/V, where V is the volume of the system. The energy of state with *N* bosons all in the same state is this quantity

multiplied by the number of possible ways of making pairs of bosons, $N(N\text{-}1)/2$. In this approximation, the energy is

$$E = \frac{N(N-1)}{2V}U_0 \approx \frac{1}{2}Vn^2U_0, \tag{2.5}$$

where $n = N/V$. Here we have assumed that $N>>1$.
The wave function of the condensed state is defined as

$$\psi(r) = N^{1/2}\phi(r), \tag{2.6}$$

and hence the density of particles is given by

$$n(r) = |\psi(r)|^2 \tag{2.7}$$

After neglecting the terms of order $1/N$, the energy of the system may be written as,

$$E(\psi) = \int dr\left[\frac{\hbar^2}{2m}|\nabla\psi(r)|^2 + V(r)|\psi(r)|^2 + \frac{1}{2}U_0|\psi(r)|^4\right] \tag{2.8}$$

In order to obtain the optimal form for ψ, we minimize the above energy with respect to $\psi(r)$ and its complex conjugate. Here we have a constraint that the total number of particles N is conserved

$$N = \int dr|\psi(r)^2|. \tag{2.9}$$

The constraint is favorably determined by the method of Lagrange multipliers. It can be written as $\delta E - \mu\delta N = 0$, where the chemical potential μ is the Lagrange multiplier. μ chemical potential ensures constancy of the particle number and the variations of ψ and ψ^* may thus be taken to be arbitrary. This process is

equivalent to minimizing the quantity $E-\mu N$ at fixed μ. Equating to zero the variation of $E-\mu N$ with respect to ψ^*(r) gives the time-independent Gross-Pitaevskii equation

$$-\frac{\hbar^2}{2m}\nabla^2\psi(r)+V(r)\psi(r)+U_0|\psi(r)|^2\psi(r)=\mu\psi(r)\,. \qquad (2.10)$$

The Gross-Pitaevskii equation is Schrödinger equation in which the potential energy is the sum of the external potential and non-linear interacting term. For non-interacting particles all in the same state the chemical potential is equal to the energy per particle, but for interacting particles it is not.

2.2.2. Thomas-Fermi Approximation

If the number of particles in a gas is very large, an accurate expression for the ground-state energy may be obtained by neglecting the kinetic energy term in the Gross-Pitaevskii equation. Let us give an example of the harmonically trap condensate [50], when the number of atoms is large and interactions are repulsive, the ratio of kinetic to interaction energy is small. Because of the large number of atoms the interatomic interaction becomes large. A better approximation for the condensate wave function for the large clouds may be obtained by solving the Gross-Pitaevskii equation, neglecting the kinetic energy term from the beginning. In this manner from Eq. (2.10) one finds

$$V(r)\psi(r)+U_0|\psi(r)|^2\psi(r)=\mu\psi(r)\,, \qquad (2.11)$$

where μ is the chemical potential. The above simple equation has the solution

$$n(r)=|\psi(r)|^2=[\mu-V(r)]/U_0 \qquad (2.12)$$

in the region where the right hand side is positive, while $\psi = 0$ outside this region. The boundary of the cloud is therefore given by

$$V(r) = \mu \tag{2.13}$$

The physical theme of the above approximation is that the energy to add a particle at any point in the cloud is the same everywhere. This energy is given by the sum of the external potential $V(r)$ and an interaction contribution $n(r)U_0$ which is the chemical potential of a uniform gas having density equal to the local density $n(r)$.

In the Thomas-Fermi approximation the extension of the cloud in the three directions is given by the three semi-axes R_i obtained by inserting the three dimensional harmonic oscillator potential, $V(x, y, z) = \frac{1}{2} m(\omega_1^2 x^2 + \omega_2^2 y^2 + \omega_3^2 z^2)$ into (2.13),

$$R_i^2 = \frac{2\mu}{m\omega_i^2}, \quad \text{i=1, 2, 3.} \tag{2.14}$$

The lengths R_i can be calculated in terms of trap parameters once the chemical potential has been determined. The normalization condition on ψ, Eq. (2.9), provides a relation between the chemical potential μ and the total number of particles *N*. For a harmonic trap with a potential given above, one finds

$$N = \frac{8\pi}{15}\left(\frac{2\mu}{m\bar{\omega}^2}\right)^{3/2} \frac{\mu}{U_0} \tag{2.15}$$

where $\bar{\omega}$ is oscillatory frequency, $\bar{\omega} = (\omega_1\omega_2\omega_3)^{1/3}$.

Solving Eq. (2.15) for μ we can find that, the chemical potential proportional with $N^{2/5}$. From the thermodynamic relation $\mu = \partial E / \partial N$, the Thomas-Fermi energy for a trapped condensate given as

$$\frac{E}{N} = \frac{5}{7}\mu . \qquad (2.16)$$

2.3. Trapping and Cooling of Atoms

The second half of the 20th century has seen the development of incredible techniques, in which light is used to control both the internal and external degrees of freedom of neutral atoms, ions, molecules, etc. These techniques are based on the fact that, just like massive objects, light carries energy, momentum, and angular momentum, and can interact strongly with matter. In any process these quantities are conserved so that they can be exchanged between the field and the matter. In brief, a good control of the light translates into a good control of the dynamics of the atoms.

The control of the polarization can be easily done with natural materials, such as crystals. In the 60's, the discovery of lasers, which provide intense and directional sources of monochromatic light, has made the control of the pulsation ω of the light and therefore, the control of its energy $\hbar\omega$ and its momentum $\hbar\omega/c$.

In this subsection, some main information is given concerning the experimental techniques to control the dynamics of the atoms. We begin with magnetic traps and then discuss the laser cooling and trapping. At last we will overview the evaporative cooling.

Magnetic traps

Magnetic trapping of neutral atoms is due to the Zeeman effect: the energy of an atomic state depends on the magnetic field, and consequently an atom in an inhomogeneous field experiences a spatially varying potential. The energy of an atom in a state *i* can then be given as

$$E_i = C_i - \mu_i B , \qquad (2.17)$$

where μ_i is the magnetic moment of the state and C_i is a constant. It is simply seen that the magnetic contribution to the energy thus provides a potential energy $-\mu_i B$ for the atom. If the magnetic moment is positive, the atom experiences a force driving it to higher field regions. If it is negative, the force is towards lower field regions. Therefore, states with a positive magnetic moment are called as *high-field seekers* and those with a negative magnetic moment as *low-field seekers*.

The energy depth of magnetic traps is determined by the Zeeman energy, $\mu_i B$. Atomic magnetic moments are of order the Bohr magneton, $\mu_B = e\hbar/2m_e$, which in temperature units is approximately 0.67K/T. Since laboratory magnetic fields are generally considerably less than 1 tesla, the depth of magnetic traps is much less than a Kelvin, and therefore atoms must be cooled in order to be trapped magnetically.

A simple magnetic field configuration in which the magnetic field vanishes at some point is the standard quadrupole one, in which the magnetic field varies linearly with distance in all directions.

Bose-Einstein condensation in dilute gases was first achieved in experiments using a modified quadrupole trap known as the time-averaged orbiting potential (TOP) trap. In this trap one superimposes on the quadrupole field a rotating, spatially-uniform, magnetic field [51]. The field used in experiment is given as

$$B = (B'x + B_0 \cos\omega t, B'y + B_0 \sin\omega t, -2B'z). \qquad (2.18)$$

Laser cooling and trapping

As we know many techniques for trapping and cooling atoms exploit the interaction of atoms with radiation fields, especially those of lasers.

The interaction between an atom and the electric field is given in the dipole approximation by

$$H' = -d \cdot \varepsilon \tag{2.19}$$

where d is the electric dipole moment operator and ε is the electric field vector.

Experiments on clouds of dilute gases exploit the forces on atoms in a laser field in a variety of ways. Let us describe the origin of these forces. The energy shift of an atom may be regarded as an effective potential V in which the atom moves. If the time-averaged electric field varies with position, the shift of the energy due to the field gives rise to a force, $F_{dipole} = -\nabla V(r)$ on an atom. This is often referred to as the dipole force. The quadrupole moment will also give rise to forces, but these will usually be much less than the dipole force.

In laser beam it is possible to create a radiation field whose intensity has a maximum in space. If the frequency of the light is detuned to the red, the energy of a ground state atom has a spatial minimum, so that it is possible to trap atoms. The depth of the trap is determined by the magnitude of the energy shift.

The situation in optical traps is quite different for trapping by magnetic fields, since the potential is then strongly dependent on the magnetic substate. With magnetic traps it is difficult to investigate the influence of the interaction energy on the spin degrees of freedom of an atomic cloud since the energy is dominated by the Zeeman term. On the other hand, optical traps are well suited for this purpose.

The main idea that led to the development of laser cooling may be understood by considering an atom subjected to two oppositely directed laser beams of the same angular frequency, ω, and the same intensity.

Evaporative cooling

The process of evaporative cooling was seen as a viable route to reaching BEC, and several groups set out to make it work. This approach was spectacularly successful, and within a few months of each other in 1995, three groups reached the regime of quantum degeneracy required for BEC. The first unambiguous demonstration of BEC in an evaporatively cooled atomic gas was reported by the NIST/JILA group [45] for doubly spin-polarized ^{87}Rb, followed by evidence of

BEC for doubly spin-polarized ^{7}Li by a group at Rice University [46] and then a demonstration of BEC at MIT [47] in the F=1, M=-1 lower hyperfine component of ^{23}Na.

Evaporation from a magnetic trap overcomes the density and the temperature limits of laser cooling. The important concepts of evaporative cooling of trapped atoms were formulated by Hess in 1986, as a method for cooling atomic hydrogen pre-cooled by cryogenic methods [52]. Evaporative cooling is performed by removing the high-energy component of the thermal distribution of atoms from the magnetic trap. The evaporated atoms carry away more than the average, so that when the remaining atoms re-thermalise (by undergoing elastic collisions) the temperature of the vapour decreases. To continue cooling one must continue to remove high-energy atoms. The essential condition for efficient evaporative cooling is a long sample lifetime compared to the collisional re-thermalisation time.

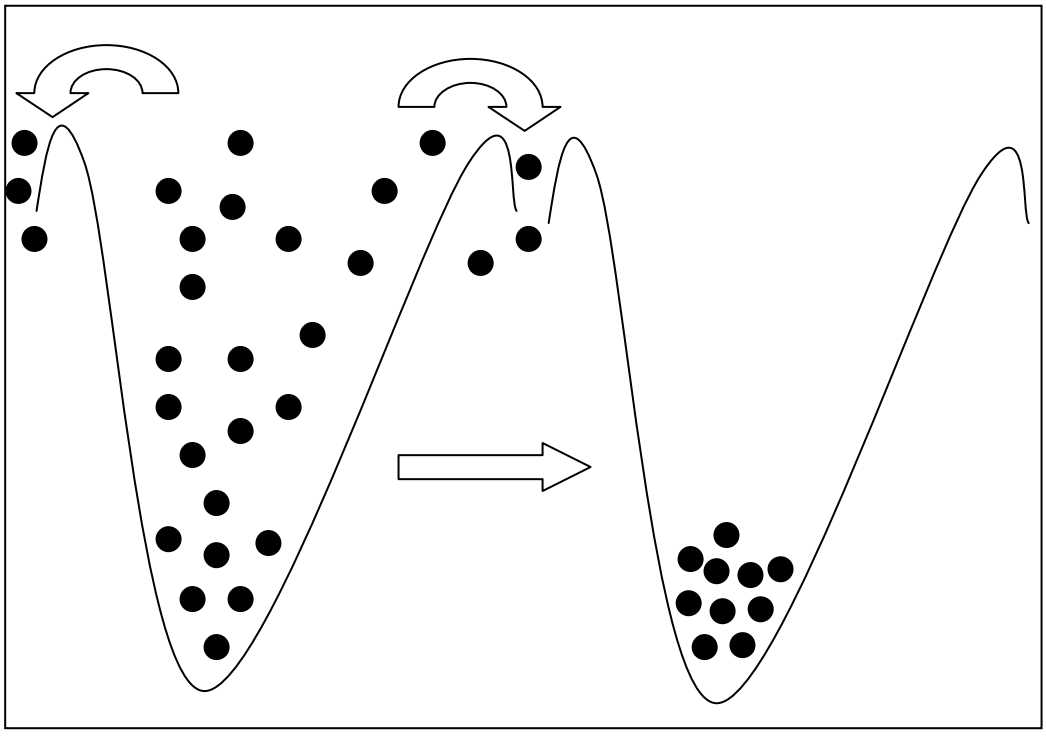

Fig. 2.1. Evaporative cooling basics-releasing high-energy atoms from a magnetic trap and subsequent re-thermalisation. It is continued cooling by lowering further the rim of the trap.

For a dilute gas confined in a magnetic potential, decreasing the temperature decreases the sample volume. Even though the total number of atoms in the sample decreases during the evaporation process, it is possible to increase the density of the remaining atoms. When this happens, the decreasing temperature leads to an increasing rate of the re-thermalisation, despite the loss of atoms. This is called runaway evaporation, because the cooling accelerates during the evaporation.

Evaporative cooling is simple to implement and relatively efficient, typically providing six orders of magnitude increase in phase space density at a cost of a factor of 10^3 reduction in the number of atoms. Outside of the laboratory, evaporative cooling is a familiar process- it' s how we cool coffee in a cup and why we feel cold after a swim.

i-) Radio-Frequency (RF) Evaporation

The evaporation could be performed conveniently if the rim of the trap was defined by a RF-resonance condition, rather than by the geometry of the magnetic field. The idea is to use an RF transition (between m_F sub-states) to remove atoms from the magnetic trap (i.e. from leak) at a distance from the trap centre defined by the Zeeman shift of the transition associated with the trapping field and the frequency of the applied RF field.

In the RF-induced evaporation, the RF radiation flips the atomic spin. As a result, the attractive force provided by the magnetic field becomes repulsive and expels the atoms from the trap. This scheme is energy-selective because the resonance frequency is proportional to the magnetic field strength, and therefore to the potential energy of the atoms. In the case of transitions between magnetic sub-levels (i.e. $\Delta m_F = 1$), the resonance condition for the magnetic field strength B is, $\mu_B g_F B = h\nu_{RF}$ where g_F is the Land g-factor and μ_B is the Bohr magneton. For example, the spacing between m_F sub-states is 0.7 MHz per gauss for transitions within the F=2 hyperfine level of ^{87}Rb frequently used to make Bose condensates. RF induced evaporation has the very nice feature that the evaporation process can be separated from the design of the magnetic trapping

potential. In particular, there is no need to weaken the trapping potential in order to lower its depth. This makes it easier to reach runaway evaporation.

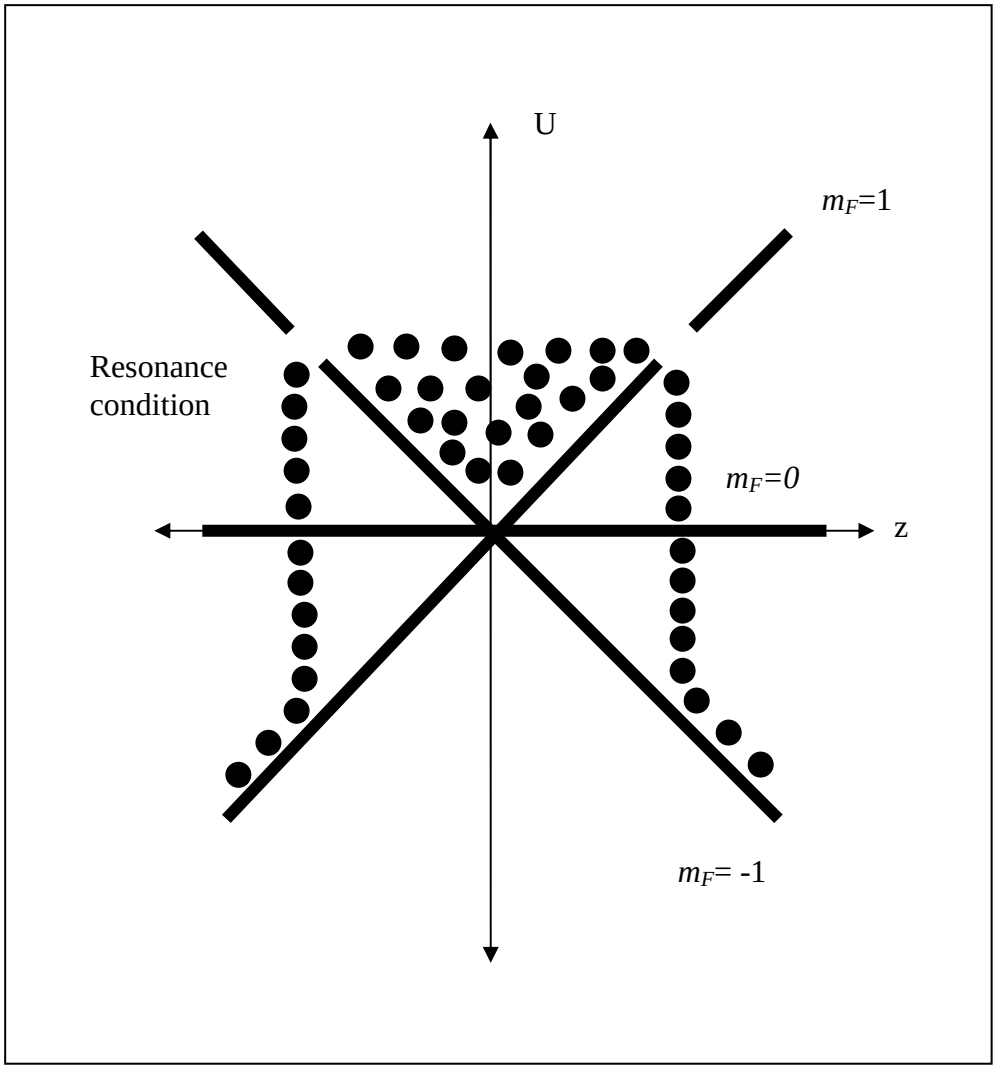

Fig. 2.2. Zeeman shift of magnetic sub-levels versus position for a simple quadrupole trap. A leak is formed at the top of the trap using a RF field, allowing high energy atoms to escape.

Idealy, RF evaporation spin-flips atoms on a 3-dimensional resonant shell (of constant magnitude magnetic field) surrounding the trapped atoms. For a static trap, such as an Ioffe-Pritchard style, the shape of the evaporation shell is an ellipsoid centred on the atomic sample (if the influence of gravity can be ignored). However, for weak confinement gravity will displace the atomic sample from the centre of the trap, potentially resulting in (less-efficient) 1D evaporation. For the

time-averaged orbiting potential (TOP) trap the evaporation geometry is more complex (see Fig. 2.3.).

ii-)Evaporation in a TOP trap

Evaporation in a TOP trap is slightly more complicated than a static trap, because of the rotating magnetic field. The spatial distribution of an atomic sample confined in a TOP trap is determined by the time-average of the magnetic potential (which is harmonic), but to understand evaporation in a TOP trap we must consider the instantaneous magnetic field.

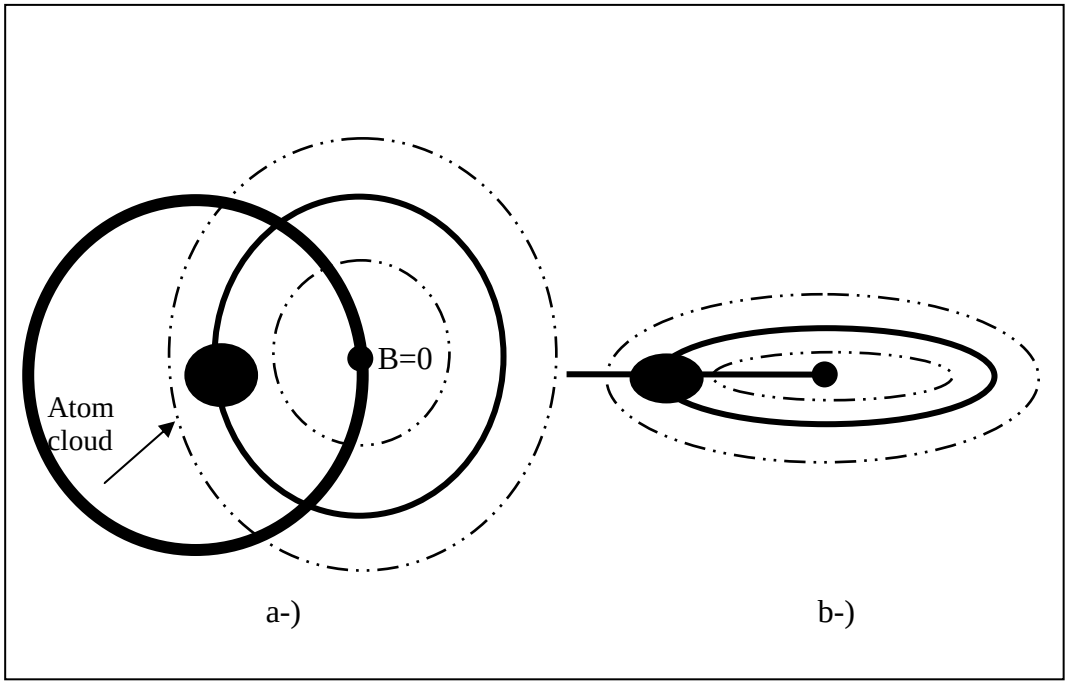

Fig. 2.3. Evaporative cooling in a TOP trap. a-) and b-) are plan and elevation view, respectively.

The region over which evaporation occurs is an ellipsoid shell with aspect ratio of 2:1 (from the quadrupolar potential), displaced from the centre of the cloud by the rotating bias field. Atoms are removed from the edge of the trap as the evaporation surface rotates about the cloud. The evaporation is essentially 2-dimensional.

With the TOP trap, in addition to RF-induced transitions, one can also induce Majorana spin flips [53] using the zero magnetic field point of the rotating quadrupole trap. Near the zero magnetic field point of the quadrupole trap the Larmor frequency becomes sufficiently small that the magnetic moment of the atoms in this region can no-longer remain aligned with the local magnetic field direction. In a quadrupole trap ejection occurs at the trap centre. However, as shown in Fig. 2.3., in a TOP trap the zero point orbits the edge of the atomic sample and the resulting spin-flips can be used for evaporative cooling.

The temperatures reached by laser cooling are impressively low, but they are not low enough to produce Bose-Einstein condensation in gases at the densities that are realizable experimentally. In the experiments, Bose-Einstein condensation of alkali gases is achieved by using evaporative cooling after laser cooling. The basic physical effect in evaporative cooling is that, if particles escaping from a system have an energy higher than the average energy of particles in the system, the remaining particles are cooled. In brief, the intent of evaporative cooling is to lower the temperature of a trapped gas while simultaneously increasing the phase-space density of the gas. The efficiency of evaporative cooling is determined by the ratio of the elastic collision rate to the rate of inelastic collisions that cause number loss and heating. For more extensive account of evaporative cooling we refer, [50, 54].

3. SHORTCUTS TO ADIABATICITY FOR TRAPPED ULTRACOLD GASES

In quantum mechanics, when the transition probabilities between the time-dependent eigenstates of H are negligible, the evolution of a system characterized by a time-dependent Hamiltonian $H(t)$ is adiabatic. This happens when H is either time-independent, or when its rate of change is slow compared to the typical time-scales involved [55]. Thinking in terms of instantaneous eigenstates is much easier than looking for the solutions of time-dependent problems.

Yet adiabatic techniques are typically slow [56], while experimentalists are often constrained by finite lifetimes or coherence times of their samples. This motivated the search for fast schemes reproducing or approaching adiabatic transformations. Some methods use minimization techniques to optimize the transition to a target state [6, 7], whereas others yield the exact same state that would have been reached after an adiabatic transformation [8, 57, 58]. The latter are referred to as shortcuts to adiabaticity.

In this chapter, we detail how such methods can be used on the motional degrees of freedom of ultracold gases confined in time-dependent harmonic traps and overview the validity of the approach of experiments. Methods to realize shortcuts to adiabaticity are detailed and overview on the experimental realization of these methods is given.

3.1. Quantum adiabatic transformations

The term adiabatic has different meanings. For this reason, it is a confusing one in physics. The word itself has a Greek root and means “not passing through” [56]. In the content of thermodynamics, it is the heat that is not passing through the walls of the container: this word describes a process in which no heat is exchanged. In that sense, thermo-dynamical adiabatic transformations need by no means be slow.

In quantum mechanics, the meaning is different: a process is adiabatic when the probabilities for the system to make transitions between the time-

dependent (or instantaneous) eigenstates of the Hamiltonian are negligible. Below we recall the adiabatic theorem of quantum mechanics.

The adiabatic theorem

The exact formulation of the adiabatic theorem was first given by Born and Fock [3]. Let us consider a varying Hamiltonian $H(t)$ of which $\{|\lambda;t\rangle\}$ is a basis of time-dependent eigenstates. The adiabatic theorem states that if this Hamiltonian is slowed down (mathematically, this is achieved by evolving the system with the new one $H_\varepsilon(t) \equiv H(\varepsilon\, t)$ with $\varepsilon < 1$) then the state $|\Phi_\varepsilon;t\rangle$ of the Schrödinger equation associated to $H_\varepsilon(t)$ satisfies,

$$|\Phi_\varepsilon;t\rangle = \exp\left\{-i\left[\omega_{\lambda_\varepsilon}(t') - i\langle\lambda_\varepsilon;t'|\frac{\partial}{\partial t'}|\lambda_\varepsilon;t'\rangle\right]dt'\right\}|\lambda_\varepsilon;t'\rangle \qquad (3.1)$$

where $|\lambda_\varepsilon;t\rangle$ is a basis instantaneous eigenstates of $H_\varepsilon(t)$ and $\hbar\omega_{\lambda_\varepsilon}(t)$ is its spectrum. This means that when the evolution is infinitely slow $(\varepsilon \to 0)$ the instantaneous eigenstates, chosen with this appropriate phase, coincide with the solutions of the Schrödinger equation.

The adiabatic theorem is not usually relevant to practical problems, because it is just valid in the limit $(\varepsilon \to 0)$. For instance, if one wants to use it in a problem involving a two-level atom interacting with a classical field oscillating with a pulsation ω, the theorem can only be applied in the limit $\omega' \equiv \varepsilon\omega \to 0$, which is not very useful since it corresponds to a static field.

However, the criteria on the rate of evolution for the transformation to be nearly adiabatic can be calculated. If the initial state is $|\lambda;t=0\rangle$ and the spectrum is discrete, the criteria generally used is given by

$$\sum_{\lambda'\neq\lambda}\left|\frac{\langle\lambda';t|\partial_t|\lambda;t\rangle}{\omega_\lambda(t)-\omega_{\lambda'}(t)}\right| \ll 1, \qquad (3.2)$$

where the sum is over all the instantaneous eigenstates other than $|\lambda;t\rangle$.

Having given the basic information about the adiabatic theorem, shortcuts to adiabatcity will be examined in detail in the next subsection.

3.2. Shortcuts to adiabaticity

The expression "shortcuts to adiabaticity" was recently introduced in Chen et al. [8], to describe protocols that speed up a quantum adiabatic process, through a non-adiabatic route.

In this subsection, the definition of a shortcut to adiabaticity and derivation of angular frequency trajectories for both non-interacting gases and interacting BECs confined in time-dependent harmonic traps will be given, and then studies up to now on this area will also be reviewed.

For a system described by a Hamiltonian H (t), a shortcut to adiabaticity is realized when another Hamiltonian $H'(t)$ can be found, such that the state obtained after a finite time of evolution with $H'(t)$ is identical to the final state of the adiabatic evolution with H (t). Shortcuts to adiabaticity are generally not adiabatic; only the final state is identical to that obtained after an adiabatic evolution. Shortcuts to adiabaticity can be realized by simply engineering the time-dependent parameters of the Hamiltonian (for example, in the case of harmonic oscillator, the angular frequency). A practical method to find a class of appropriate angular frequency $\omega(t)$ was detailed by Chen et al. [8]. Let us overview this method. In that case, the Hamiltonian is chosen to be time-independent (but with different frequencies) outside the time interval $t \in [0, t_f]$. An invariant is engineered to commute with the Hamiltonian outside this interval. This yields a specific $\omega(t)$ for which all the eigenstates of H ($t \leq 0$) are exactly mapped to the corresponding ones of H ($t \geq t_f$) after the evolution for $t \in [0, t_f]$. Up to a global phase and a rescaling of the energies and lengths, the final state (at time $t = t_f$) is identical to the initial one ($t = 0$), i.e. if the initial state is

$$|\psi; t \le 0\rangle = \sum_n c_n |n\ ; t = 0\rangle e^{-i\omega_n(0)t}, \tag{3.3}$$

where $\{|n;t\rangle, n \in \mathbb{N}\}$ is a basis of instantaneous eigenstates of $H(t)$, with $\{\hbar\omega_n(t)\}$ the corresponding eigenvalues, and $\sum_n |c_n|^2 = 1$, the final state is

$$|\psi; t \ge t_f\rangle = e^{i\phi} \sum_n c_n |n\ ; t_f\rangle e^{-i\omega_n(t_f)t - i\delta_n}, \tag{3.4}$$

where

$$\begin{aligned} \delta_n &= (\lambda_n / \hbar) \int_0^{t_f} dt' / b^2, \\ \ddot{b} + \omega^2(t) b &= \frac{\omega_0^2}{b^3}, \\ \lambda_n &= \left(n + \frac{1}{2}\right)\hbar\omega_0. \end{aligned} \tag{3.5}$$

This is true even if the initial state is not an equilibrium state.

In the following, firstly we will give fundamental info for frequency trajectories for a non-interaction gas and for an interaction BEC.

Frequency trajectory for a non-interacting gas

The Hamiltonian is assumed to have the form

$$H = \frac{p^2}{2m} + \frac{1}{2} m\omega^2(t) q^2 + mgq. \tag{3.6}$$

It is a single particle Hamiltonian for a harmonic trap and a constant force. The angular frequency $\omega(t)$ is assumed to be constant outside the interval $t \in [0, t_f]$. During this interval, the problem is to find the appropriate frequency trajectory

$\omega(t)$, connecting the initial trap of initial frequency $\omega(0)$ to a final trap of frequency $\omega(t_f)$, for the decompression [or compression if $\omega(0) < \omega(t_f)$] to implement a shortcut to adiabaticity.

In order to best understand the frequency trajectory for a non-interacting gas, let us overview the strategy introduced by Chen et al. [8]. If the invariant commutes with the Hamiltonian

$$[I, H] = 0 \tag{3.7}$$

for $t \leq 0$ and $t \geq t_f$, and provided that the function b is sufficiently continuous, the stationary states of $H(t \leq 0)$ will be transferred to the corresponding ones of $H(t \geq t_f)$. An effectively one dimensional time-dependent harmonic oscillator,

$$H = \hat{p}^2 / 2m + m\omega^2(t)\hat{q}^2 / 2, \tag{3.8}$$

with an initial angular frequency $\omega(0) > 0$ at time t=0 and final frequency $\omega_f = \omega(t_f) < \omega(0)$ at time t_f is considered. The main tool to engineer $\omega(t)$ and the state dynamics will be the solution of the corresponding Schrödinger equation based on the existence of invariants of motion of the form

$$I(t) = 1/2[(1/b^2)\hat{q}^2 m\omega_0^2 + (1/m)\hat{\pi}^2], \tag{3.9}$$

where

$$\hat{\pi} = b(t)\hat{p} - m\dot{b}\hat{q} \tag{3.10}$$

plays the role of a momentum conjugate to $\hat{q}/b$, the dots are derivatives with respect to time, and ω_0 is in principle an arbitrary constant. The scaling, dimensionless function b=b (t) satisfies the condition

$$\ddot{b} + \omega^2(t)b = \omega_0^2 / b^3 . \qquad (3.11)$$

This is an Ermakov equation where real solutions must be chosen to make I Hermitian. An alternative, successful strategy put forward in [8], inspired in inverse scattering techniques for complex potential optimization [59], is to leave $\omega(t)$ undetermined at first and impose properties on b and its derivatives at the boundaries, $t = 0$ and t_f, to assure: (i) that any eigenstate of $H(0)$ evolves as a single expanding mode and that (ii) this expanding mode becomes, up to a position-independent phase factor, equal to the corresponding eigenstate of the Hamiltonian $H(t_f)$ of the final trap. This keeps the populations in the instantaneous basis equal at the initial and final times. After $b(t)$ and its derivatives are fixed at the boundaries, $b(t)$ may be chosen as a real function satisfying the boundary conditions, for example as a polynomial or some other convenient functional form with enough free parameters. Once b(t) has been determined, the physical frequency $\omega(t)$ is obtained from the Eq. (3.11).

Firstly, the conditions at t=0 are discussed. By choosing $b(0) = 1$, $\dot{b}(0) = 0$, $H(0)$ and $I(0)$ commute and have common eigenfunctions initially. We set $\omega_0 = \omega(0)$ from now on so that $\ddot{b}(0) = 0$ must hold as well. These boundary conditions imply that any initial eigenstate of H (0) will evolve according to the expanding mode for all later times. In general, H (t) and I (t) will not commute for, $t > 0$, so that the expanding mode may have more than one component in the "adiabatic basis" of instantaneous eigenstates of H (t). At time t_f, we want the expanding mode to be proportional to the corresponding eigenstate of the final trap. Thus, at time t_f, it is secondly imposed that the following conditions $b(t_f) = \gamma = [\omega_0 / \omega_f]^{1/2}$, $\dot{b}(t_f) = 0$, $\ddot{b}(t_f) = 0$ must satisfies.

Substituting the simple polynomial ansatz

$$b(t) = \sum_{j=0}^{5} a_j t^j , \qquad (3.12)$$

into the six boundary conditions gives six equations that can be solved to provide the coefficients, $b(t) = 6(\gamma-1)\left(\frac{t}{t_f}\right)^5 - 15(\gamma-1)\left(\frac{t}{t_f}\right)^4 + 10(\gamma-1)\left(\frac{t}{t_f}\right)^3 + 1$.

As an example, we will overview the method described in [60]. This method is proposed to design the time dependence of the trap frequency and achieve in a short time an adiabatic-like (frictionless) evolution of Bose-Einstein condensates. In order to manipulate Bose-Einstein condensates for different applications, it is important to study and control their response to time-dependent changes of the confining fields. A general approach to avoid undesired excitations is to modify the trap adiabatically, i.e., very slowly, so that, if the initial state is in the ground state the final state will be the ground state as well. However, this may require very long times and become impractical. Faster changes are thus a desirable objective but they will in general induce excitations and oscillations, so that the proportion of the ground state in the final state may be small [61-63]. These difficulties has motivated the researchers to find the answer of this question: Is it possible to change the trap in a very short time, taking the condensate, up to a global phase, to the same state that would be reached after a slow (adiabatic) process? This question has been answered recently in the affirmative for cooling expansions within the framework of the linear Schrödinger equation [8]. The method used to design the time-dependence of the trap frequency was based on Lewis-Riesenfeld invariants of motion [64] and simple inverse scattering techniques that had been applied for complex potential optimization [59].

The starting point is the effective one dimensional Gross-Pitaevskii equation for the longitudinal *x*-direction in an elongated cigar trap,

$$i\hbar\frac{\partial\psi}{\partial t} = \left[-\frac{\hbar^2}{2m}\frac{\partial^2}{\partial x^2} + \frac{1}{2}m\omega^2(t)x^2 + g|\psi|^2\right]\psi \qquad (3.13)$$

g being the coupling parameter. To solve the problem, let us use the following ansatz [60],

$$\psi(x,t) = e^{-\beta(t)} e^{-\alpha(t)x^2} \phi(x,t) \tag{3.14}$$

where time-dependent functions $\alpha(t)$ and $\beta(t)$ to be determined later. After substituting this ansatz into Eq. (3.13), and using the scaling $\rho = x/b$, we obtain

$$i\hbar \frac{\partial \phi}{\partial t} = -\frac{\hbar^2}{2m}\frac{1}{b^2}\frac{\partial^2 \phi}{\partial \rho^2} + \left[\frac{1}{2} m\omega(t)^2 + i\hbar\dot{\alpha} - \frac{2\hbar^2}{m}\alpha^2\right] b^2 \rho^2 \phi$$

$$+ \left[g e^{-(\alpha+\alpha^*)x^2} e^{-(\beta+\beta^*)} |\phi|^2\right]\phi + \left[i\hbar\dot{\beta} + \frac{\hbar^2\alpha}{m}\right]\phi$$

$$+ \left[2\frac{\hbar\alpha}{m} + i\frac{\dot{b}}{b}\right]\hbar\rho\frac{\partial \phi}{\partial \rho}, \tag{3.15}$$

where the dot means derivative with respect to time. Let us now impose that the coefficients in square brackets […] of the last two terms vanish. This gives us the equation for α and β (it is assumed that *b* is real)

$$\beta = \frac{1}{2}\ln b$$

$$\alpha = -\frac{im\dot{b}}{2\hbar b}, \tag{3.16}$$

and $e^{-(\alpha+\alpha^*)x^2} e^{-(\beta+\beta^*)} = b^{-1}$. With the help of the boundary conditions given below,

$$b(0) = 1, \qquad \dot{b}(0) = 0, \qquad \ddot{b}(0) = 0$$

$$b(t_f) = (\omega_0/\omega_f)^{1/2}, \quad \dot{b}(t_f) = 0 \qquad \ddot{b}(t_f) = 0 \tag{3.17}$$

b (t) is then interpolated with some functional form, e.g. a polynomial with enough coefficients to satisfy all conditions, and finally $\omega^2(t)$ is calculated from the Ermakow equation. Hence, frictionless trajectories for $\omega(t)$ is found.

In brief, it was shown in [60] that it is possible to take a Bose–Einstein condensate in a very short time from an initial harmonic trap to a final one without excitations, by choosing the time dependence of the frequency according to the Ermakov equation or its modifications after matching the time-dependence of a scaling factor to suitable boundary conditions. In [60], it was also shown that in 1D and 3D traps, this requires either a simultaneous change of the time dependence of the coupling, or a Thomas–Fermi type of regime. 2D traps are privileged in this respect and do not require either of these conditions.

Shortcut to adiabaticity for an interacting Bose-Einstein condensate

The rapid shortcut decompression of a 3D interacting BEC confined in an anisotropic harmonic trap was investigated in [14]. Here in order to understand the system we prefer to follow that study. The system was described by a zero-temperature BEC plus a thermal cloud, assumed to behave independently. The BEC component thus obeys the 3D Gross-Pitaevskii equation

$$i\hbar\frac{\partial}{\partial t}\psi(r,t)=\left[-\frac{\hbar^2}{2m}\nabla^2+V(r,t)+UN\left|\psi(r,t)\right|^2\right]\psi(r,t) \qquad (3.18)$$

where $\psi(r,t)$ is the wave function of the condensate, m is the mass, N is the number of particles, and U is the interaction potential. The time-dependent trapping potential V has a cylindrical symmetry along the horizontal axis (y)

$$V(r,t)=\frac{1}{2}m\omega_{\perp}(t)^2(x^2+z^2)+\frac{1}{2}m\omega_{\parallel}(t)^2y^2+mgz\,, \qquad (3.19)$$

It was emphasized that decreasing the trap frequencies not only decompresses the BEC but also translates the harmonic potential minimum vertically by $\Delta z = -g(1/\omega_{0\perp}^2 - 1/\omega_{f\perp}^2)$, where ω_0 is initial angular frequency and ω_f is final angular frequency. The objective is to engineer a trajectory $\omega_{\perp,\parallel}(t)$ connecting the equilibrium states in the initial and final potentials. It was stressed that the BEC is not at equilibrium at any time during the trajectory, but only at t = 0 and t=t_f . Thus with using a specifically designed parameter trajectory for the harmonic trapping potential, shortcut to adiabaticity transformations on a 3D interacting BEC was performed [14]. It is well known that all we have to do is to engineer the time-dependent parameter of the Hamiltonian and find the convenient trajectories to obtain shortcut to adiabaticity.

So far, the main information of shortcuts to adiabaticity for both non-interacting gas and interacting BEC with harmonic trap are shortly given. After giving the main structure of these systems, in the following we will briefly mention about the studies on shortcuts to adiabaticity up to now [9-31, 57, 58, 65-82] and we will overview the methods used in this field. In [8], the Lewis-Riesenfeld invariants were used to inverse engineer the time dependence of a harmonic oscillator frequency between predetermined initial and final values so as to avoid final excitations. That paper and its fellow on Bose-Einstein condensates [60] have indeed initiated a surge of activity, not only for harmonic expansions [13, 14, 18, 20, 23, 25-27, 67, 71, 75,79] but for atom transport [9, 66, 78], optical lattice expansions [17, 65], wavepacket splitting [71], internal state control [68, 73, 74, 77], many-body state engineering [24, 26, 72] and other applications such as sympathetic cooling of atomic mixtures [16, 82], or cooling of nanomechanical resonators [22]. Actually several works had previously or simultaneously considered to speed up adiabatic processes making use of different techniques. For example, Masuda and Nakamura [12] developed a fast-forward technique for several manipulations on wavepackets such as expansions, transport or splitting of Bose-Einstein condensates.

Expansion of trapped particles

Obtaining fast expansions of trapped cold atoms without losing or exciting them is important for many applications: for example to reduce velocity dispersion and collisional shifts in spectroscopy and atomic clocks, decrease the temperature, adjust the density to avoid three body losses. As well as trap compressions are also quite common.

There are several methods used in investigation of shortcuts to adiabaticity. Let us mention about most using ones, (i) inverse engineering of the external driving [8, 9, 11, 13, 14, 60, 70] based on Lewis-Riesenfeldt invariants [64] which has been applied in several expansion experiments [13, 14]; (ii) transitionless tracking algorithm that adds to the original Hamiltonian extra terms to cancel transitions in the adiabatic bases [1, 10, 15], (iii) optimal control methods [6, 18, 78]; (iv) the fast-forward approach advocated by Masuda and Nakamura [12, 19]. In [75], the connection between fast-forward approach and inverse engineering method based on Lewis-Reisenfeld invariants was investigated. The fast-forward approach by Masuda and Nakamura [19] generates driving potentials to accelerate slow quantum adiabatic dynamics. The objective of the method is to accelerate a standard system subjected to a slow variation of external parameters. A streamlined version of the formalism that produces the main results was presented in a few steps and more general applications exemplified by wave splitting processes were also discussed [75]. Firstly, a streamlined construction of local and real fast-forward potentials were provided. The starting point is the 3D time-dependent Gross-Pitaevskii equation where the Hamiltonian H is the sum of the kinetic energy T, the external potential $V(t)$, and the mean field potential $G(t)$. It was assumed that an external local potential, where local means here $\langle x|V(t)|x'\rangle = V(x,t)\delta(x-x')$. Then, by solving 3D time-dependent Gross-Pitaevskii equation in coordinate space, $V(x, t)$ may be written as

$$V(x,t) = \frac{i\hbar\langle x|\partial_t\psi(t)\rangle - \langle x|T|\psi(t)\rangle - \langle x|G(t)|\psi(t)\rangle}{\langle x|\psi(t)\rangle} \qquad (3.20)$$

with $\langle x|\psi(t)\rangle = \psi(x,t)$. The kinetic and mean field terms in the coordinate representation have the usual forms

$$\langle x|T|\psi(t)\rangle = -\frac{\hbar^2}{2m}\nabla^2\psi(x,t),$$
$$\langle x|G(t)|\psi(t)\rangle = g|\psi(x,t)^2|\psi(x,t), \qquad (3.21)$$

g being the coupling constant of the BEC. By introducing the ansatsz into Eq. (3.20)

$$\langle x|\psi(t)\rangle = r(x,t)e^{i\phi(x,t)}, \qquad r(x,t),\phi(x,t)\in\mathbb{R}, \qquad (3.22)$$

gives us the below equation where r is the modulus and ϕ is the phase,

$$V(x,t) = i\hbar\frac{\dot{r}}{r} - \hbar\dot{\phi} + \frac{\hbar^2}{2m}\left(\frac{2i\nabla\phi\bullet\nabla r}{r} + i\nabla^2\phi - (\nabla\phi)^2 + \frac{\nabla^2 r}{r}\right) - gr^2, \qquad (3.23)$$

where the dot means time derivative. The real and imaginer parts are

$$\mathrm{Re}[V(x,t)] = -\hbar\dot{\phi} + \frac{\hbar^2}{2m}\left(-(\nabla\phi)^2 + \frac{\nabla^2 r}{r}\right) - gr^2 \qquad (3.24)$$

$$\mathrm{Im}[V(x,t)] = \hbar\frac{\dot{r}}{r} + \frac{\hbar^2}{2m}\left(\frac{2\nabla\phi\bullet\nabla r}{r} + \nabla^2\phi\right). \qquad (3.25)$$

The purpose now is to design a local and real potential such that an initial eigenstate of the initial Hamiltonian, typically the ground state but it could be otherwise, evolves in a time t_f into the corresponding eigenstate of the final Hamiltonian. It was assumed that the full Hamiltonian and the corresponding eigenstates are known at the boundary times. If we impose that Im[V (x, t)] = 0, i.e.

$$\frac{\dot{r}}{r}+\frac{\hbar}{2m}\left(\frac{2\nabla\phi\bullet\nabla r}{r}+\nabla^2\phi\right)=0\,, \tag{3.26}$$

then a local and real potential have been obtained.

In the inversion protocol, firstly $r(x,\ t)$ is designed, then solved for ϕ in Eq. (3.26), and finally the potential V is obtained from Eq. (3.24). If at the boundary times, $\dot{r}=0$ is imposed, Eq. (3.26) has solutions $\phi(x,t)$ fulfilling that $\phi(x,t)$ is independent of x at t = 0 and $t = t_f$. Using this in Eq. (3.51) at t = 0, and multiplying by $e^{i\phi(0)}$, we get

$$\left[-\frac{\hbar^2}{2m}\nabla^2+V(x,0)+g\left|\psi(x,0)\right|^2\right]\psi(x,0)=-\hbar\dot{\phi}(0)\psi(x,0)\,, \tag{3.27}$$

The initial state is thus an eigenstate of the stationary Gross-Pitaevskii equation at t = 0, and $-\hbar\dot{\phi}(0)=E(0)$ is the energy of the eigenstate $\psi(x,0)$. Note that the above solution of ϕ (with $\dot{r}=0$ at boundary times) admits the addition of an arbitrary function that depends only on time and modifies the zero of energy. A similar result is found at t_f . After giving the basic information, let us mention about the connection with the fast-forward approach and the connection with the invariant based inverse engineering approach. Firstly, the above results are reformulated to connect them with the fast-forward approach [12]. And after some algebra, fast-forward potential is determined as,

$$V(x,t)=V_0(x,R(t))-\hbar\varepsilon\dot{\alpha}(t)\theta(x,R(t))-\hbar\varepsilon^2\alpha^2(t)\frac{d\theta(x,R(t))}{dR}$$
$$-\frac{\hbar^2}{2m}\varepsilon^2\alpha^2(t)[\nabla\theta(x,R(t))]^2\,. \tag{3.28}$$

where $R(t)$ is in general function of its argument, ε is small parameter and α is magnification factor. Secondly, inverse engineering approach was examined. In the inverse approach based on quadratic-in-momentum invariants, the

Hamiltonian is assumed to have the form given in Eq. (3.29), at all times and in particular at initial and final instants.

$$V(x,t) = -F(t)\cdot x + \frac{1}{2}m\omega^2(t)|x|^2 + \frac{1}{\delta^2}U(\sigma) + h(t), \tag{3.29}$$

where $\omega(t)$, $F(t)$ and $h(t)$ are arbitrary functions of time and $U(\sigma)$ is an arbitrary function of its argument $\sigma = \sigma(t) = (x-\alpha)/\rho$. The time-dependent functions $\rho(t)$ and $\alpha(t)$ must satisfy the auxiliary equations

$$\frac{\omega_0^2}{\rho^3} = \ddot{\rho} + \omega^2(t)\rho, \tag{3.30}$$

$$\frac{F(t)}{m} = \ddot{\alpha} + \omega^2(t)\alpha, \tag{3.31}$$

with ω_0 an arbitrary constant. Up to now, we have mentioned about the general content of the study given in [75]. Let us continue with the results and future works related with that problem. Briefly, we can say that the inverse engineering approach and fast-forward approach are connected via the simple inversion method [75]. An example of a 3D harmonic expansion produced with the inverse engineering approach based on invariants and with the fast forward technique were also discussed to illustrate the links between the two methods (see [75] for more details). Consequently, it was shown that the connection between fast-forward approach and inverse engineering based on Lewis-Riesenfeld invariants. In [75], they identified in this manner applications in which the engineered potential does not depend on the initial state. Firstly, from the imposing set of equations of the fast-forward formalism as originally presented a streamlined version was extracted. And the second aim was to relate it to other inverse engineering methods. In a previous publication, the inverse-engineering method based on invariants, was related to the transitionless tracking algorithm, and the potential equivalence was demonstrated [77]. Similarly, in [75] the connection between the fast-forward method and the invariant-based method was established

for quadratic-in-momentum invariants. It was stated that these relations do not imply the full identity of the methods but their overlap and equivalence in a common domain. They are still useful heuristically as separate approaches since they are formulated in rather different terms [19, 77]. Moreover, they facilitate extensions beyond their common domain, as exemplified by the wave-splitting processes discussed in the content of [75]. It was pointed out that further extensions are left for separate analysis: for example the possibility to transfer an excited state into the ground state or adversely, or combining the fast-forward approach with optimal control theory without including the final fidelity in the cost function as in [6]. As a future work, it was stated that it is interesting to consider complex potentials, either as solutions to the shortcut dynamics, as in the quantum brachistochrone [83], or as an effective description of the system dynamics to be accelerated [11].

As a part of shortcut to adiabaticity with expansion of trapped particles, let us continue with the study of transitionless quantum drivings for the harmonic oscillator given in [79]. Muga et al. [79] compared and distinguished two different methods: a transitionless-tracking algorithm, and an inverse-invariant method, to achieve transitionless dynamics for a fast frequency change of a quantum harmonic oscillator. The first method, a transitionless-tracking algorithm, makes use of a generalized harmonic oscillator and a non-local potential. The second method, based on engineering an invariant of motion, only modifies the harmonic frequency in time, keeping the potential local at all times. To achieving transitionless dynamics for a fast frequency change of a quantum harmonic oscillator different driving Hamiltonians were implied. The one in the inverse-invariant method can be implemented for ultracold atoms or ions in the laboratory by varying the trap frequency in time along a certain trajectory, and a generalization to Bose Einstein condensates was worked out [60], but its extension to other potentials or systems may be difficult and remains an open question. By contrast, Muga et al. [79] found some difficulties to realize the transitionless-tracking Hamiltonian for the harmonic oscillator, but the transitionless-tracking method has the advantage of being, at least formally, more generally applicable. The feasibility of the actual realization is quite another

matter and has to be studied in each case. An example of application was provided in [1] for two-level systems.

Researchers are so much concern with a surge of interest to shorten the time it takes to change the state of a quantum system adiabatically. One of the studies on this area is related about the transient energy excitation in shortcuts to adiabaticity. Chen and Muga [70] dealt with this problem and studied the transient energy excitation in time-dependent quantum harmonic oscillators engineered so that the level populations at a final time are the same as the initial populations. Implications for the limits imposed to the process times and for the principle of unattainability of the absolute zero, in a single expansion or in quantum refrigerator cycles, were drawn. Simple processes in which the only external manipulation consists in shaping angular frequency $\omega(t)$ were considered. The populations of the instantaneous levels at intermediate times are, however, not preserved, so the transient excitation should be understood and possibly controlled. Bounds were obtained and examples can be found in [70]. In a realistic application, the oscillator will not be perfectly harmonic and it is natural to set some maximum value to the allowed excitation. Then the minimal time required for a fast expansion scales with the final frequency as $t_f \propto \omega_f^{-1/2}$, where t_f is the total time. The results presented in [70] provide strong support for the validity of this conjecture within the set of processes defined exclusively by time-dependent frequencies. It was also stated that these results call for further studies [70].

The discussion mentioned previously is limited to 1D but real traps are three-dimensional and at most effectively 1D. Torrontegui et al. [67] found out the theory and performed numerical simulations of fast expansions of cold atoms in a three-dimensional Gaussian-beam optical trap. Specifically, a simple physical realization based on an elongated cigar-shaped optical dipole trap with cylindrical symmetry was modeled. This trap is formed by a single laser which is red detuned with respect to an atomic transition to make the potential attractive, and is characterized in the harmonic approximation by longitudinal and radial frequencies. While magnetic traps allow for an independent control of longitudinal and radial frequencies [13], this is not the case for a simple laser trap

that therefore requires a special study. It was assumed that the time-dependence of the longitudinal frequency is engineered to avoid final excitations with a simple 1D harmonic theory and the final fidelity in the actual trap was analyzed. Even though for full 3D-results a purely numerical calculation was also performed. An understanding of the effects involved was achieved first by analyzing separately longitudinal and radial motions. Let us overview the model of the system given in [67]. The intensity profile of a Gaussian laser beam in the paraxial approximation is given by

$$I(r,z,t) = I_0(t)e^{-2r^2/\omega^2(z)}\frac{1}{1+z^2/z_R^2} \tag{3.32}$$

where r and z are the radial and longitudinal coordinates respectively, and the variation of the spot size ω with z is given by

$$\omega(z) = \omega_0\sqrt{1+\left(\frac{z}{z_R}\right)^2}\,, \tag{3.33}$$

where $z_R = \pi\omega_0^2/\lambda$ is the Rayleigh range, ω_0 is the waist, and λ is the laser wave length. Thus cold atoms in optical Gaussian beam traps are exposed to the potential of the form

$$V(r,z,t) = -V_0(t)e^{-2r^2/\omega^2(z)}\frac{1}{1+z^2/z_R^2} + V_0(t)\,, \tag{3.34}$$

where

$$V_0(t) = I_0(t)\hbar\Gamma^2/(8\delta\, I_{sat})\,. \tag{3.35}$$

Saturation intensity is given by $I_{sat} = \pi hc/(3\lambda^3\tau)$, and δ is the detuning, the inverse of the lifetime of the excited state is given by $\Gamma = \tau^{-1}$. In the following the

aim is to solve the time-dependent Schrödinger equation associated with the potential given in Eq. (3.34). The Schrödinger equation can be solved with standard separation of variables method in the cylindrical coordinates [67]. In [67], three different methods to avoid final motional excitation were compared: inverse engineering using Lewis-Riesenfeld invariant, which provides the best overall performance, a bang-bang approach, and a fast adiabatic approach. Transitionless expansion was analyzed, taking into account anharmonicities, radial-longitudinal couplings and the radial-longitudinal frequency mismatch. It was concluded that the transitionless expansions in optical traps are feasible under realistic conditions. Despite the relation between the longitudinal and transversal trapping frequencies through the intensity, the different timescales enable them to design fast expansions with high fidelities with respect to the ideal results using the invariant-based inverse engineering method, which is particularly suitable compared to the two other approaches examined. Detailed analysis of radial and longitudinal motions reveals the weakest points of each approach: for the inverse engineering, the main perturbation is due to the possible adiabaticity failure in the radial direction, which can be suppressed or mitigated by increasing the laser waist. This waist increase would also reduce smaller perturbing effects due to longitudinal anharmonicity or radial-longitudinal coupling. The simple bang-bang approach fails because the time for the radial expansion is badly mismatched with respect to the ideal time, and the fast adiabatic method fails at short times because of the adiabaticity failure in the longitudinal direction. In [67], complications such as perturbations due to different noise types, and consideration of condensates, gravity effects, or the transient realization of imaginary trap frequencies were left for separate works.

In the following, we will continue with the discussion of the possibility of realizing shortcut to adiabaticity in a harmonically trapped Bose-Einstein condensate using a scaling ansatz. One of the studies can be found in [25]. The conditions for quantum phase fluctuations to be negligible for shortcut to adiabaticity were discussed in that study. In [25] fast frictionless dynamics was exploited as a tool-box to manipulate and control low-dimensional quantum gases. Implementing a fast frictionless dynamics in a low-dimensional Bose-Einstein

condensate by engineering the time dependence of the transverse confining potential in a highly anisotropic trap a method was proposed in [25]. This method tunes the non-linearity of the effective low-dimensional (1D and 2D) dynamics of an anisotropic Bose-Einstein condensate by engineering the time-modulation of the transverse confinement. The purpose of that study is to find a 1D effective non-linear Schrödinger equation or Gross-Pitaevskii equation for a BEC in an elongated trap in which the transverse confinement is modulated in time. The corresponding potential used in [25] is similar with Eq. (3.19) and given as

$$V(r,t) = V(z) + \frac{m}{2}[\omega_x(t)^2 x^2 + \omega_y(t)^2 y^2], \tag{3.36}$$

where ω_x and ω_y are the frequencies along *x* and *y* direction, respectively. Here differently from Eq. (3.19) there is no trap due to gravity. The starting point is 3D time-dependent GPE given in Eq. (3.18) which governs the dynamics of the order parameter. It was stated that dimensional reduction of the 3D Gross-Pitaevskii equation is possible. After some algebra, the 3D Gross-Pitaevskii equation becomes a linear Schrödinger equation for the radial degrees of freedom and after dimensional reduction, the following effective 1D Gross-Pitaevskii equation with a time-dependent non-linearity is derived as

$$i\hbar \frac{\partial \psi(z,t)}{\partial t} = [H_z + g_{1D}(t)|\psi(z,t)|^2]\psi(z,t) \tag{3.37}$$

where

$$H_z = -\frac{\hbar^2}{2m}\frac{\partial^2}{\partial_z^{\,2}} + V(z) - \alpha(t), \tag{3.38}$$

and the term $\alpha(t)$

$$\alpha(t) = \sum_{j=x,y} \left\langle H_j(t) \right\rangle_0 = \sum_{j=x,y} \hbar(4\omega_0)^{-1}(\dot{b}_j^2 + \omega_j(t)^2 b_j^2 + \omega_0^2 / b_j^2) \qquad (3.39)$$

can be removed with a unitary transformation. Formerly, we have mentioned in detail about the scaling factor b=b (t) which is the solution of the Ermakov differential equation. As we have indicated before control of low dimensional quantum gases studied in [25], for that purpose inducing fast frictionless dynamics in quasi-1D BEC and quasi-2D BEC were investigated and finally the trajectories were found as

$$\omega_\perp^2(t) = \left(\frac{\omega_\perp(0)}{b_z}\right)^2 + \left(\frac{\dot{b}_z}{2b_z}\right)^2 - \frac{\ddot{b}_z}{2b_z}, \qquad (3.40)$$

and

$$\omega_x^2(t) = \omega_x^2(0)\left(\frac{g_{2D}}{g_{2D}(0)}\right)^4 + \frac{\ddot{g}_{2D}}{g_{2D}} - 2\left(\frac{\dot{g}_{2D}}{g_{2D}}\right)^2,$$

$$g_{2D}(t) = g_{2D}(0)/\sqrt{1+\omega_x^2(0)t^2} \qquad (3.41)$$

respectively. The trajectory given for the quasi-1D BEC induces a frictionless dynamics in the transverse direction, which modulates the three-dimensional density, and ultimately the effective axial non-linearity in the required way $g_{1D}(t) = g_{1D}(0)/b_z(t)$ for the axial dynamics to be self-similar. It was shown that a modulation of the transverse trapping frequency can be used to tune the axial effective coupling constant without the need to use a Feschbach resonance [25]. Briefly, a scheme to implement a fast frictionless dynamics of a low-dimensional Bose-Einstein condensate was presented, in which spurious excitations are avoided without the need to fulfill adiabaticity constraints. Exploiting the self-similar dynamics in the strongly confined degrees of freedom, it was shown that this can be achieved by engineering the modulation of the transverse confinement of the cloud in an elongated trap. As a result, it is possible to tune the amplitude of

non-linear interactions in these systems. The method to preserve short-range correlations in time of flight, assist shortcuts to adiabatic expansions in quasi-1D interacting BEC, and implement nearly sudden interaction quenches were also applied. More generally, it was discussed that inverting the equations associated with self-similar scaling laws allows to determine the trajectory of the control parameter for different processes, and constitute a powerful toolbox for the manipulation of ultracold atoms.

An important simplification occurs in the Thomas-Fermi regime, where the mean-field energy dominates over the kinetic part. In the Thomas-Fermi regime, it is possible to engineer a shortcut exactly, while keeping the coupling strength g_D constant [60]. Optimal control theory has been recently applied in this regime to find optimal protocols with a restriction on the allowed frequencies [80].

Studies mentioned up to now were focused on single-particle systems and a meanfield description of Bose-Einstein condensates. It has been obviously seen that the inversion of scaling laws is a powerful technique to design shortcut to adiabaticity in those processes where the dynamics is self-similar, e.g. expansions, or transport. In the following, we shall mention about the engineering of shortcut to adiabaticity in strongly correlated quantum fluids of relevance to ultracold gases experiments. For example, in [24] a method was proposed to spatially scale up a trapped ultracold gas while conserving the quantum correlations of the initial many-body state. A fairly general model in dimension D consisting of N indistinguishable particles with coordinates $x_i \in \mathbb{R}^D$, trapped in a time-dependent isotropic harmonic potential of frequency $\omega(t)$ and interacting with each other through a two-body potential $V(x_i - x_j)$ was considered. The many-body Hamiltonian describing this system reads [24]

$$H = \sum_{i=1}^{N}\left[-\frac{\hbar^2}{2m}\Delta_i + \frac{1}{2}m\omega^2(t)x_i^2\right] + \varepsilon\sum_{i<j}V(x_i - x_j) \qquad (3.42)$$

where Δ_i is the D-dimensional Laplacian operator for the x_i variable, and $\varepsilon = \varepsilon(t)$ is a dimensionless time-dependent coupling strength satisfying $\varepsilon(0) = 1$ (for more detail see [24]). In brief, the possibility of scaling up the system while preserving

quantum correlations constitutes a new type of microscopy of quantum correlations in quantum fluids was studied in [24, 26]. In [26], ultracold gas trapped in a box potential was studied. del Campo and Boshier [26] recently presented a method to drive an ultrafast dynamics in a time-dependent box trap which reproduces the adiabatic result at the end of the evolution. As illustrative examples, the ultrafast expansion of a Tonk-Girardeau gas and of BECs in different dimensions, where the method exhibits an excellent robustness against different regimes of interactions and the features of an experimentally realizable box potential were considered. The method is assisted by an auxiliary external harmonic potential which provides the speed-up. This method is also applicable to a large family of many-body systems supporting dynamical scaling laws, where it not only leads to a robust expansion of the density but also preserves the non-local correlation functions of the initial state, up to an expansion factor. This method is applicable to realistic box potentials and can be implemented in the laboratory with well-established technology. Its applications range over all scenarios requiring a shortcut to adiabaticity, i.e., probing strongly correlated phases, preventing decoherence, the effect of perturbations and atomic losses. It was stated that the method could be directly applied as well to ultrafast population-preserving cooling methods and quantum heat engines and refrigerators [26]. Finally, it was shown that shortcut to adiabaticity is robust in the presence of interactions and experimental imperfections.

The time-dependent frequency of a harmonic trap expansion based on invariants can be optimized with respect to time or to transient excitation energy, restricting the allowed transient frequencies [18, 23]. The efficient cooling of trapped atoms has an important place in modern quantum technology, since it has created the ultimate physical systems so far precision spectroscopy, frequency standards, and even tests of fundamental physics. In [23], a time-optimal control problem related to the frictionless cooling of atoms in a time-dependent harmonic potential was studied. It is well known that frictionless atom cooling in a harmonic trapping potential is defined as the problem of changing the harmonic frequency of the trap to some lower final value, while keeping the populations of the initial and final levels invariant, thus without generating friction and heating. Traditionally, an

adiabatic process is used where the frequency is changed slowly and the system follows the instantaneous eigenvalues and eigenstates of the time-dependent Hamiltonian. It has been mentioned before that the obstacle of this method is the necessary of longer times which may make it impractical. A way to bypass this problem is to use the theory of the time-dependent quantum harmonic oscillator [64] to prepare the same final states and energies as with the adiabatic process at a given final time, without necessarily following the instantaneous eigenstates at each moment. Achieving this goal in minimum time has many important potential applications. For example, it can be used to reach extremely low temperatures inaccessible by standard cooling techniques [84], to reduce the velocity dispersion and collisional shifts for spectroscopy and atomic clocks [85], and in adiabatic quantum computation [86]. It is also closely related to the problem of moving in minimum time a system between two thermal states [87]. It was initially proved that minimum transfer time for the previously mentioned problem can be achieved with “bang-bang” real frequency controls [87]. Later, it was shown that when the restriction for real frequencies is relaxed, allowing the trap to become an expulsive parabolic potential at some time intervals, shorter transfer times can be obtained, leading to a shortcut to adiabaticity [8]. In Stefanatos et al.’ s work [18], frictionless atom cooling was formulated as a minimum-time optimal control problem, permitting the frequency to take real and imaginary values in specified ranges. It was shown that the optimal solution has again a “bang-bang” form and this fact was used to obtain estimates of the minimum transfer times for various numbers of switching. In [23], solution of the corresponding time-optimal control problem was found and the optimal syntheses were obtained. A time-optimal control problem related to the frictionless cooling of atoms trapped in a time-dependent harmonic potential was studied. It was stated that the results presented in the article [23] can be immediately extended to the frictionless cooling of a two-dimensional Bose-Einstein condensate confined in a parabolic trapping potential [60] and even to the implementation of a quantum dynamical microscope, an engineered controlled expansion that allows to scale up an initial many-body state of an ultracold gas by a desired factor while preserving the quantum correlations of the initial state [24]. The techniques given above are not

restricted to atom cooling but are applicable to areas as diverse as adiabatic quantum computing and finite time thermodynamic processes.

Kosloff and coworkers [31] have applied optimal control theory to minimize the expansion time with frictionless conditions, i.e., taking an initial thermal equilibrium at one temperature into thermal equilibrium at another temperature in a cooling cycle, using real or imaginary bang-bang intermediate trap frequencies, see e.g. [31, 87].

Inverse engineering expansions using invariant theory or scaling laws have been applied in several contexts. For example, Choi et al. [16] discussed the possibility of achieving deep degeneracy of Fermi gases via sympathetic cooling by changing the trapping frequency of another species to keep constant the Lewis-Riesenfeld invariant. The defined advantages are the maximal heat capacity retained by the coolant due to the conservation of the number of atoms, and the preservation of its phase-space density in the nondegenerate regime. The limits of the approach are set by the transient excitation that should be kept below some allowed threshold, and by the spreading of the cooling cloud which reduces the spatial overlap with the Fermionic cloud. The method is found to be quite robust with respect to broadband noise in the trapping frequency [82]. The squeezing and robustness of frictionless cooling strategies studied in that study. Choi et al. have rephrased the cooling features in terms of a peculiar squeezing effect and used it to parameterize the robustness of frictionless cooling techniques with respect to noise-induced deviations from the ideal time-dependent trajectory for the trapping frequency [82]. Qualitative issues for the experimental implementation of this scheme were also discussed using bichromatic optical traps and lattices, which seem especially suitable for cooling Fermi-Bose mixtures and for investigating equilibration of negative temperature states, respectively. It was stated that a scheme using a single-frequency optical dipole trap with a continuously tunable laser is not feasible due to the large atom losses expected in crossing the dominant atomic transition from the red-detuned to the blue-detuned side to achieve antitrapping [82]. However, this issue may be circumvented through the use of two laser beams at constant frequencies, opposite detunings, and a variable power ratio, such as the bichromatic optical dipole traps proposed in [88] to allow optimal heat

capacity matching between Bose and Fermi gases [89, 90]. The presence of noise related to power fluctuations and beam-pointing stability for both laser beams in this configuration makes the discussion presented in [82] quite relevant for implementing frictionless cooling in bichromatic optical dipole traps. Bichromatic trapping has been recently implemented at the magneto-optical trap stage for a single species [91], and two-species selective trapping and cooling have been demonstrated with hybrid traps involving magnetic and optical confinement [92, 93]. Therefore experiments involving trapping and cooling of two species in a bichromatic trap should be within reach in the near-future. It was claimed that this study should also be relevant to the case of frictionless cooling in optical lattices, as recently discussed [17]. Dynamically variable spacing (the so-called accordion lattice) allows for a continuous increase in lattice periodicity, as experimentally demonstrated in [94-96], without the need to change the laser frequency. In that case an additional source of noise during the trap expansion is due to the presence of acousto-optic deflectors, adding up to the beam-pointing stability of the lasers. Implementing frictionless cooling with a negative square frequency stage via a bichromatic optical lattice could also be of great relevance to investigate fundamental issues of statistical mechanics as the approach to equilibrium in atomic systems at negative temperatures [97-99]. It was found that the robustness of the frictionless cooling method to noise can be analyzed by characterizing the final quantum state in terms of the amount of squeezing as well as a variant of the usual fidelity [82]. Another result is about the robustness, it was found that the method is quite robust to the presence of broadband noise in the trapping frequency, and further analysis involving monochromatic sinusoidal modulation has allowed us to resolve its response. The dependence of the squeezing and fidelity were studied on the deviation from the expected Ermakov trajectory and identified the role of measures such as the temporal density of states and characteristic angular frequency. Numerical simulations indicate that, despite the perception that a short t_f could mean less time for noise to interfere with the system, too short a t_f is best avoided in practice. Additively, a way to reduce the maximum temperature and hence atom losses was found by adding a high

frequency sinusoidal modulation, which helps to mitigate one of the limitations of this scheme.

In [22], a scheme to cool down a mechanical resonator in a three-mirror cavity optomechanical system was proposed. The dynamics of the mechanical resonator and cavities is reduced to that of a time-dependent harmonic oscillator, whose effective frequency can be controlled through the optical driving fields.

Most of the investigations so far have been based on ultracold atoms in a harmonic trap. Differently from the literature the case of frictionless cooling in optical lattices is firstly studied in [17]. It was shown that a correct choice of expansion trajectory allows us to get a final adiabatic state in a non-adiabatic way. The method of obtaining fast adiabatic transformation is to use a lattice with dynamically variable spacing (accordion lattice). The conditions for fast frictionless expansion for an accordion lattice were given. Such accordion lattices are useful since the final lattice spacing can be made large enough to be resolved in an experiment. It was assumed that the transverse motion of the atoms is frozen (i.e., a 1-D problem). The description of a condensate in 1-D is based on the Hamiltonian

$$H = \frac{p^2}{2m} + V(t)\cos\left(2k_L \frac{x}{\Lambda(t)}\right) + \frac{m\omega^2(t)}{2}x^2 \tag{3.43}$$

where m is the atomic mass, $V(t)$ is the lattice depth, k_L is the optical lattice wave number, $\omega(t)$ is the time-dependent angular frequency and $\Lambda(t)$ is the scale parameter describing the expansion of the lattice. To find the time evolution of the state for the Hamilton given above, a transformation on the wave function is introduced as

$$\Psi(x,t) = \exp\left(-igx^2/2 + \frac{\hbar}{2m}\int g dt\right)\Phi(x,t), \tag{3.44}$$

with a subsequent transformation on the coordinate

$$x = z\Lambda(t), \tag{3.45}$$

where $g(t)$ is a time-dependent function to be determined later. After these transformations let us choose $g(t)$ as

$$g(t) = -\frac{m}{\hbar}\frac{\dot{\Lambda}}{\Lambda}, \tag{3.46}$$

then the corresponding Hamiltonian takes form as

$$i\hbar\frac{\partial\Phi}{\partial t} = -\frac{\hbar^2}{2m\Lambda^2}\frac{\partial^2\Phi}{\partial z^2} + V\cos(2k_L z)\Phi + \frac{m}{2}\Omega^2 z^2\Phi \tag{3.47}$$

where Ω^2 is given by

$$\Omega^2(t) = \Lambda^2\omega^2 + \Lambda\frac{\partial^2\Lambda}{\partial t^2}. \tag{3.48}$$

The effects of lattice spacing are equivalent to the effective time-dependent mass and the effective parabolic potential in the stationary frame. The first term in the right-hand side of the above equation is due to the external one while the second one is induced due to the variation of the lattice spacing. In the following, the convenient trajectories will be investigated. In [17], it was found that the fast frictionless expansion can be achieved by engineering the potential depth as

$$V(t) = \frac{V_0}{\Lambda^2(t)}. \tag{3.49}$$

As can be obviously seen from the above equation, the potential depth decreases as the system expands. On the other hand, the time-dependent angular frequency was chosen as

$$\omega^2(t) = -\Lambda^{-1}\frac{\partial^2\Lambda}{\partial t^2}. \qquad (3.50)$$

The Eq. (3.47) is reduced with the help of Eq. (3.49) and (3.50), and the Schrödinger equation with constant potential depth is obtained. Finally, three different solutions of $\Lambda(t)$ were examined, and the corresponding external frequency ω^2 was plotted [17]. For a harmonically trapped system, the harmonic potential is needed to achieve the expansion of the system. However, expansion for an optical lattice can be achieved either by changing the wavelength of the laser or the relative angle between the two laser beams. The external parabolic potential is necessary for an optical lattice to cancel the induced parabolic potential, which pushes the atoms out of the condensate since it becomes expulsive for certain time intervals. Shortly, it was shown that the non-adiabatic expansion of an optical lattice in few milliseconds may lead to adiabatic final states provided that the expansion trajectory satisfies the boundary conditions and the external parabolic potential is applied to the system. It was claimed in [17] that this prediction can be tested through experiments similar to the one described in [96]. Since the non-adiabatic protocol preserves the initial state after expansion, it allows straightforward imaging of the optical lattice. The effect of additional forces and harmonic trapping was also discussed. It was also argued that fast frictionless expansion is possible even in the presence of nonlinear interaction provided that the scattering length is decreased proportionally to lattice scaling Λ^2 via Feshbach resonance [17].

Due to the need to control and implement quantum resources, there has been an increasing interest in developing fast protocols to shortcut adiabatic following [8, 9, 72, 77]. In [72], methods for fast production of highly coherent-spin-squeezed many-body states in bosonic Josephson junctions were described. The starting point is the known mapping of the two-site Bose-Hubbard Hamiltonian to that of a single effective particle evolving according to a Schrödinger-like equation in Fock space. Since, for repulsive interactions, the effective potential in Fock space is nearly parabolic, recently derived protocols for shortcuts to adiabatic evolution in harmonic potentials to the many-body Bose-Hubbard Hamiltonian were

extended in [72]. As we mentioned before, the key purpose to obtain the shortcuts to adiabaticity is to engineer procedures to drive, in a finite time, a system from some initial state to a final state that could be reached with an adiabatic, slow process. It should be noted that, in general, the initial and final states are not necessarily required to be the ground states of the system. In [72], however, in transitions between ground states corresponding to different values of the model parameters which can be controlled externally were concentrated. The proposed method was designed so that the desired final state is produced as a stationary eigenstate of the Hamiltonian, with no need to stop or freeze the dynamics. Analytical formulas to perform this type of processes exist for the harmonic oscillator [8]. It was shown that they can be extended to many-body states described by the Bose-Hubbard Hamiltonian. That allows to computing the dependence of the attainable coherent spin squeezing in the system on the number of atoms, *N*, the atom-atom interactions, *U*, and the linear coupling *J*. To do so, the explicit mapping was used, in the large *N* limit, between the two-site Bose-Hubbard model and an effective single particle system described by a Schrödinger-like equation. The dynamics of a bosonic Josephson junction can be well described by a quantized two-mode model [100]. For the external bosonic Josephson junctions case [100], or by construction in the internal bosonic Josephson junctions case, the system of ultra-cold bosons is well modeled by the Bose-Hubbard Hamiltonian

$$H_{BH} = -2J\ \hat{J}_x + U\ \hat{J}_z^2, \tag{3.51}$$

where the pseudo-angular momentum operator $\hat{J} \equiv \{\hat{J}_x, \hat{J}_y, \hat{J}_z\}$ is defined as

$$\begin{aligned} \hat{J}_x &= (1/2)(\hat{a}_1^\dagger \hat{a}_2 + \hat{a}_2^\dagger \hat{a}_1) \\ \hat{J}_y &= (1/2i)(\hat{a}_1^\dagger \hat{a}_2 - \hat{a}_2^\dagger \hat{a}_1) \\ \hat{J}_z &= (1/2)(\hat{a}_1^\dagger \hat{a}_1 - \hat{a}_2^\dagger \hat{a}_2), \end{aligned} \tag{3.52}$$

and $\hat{a}_j^\dagger$ creates a boson in site j, $[\hat{a}_i, \hat{a}_j^\dagger] = \delta_{i,j}$, J is the hopping strength, taken positive, and U is the non-linear coupling strength proportional to the atom-atom s-wave scattering lenhtg. In [72], repulsive interactions were considered, $U>0$. Time-dependent $U(t)$ was used, keeping J and N fixed during the time evolution. The method used in that article for fast adiabatic-like preparation of a given ground state requires control of the atom-atom interaction at time scales of fractions of the Rabi time. So we can conclude that protocols for fast generation of very coherent-spin-squeezed states in bosonic Josephson junctions were proposed [72]. The attained squeezing is the one corresponding to the ordinary adiabatic evolution in the case of repulsive atom-atom interactions, but requires much shorter preparation times. Thus, practical methods of fast-adiabatic driving have important advantages for any future implementation of bosonic Josephson junctions where they are used as resources of entanglement/spin squeezing. The best scaling of the squeezing parameter for large number of atoms N is $\xi_S^2 \approx 1/N$ [for more detail see 72]. Procedures, given in [72], require a good control of the time variation of the atom-atom scattering length during the desired period, from now on a possibility at hand in that experimental setups for internal bosonic Josephson junctions. In [72], the methods were obtained by extending recently developed protocols for fast-adiabatic evolution in the case of a single particle evolving in a time-dependent Harmonic oscillator potential, to the Bose-Hubbard Hamiltonian. It was claimed that if confirmed experimentally, the proposal presented in [72] represents a useful step forward in the fast-adiabatic preparation of many-body entangled quantum resources.

As we know, adiabatic methods are ubiquitous in cold atoms and atomic physics laboratories to control and prepare atomic states in a robust way. An explicit obstacle is that the times required may be too long for practical applications. Furthermore the ideal robustness may be damage by the accumulation of perturbations and decoherence due to noise and undesired interactions. Studies and experiments to speed up adiabatic processes have been realized for wave splitting [71], transport [9, 66], expansions and compressions [13, 14, 18, 8, 60, 79], or internal state control [1, 77]. These researches have been performed for Hermitian Hamiltonians, but many systems admit an effective non-Hermitian

definition. Shortcuts to adiabaticity techniques for non-Hermitian Hamiltonian systems were generalized and provided application examples in [11]. Specifically, the inverse engineering method proposed by Berry [1] and the one based on dynamical invariants [8] were generalized. A two-level decaying atom and the motion of a classical particle in a harmonic oscillator with time-dependent frequency were discussed. In order to understand the system, we will continue with the revision of the methods given in [11]. Fisrtly, the non-Hermitian Hamiltonian was defined. Non-Hermitian Hamiltonians typically describe sub-systems of a larger system. It was assumed that a non-Hermitian time-dependent Hamiltonian $H_0(t)$ with N non-degenerate right eigenstates $\{|n(t)\rangle\}$, $n = 1,2...,N$,

$$H_0(t)|n(t)\rangle = E_n(t)|n(t)\rangle, \tag{3.53}$$

and biorthogonal partners $\{|\hat{n}(t)\rangle\}$,

$$H_0{}^{\dagger}(t)|\hat{n}(t)\rangle = E_n{}^{*}(t)|\hat{n}(t)\rangle, \tag{3.54}$$

where the star means complex conjugate and the dagger denotes the adjoint operator. The time-dependent Schrödinger equations for a generic state $|\Psi(t)\rangle$ and for its biorthogonal partner $|\hat{\Psi}(t)\rangle$ satisfying $\langle\hat{\Psi}(t)|\Psi(t)\rangle = 1$ are

$$\begin{aligned} i\hbar\partial_t|\Psi(t)\rangle &= H_0(t)|\Psi(t)\rangle, \\ i\hbar\partial_t|\hat{\Psi}(t)\rangle &= H_0{}^{\dagger}(t)|\hat{\Psi}(t)\rangle. \end{aligned} \tag{3.55}$$

After obtaining the main equations, the methods mentioned above are generalized to the system. M. V. Berry proposed a method to design a Hermitian Hamiltonian H (t) for which the approximate adiabatic dynamics driven by the Hermitian Hamiltonian H_0 (t) becomes exact [1]. This method was generalized for non-Hermitian Hamiltonians, too. In this instance the adiabatic approximation when

H_0 (t) is non-Hermitian is needed [101]. Lewis and Riesenfeld [64] proposed the use of dynamical invariants of a quantum mechanical system to perform expansions of arbitrary time-dependent wave functions by superposition of eigenstates of the invariant. Inverse engineering techniques rely on designing the invariant eigenvectors and phase factors first, possibly taking into account partial information on the structure of the Hamiltonian, and then deducing the Hamiltonian from them. To understanding shortcut to adiabaticity for non-Hermitian Hamiltonian systems, the inverse engineering method proposed by Berry [1] and the one based on dynamical invariants [8] were used. While these methods are intimately connected as shown in [77] and may in fact be considered potentially equivalent, they are used in different ways in standard applications. These methods provide different answers so they are separately considered (for more detail see [11]).

There is a plenty of methods to preserve the initial state populations in a fast harmonic trap expansion. Some of them were examined and compared in the review articles given in [57, 69]. The design of the time dependence of the frequency using inverse techniques presents advantages over the slow adiabatic approach, band-bang methods, or the non-local transitionless tracking algorithm. The inverse-invariant method was compared with adiabatic and bang-bang techniques [69]. This method was also applied to Bose-Einstein condensates governed by the Gross-Pitaevskii (GP) equation. The other examined method is the transitionless tracking method [69]. It was shown that the inverse-invariant method is able to cool down atoms in a harmonic trap without phase-space compression as in a perfectly slow adiabatic expansion but in a much shorter time by a special design of the time dependence of the frequency. For very short total expansion times, this may require that the harmonic trap becomes temporarily an expulsive parabolic potential. There is a possibility to take a Bose-Einstein condensate in a very short time from an initial harmonic trap to a final one without excitations, by the same technique. In [57], different theoretical techniques proposed to engineer the shortcuts, the experimental results and the prospects were reviewed.

Transport

The efficient transport of atoms and ions by moving the confining trap is a necessary fundamental requirement for many applications. The transport should ideally be lossless and fast, i.e. the final state should be equal to the initial one apart from the translation and possibly phase factors. Many different experimental approaches have been implemented. Neutral atoms have been transported individually, as thermal atomic clouds, or condensates, using optical or magnetic traps. Usually, a way to avoid spilling or excitation of the atoms is to perform a sufficiently slow (adiabatic) transport. But for many applications the total processing time is limited due to decoherence and an adiabatic transport may turn out to be too long. As a part of quantum information processing, transport could occupy most of the operation time of realistic algorithms, so transport times need to be minimized. In short, we have significant reasons to reduce the transport time, and varied theoretical and experimental works have studied ways to make fast transport [7, 9, 12, 66, 78]. As well as expansions, shortcut techniques can be applied to perform fast atomic transport without final vibrational heating by combining dynamical invariants and inverse engineering. Two basics scenarios can be dealed in this way: first one is shortcuts for the transport of a harmonic trap and the other is shortcuts for the transport of an arbitrary trap.

In [78], optimal harmonic-trap trajectories were designed to transport cold atoms without final excitation, combining an inverse engineering technique based on Lewis-Riesenfeld invariants with optimal control theory. Optimal trajectories with bang-bang and bang-off-bang forms were respectively obtained for time minimization and displacement-minimization with constrained displacement between the trap center and the center of mass of the particle density. The transient energies for bounded and unbounded displacement were also minimized in [78]. In the time-optimal problem, the minimal time corresponds to a fixed constraint δ. Consistently with this, no solutions were found for displacement and energy minimization problems for transport times shorter than the minimal time, i.e. for $t_f < (2/\omega_0)\sqrt{d/\delta}$, where ω_0 is the constant harmonic frequency and *d* is the final position of the center of the harmonic trap. To achieve fast and

faithful transport in shorter times, an energy price must be paid by increasing δ which, in real traps, will also produce errors because of anharmonicities. The relation between the minimal (time-averaged) energy and the transport time t_f obtained in [78] is not at all trivial, in particular they are not simply inversely proportional, see e.g. Eqs. (56) or (70) in [78], as one might naively expect from the form of time-energy uncertainty relations. The scaling laws found are also peculiar of transport. In a previous work on invariants and transport [9], the energy bound for the time-averaged potential energy $\overline{E}_p$ was found using the Euler-Lagrange equation. In [78], it was shown that how to realize this bound by allowing the discontinuous acceleration of the trap at $t = 0$ and $t = t_f$ in the unbounded control optimization. As a principle these and other discontinuities found could be avoided by imposing appropriate bounds and using a powerful pseudospectral numerical optimization method [18] to address the corresponding more complex optimal control problem. Anharmonicity could be dealt with in a completely different way using the protocols for anharmonic transport described in [9], which require a compensation of inertial forces in the frame of the trap. It was suggested in [77] that, the results may be extended to Bose-Einstein condensates which can be found in [66].

Now let us overview the study of [66] which concentrate with the adiabatic transport of a Bose Einstein condensate. An inverse method to accelerate without final excitation of the Bose-Einstein condensate was proposed in [66]. The method, applicable to arbitrary potential traps, is based on a partial extension of the Lewis-Riesenfeld invariants, and provides transport protocols that satisfy exactly the no-excitation conditions without constraints or approximations. Inverse method, used in [66], was complemented by optimizing the trap trajectory with respect to different physical criteria and by studying the effect of noise. The effect of noise in harmonic transport was also investigated in [66]. It was assumed that the center of physical trap is randomly perturbated. For the shifted trap center, the equation for trajectory was solved (for more detail see Ref. [66]). The inverse method was also applied to anharmonic transport. For an arbitrary trap and a final time t_f =20ms, the maximal compensating acceleration found as 23.1m/s^2. The

results were given as a graphic and the average fidelity of harmonic transport versus noise amplitude was plotted in [66]. The inverse method can also be applied to anharmonic transport of condensates by means of a compensating force [9]. In either scheme this method does not require that t_f satisfies any discretization condition, as it occurs with other approaches [66], and t_f can in principle be made as small as desired. In practice there are of course technical and fundamental limitations[9]. Smaller values of t_f increase the distance from the condensate to the trap center, and the effect of anharmonicity.

Internal State engineering

Directing the internal state of a quantum system with time-dependent interacting fields is the basis of quantum information processing and many other fields. In those studies, we need an accurate control of pulse phase and intensity. Adiabatic passage is robust towards parameter variations but slow. It is moreover leaning to decoherence because of the effect of noise over the long times required. This motivates the investigation for fast and robust shortcuts, with respect to parameter variations and noise. Several methods to find shortcut to adiabaticity have been proponed for two- and three-level atomic systems. Among them, methods that we have mentioned previously above, like the transitionless driving, invariant-based engineering, or optimal control theory.

Adiabatic passage techniques in two-level and three-level atoms were also studied with different ways in [10, 68, 76]. A method to speed up adiabatic passage techniques in two-level and three-level atoms extending to the short-time domain their robustness with respect to parameter variations was proposed in [10]. The method used in [10], substitutes the standard laser beam setups with auxiliary pulses that steer the system along the adiabatic path. Compared to other strategies, such as composite pulses or the original adiabatic techniques, it provides a fast and robust approach to population control.

Manipulating the state of a quantum system with time-dependent interacting fields is a fundamental operation in atomic and molecular physics. Shortcut to adiabaticity in two-level systems can be also found making use of Lewis-

Riesenfeld invariants [27, 68]. For two-level systems there are several approaches proposed to attain a complete population transfer, for example, π pulses, composite pulses, adiabatic passage and its variants. A robust option is in principle adiabatic passage, which is however leaning to decoherence because of the effect of noise over the long times required. As we have mentioned before, a compromise is to use speeded-up shortcuts to adiabaticity, which may be obviously defined as the processes that lead to the same final populations than the adiabatic approach in a shorter time. In that sense, population inversion in two-level quantum systems has been recently studied in [68]. A key aspect to choose among the many possible shortcuts is their stability or robustness towards different perturbations. The stability versus different types of perturbations of recently proposed shortcuts to adiabaticity to speed up the population inversion of a two-level quantum system was examined [68]. Optimally robust processes using invariant based engineering of the Hamiltonian were found. The main aim of that study is to find optimal protocols with respect to systematic errors. The optimality was determined by minimizing properly defined sensitivities. It turns out that the perturbations due to noise and systematic errors require different optimal protocols, and invariant-based inverse engineering was used to find them. Hamiltonian for the two-level system is given as

$$H_0(t) = \frac{\hbar}{2}\begin{pmatrix} -\Delta(t) & \Omega_R(t) - i\Omega_I(t) \\ \Omega_R(t) + i\Omega_I(t) & \Delta(t) \end{pmatrix}, \tag{3.56}$$

where $\Omega(t) = \Omega_R(t) + i\Omega_I(t)$ is the complex Rabi frequency (where Ω_R and Ω_I are the real and imaginary parts, respectively) and Δ is the time-dependent detuning between laser and transition frequencies. Firstly, the definition of the Hamiltonian for two level system was given, and then different schemes to achieve a population inversion were reviewed. For example, a simple scheme to achieve population inversion is a π pulse. Secondly, different types of possible error sources were discussed. The stability of different fast protocols was examined for exciting a two-level system with respect to amplitude-noise error and systematic errors. First the noise error alone was studied and noise sensitivity

introduced. It was shown that a special type of π pulse is the optimal protocol with minimal noise sensitivity. Then the systematic error alone was studied and systematic error sensitivity was introduced. It was shown that there are protocols for which this sensitivity is exactly zero. Finally, the general case with noise and systematic errors together were investigated. And at the end of that study, it was also mentioned about future work. Future work will involve extending the present results to different types of noise and perturbations. The existence of a set of optimal solutions for systematic errors also opens the way to further optimization with respect to other variables of physical interest.

The transitionless driving for stimulated rapid adiabatic passage from level 1 to level 3 in a lambda configuration with an intermediate state 2 making use of a pumping and a Stokes laser was studied in [10]. The fast-driving field connects levels $|1\rangle$ and $|3\rangle$. This implies in general a weak magnetic dipole transition, which limits the ability of the field to shorten the times. Invariant- based engineering solves the problem by providing alternative shortcuts that do not couple directly levels $|1\rangle$ and $|3\rangle$ [76], as discussed below.

Our investigation continue with the study of Chen and Muga which is about fast and robust population transfers in three-level systems [76]. They have designed resonant laser pulses to perform fast population transfers in three-level systems by invariant-based inverse engineering. In a three-level system as the one depicted in Fig. 3.1, stimulated Raman adiabatic passage allows to transfer the population adiabatically from the initial state $|1\rangle$ to the target state $|3\rangle$. To speed up the process, a fast-driving counterdiabatic field connecting levels $|1\rangle$ and $|3\rangle$ may be used [10]. In general, though, this implies a weak magnetic dipole transition, which limits the ability of the counterdiabatic field to shorten the times [10]. This can be solved by invariant-based engineering, which provides alternative shortcuts without coupling directly levels $|1\rangle$ and $|3\rangle$. Let us overview the results of [76]. The laser intensities to improve the fidelity or to achieve a perfect transfer were examined for different protocols. Two different single-mode protocols were applied first in which the fidelity is linked to the laser intensity. Shortening the time also implies an energy cost. Interestingly, to achieve the same fidelity, it was

found that less intensity is required when the intermediate level $|2\rangle$ is populated. A variant of the first protocol in which the initial state is simply the bare state $|1\rangle$ and the dynamics is driven by a multi-mode wave-function provides a stable and less costly shortcut.

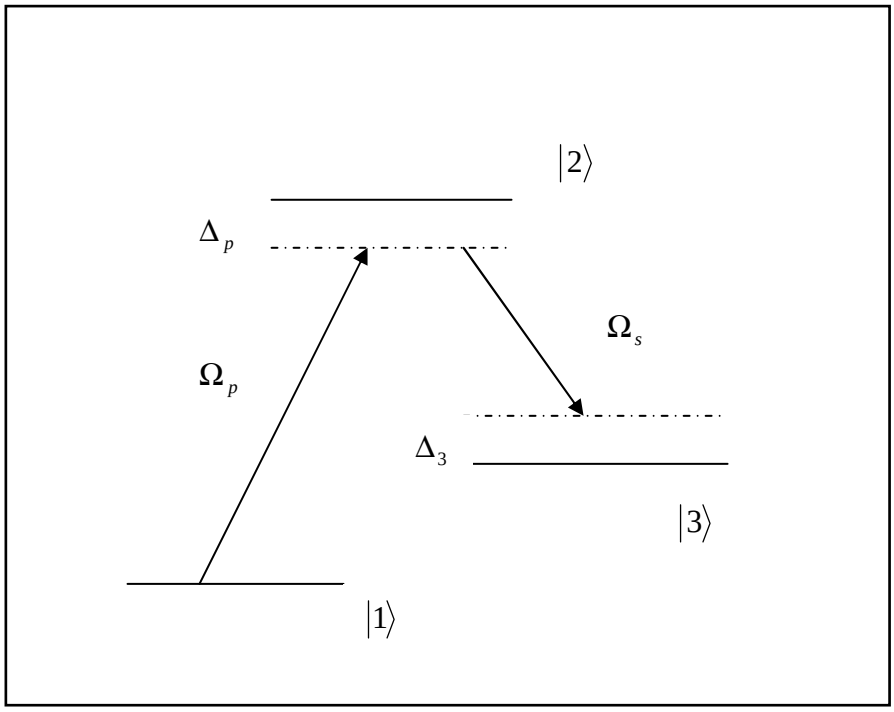

Fig. 3.1. Level scheme of stimulated Raman adiabatic passage for a Λ level configuration. Ω_p and Ω_s are the Rabi frequencies for the interactions with the pump and Stokes fields respectively, and Δ_p and Δ_3 are the detunings from the resonances.

In [76], it was stated that further exploration of the multi-mode approach in this and other systems is left for a separate study. The population of the intermediate level is usually problematic when its time decay scale is smaller than the process time. While this may be a serious drawback for an adiabatic slow process, it need not be for a fast shortcut. Protocols that populate level 2 may thus be considered as useful alternatives for sufficiently short process times. It is widely known that, different techniques to find shortcuts to adiabaticity are strongly related, or even equivalent. Lastly, the invariant-based inverse method presented in [76] was

compared to the optimal control approach used in [102]. In the optimal control method [102], the system of control differential equations are the same in the invariant method. The ultimate reason is that these equations are in fact equivalent to the Schrödinger equation for a given wave-function parameterization. The invariant dynamics provide thus a complementary understanding of the optimal control approach, whereas optimal-control techniques also help to optimize the results given by the invariant-based inverse engineering.

Now let us overview another interesting investigation of shortcuts to adiabaticity. Coherent spin manipulation in quantum dots is the key element in the state-of-the-art technology of spintronics. Ban et al. proposed another all-electrical technique to flip spin with high fidelity via shortcuts to adiabaticity, in a much shorter than any decoherence time [74]. A fast and robust method to flip electron spin in a quantum dot with spin-orbit coupling and weak perpendicular magnetic field was proposed. An invariant-based inverse engineering method was applied to control by time-dependent electric fields electron spin dynamics in a quantum dot with spin-orbit coupling in a weak magnetic field. The invariant-based inverse engineering approach was chosen, because Ban et al. claimed that it is better suited than the transitionless driving to be produced by the desired all-electrical means. Let us shortly give the definition of the model used in [74]. Consider the electric control of electron spin in a quantum dot formed in the *x*-*y* plane of a two-dimensional electron gas confined in the *z*-direction by the coordinate-dependent material composition, under a weak magnetic field $B_0 \| z$, as shown in [74]. The total Hamiltonian *H* of the electron interacting with the external electric field $E(t) = -\partial A/c\partial t$ is $H = H_0 + H_{so} + H_{\text{int}}$, with [103]

$$H_0 = \frac{p_x^2 + p_y^2}{2m} + U(x,y) + \frac{\Delta_z}{2}\sigma_z,$$

$$H_{so} = (-\alpha\sigma_y + \beta\sigma_z)p_x + \alpha\sigma_x p_y,$$

$$H_{\text{int}} = -\frac{e}{c}A(t)\bullet v, \qquad (3.57)$$

where m is the electron effective mass and σ_i ($i = x, y, z$) are the Pauli matrices. H_0 represents the kinetic energy, the potential $U(x, y)$, and the Zeeman $\Delta_z = g\mu_B B_0$, where μ_B is the Bohr magneton, and g is the Land´e factor. The eigenfunctions of H_0 are $\psi_j(x,y)|\sigma\rangle$, where $|\sigma\rangle = |\pm 1\rangle$ is the eigenstate of σ_z, and the spectrum is given by $E_j \pm \Delta_z / 2$, where E_j are the orbital eigenenergies in the confinement potential. The spin-orbit coupling is the sum of structure-related Rashba (α) and bulk-originated Dresselhaus (β) terms for growth axis. The vector potential $A(t)$ is in the (x, y)-plane and corresponding spin-dependent velocity operators are

$$v_x = p_x / m + \beta\sigma_z - \alpha\sigma_y,$$
$$v_y = p_y / m + \alpha\sigma_x. \qquad (3.58)$$

The designed electric fields provide a shortcut to adiabatic processes that flips the spin rapidly, thus avoiding decoherence effects. It was shown that this approach, being robust with respect to the device-dependent noise, can open new possibilities for the spin-based quantum information processing.

Multiple Schrödinger pictures and dynamics in shortcuts to adiabaticity were also discussed in [73]. Ibáñez et al., proposed schemes for which different interaction and Schrödinger picture dynamical equations represent different physical processes and interactions [73]. These schemes were later combined and exemplified to produce better, realizable shortcuts to adiabaticity for population inversion protocols, and for expansions and compressions. As it is widely known, the Schrödinger equation may be transformed by unitary operators into dynamical equations in different interaction pictures which share with it a common physical frame, i.e., the same underlying interactions, processes and dynamics. In contrast to this standard scenario, other relations are also possible, such as a common interaction-picture dynamical equation corresponding to several Schrödinger equations that represent different physics. This enabled to design alternative and feasible experimental routes for operations that are a priori difficult or impossible to perform. The power of this concept was exemplified by engineering Hamiltonians that improve the performance or make realizable several shortcuts

to adiabaticity. It was stated that, similar manipulations may be applied as well to facilitate or improve shortcuts to adiabaticity for other operations such as controlled atomic transport [9]. In fact, the idea of designing the pictures to generate alternative, easier to handle physics, is applicable to a fullness of quantum systems, especially, in the fields of quantum simulations, quantum control, or quantum information, where developing techniques to externally drive the systems for specific goals is a central objective.

While keeping on investigation of the studies on shortcuts to adiabaticity, we will examine the study of [77] which pointed out the connection between Lewis-Riesenfeld invariants and transitionless tracking algorithm. Different methods have been recently put forward and implemented experimentally to inverse engineer the time-dependent Hamiltonian of a quantum system and accelerate slow adiabatic processes via non-adiabatic shortcuts. In the transitionless tracking algorithm proposed by Berry, shortcuts Hamiltonians are designed so that the system follows exactly, in an arbitrarily short time, the approximate adiabatic path defined by a reference Hamiltonian. A different approach is based on designing first a Lewis-Riesenfeld invariant to carry the eigenstates of a Hamiltonian from specified initial to final configurations, again in an arbitrary time, and then constructing from the invariant the transient Hamiltonian connecting these boundary configurations. It was shown in [77] that the two approaches, apparently quite different in form and so far in results, are in fact strongly related and potentially equivalent, so that the inverse-engineering operations in one of them can be reinterpreted and understood in terms of the concepts and operations of the other one. As explicit examples the expansions of time-dependent harmonic traps and state preparation of two level systems were studied. In short, the study in [77] provides a significant step towards a deeper understanding of shortcut-to-adiabaticity methods that will help to choose the most adequate approach in atomic transport, quantum gates, and generally atomic manipulation and control applications.

Wavepacket Splitting

Splitting a wavefunction without excitation is important in matter wave interferometry. For linear waves, described by the Schrödinger equation, it is a peculiar operation, as adiabatic following is not robust but unstable with respect to a small external potential asymmetry [71]. The ground-state wavefunction collapses into the slightly lower well so that a very slow trap potential bifurcation fails to split the wave except for perfectly symmetrical potentials. A fast bifurcation with a rapidly growing separating potential succeeds to split the wave but at the price of a strong excitation. Shortcuts to adiabaticity that speed up the adiabatic process along a non-adiabatic route overcome these problems [71]. Numerical modeling shows that the wave splitting via shortcuts is significantly more stable than the adiabatic following with respect to asymmetric perturbations and avoids the final excitation. Specifically, Torrontegui et al. [71] use the streamlined version [75] of the fast-forward technique of Masuda and Nakamura [12], which applied to Gross-Pitaevskii or Schrödinger equations after having found some obstacles to apply the invariants-based method and the transitionless-driving algorithm. The following discussion refers to the Schrödinger equation except for a final comment on the Gross-Pitaevskii equation.

We will continue with the engineering fast and stable splitting of matter waves. Shortcut to adiabaticity is engineered to speed up the adiabatic process through non-adiabatic transients, provide instead quiet and robust fast splitting. Torrontegui et al. have been recently discussed the wave splitting via shortcuts to adiabaticity in [71]. The wave splitting via shortcuts avoids the final excitation and turns out to be significantly more stable than the adiabatic following with respect to the asymmetric perturbation. For that purpose, a simple inversion method was specifically used: a streamlined version of the fast-forward technique of Masuda and Nakamura [12] applied to Gross-Pitaievski or Schrödinger equations. Let us shortly mention about the fast-forward approach. The fast-forward method [12] may be used to generate external potentials to drive the matter wave from the initial single well to a final symmetric double well. The starting point of the streamlined version is the 3D time-dependent GP equation

$$i\hbar \frac{\partial |\psi(t)\rangle}{\partial t} = (T + G(t) + V(t))|\psi(t)\rangle, \qquad (3.59)$$

where T is the kinetic energy, V is the external potential and the G is the mean field potential. Here it is assumed that V is local, $\langle x|V(t)|x'\rangle = V(x,t)\delta(x-x')$. Based on this approach simple Y-shaped potentials have been generated. Simple Y-shaped (position and time dependent) potential trap bifurcations was designed to split matter waves rapidly without final excitation, avoiding the intrinsic instability of the adiabatic approach with respect to slight asymmetries. By the way, it also avoided or mitigated in this manner the decoherence effects that affect slow adiabatic following. The bifurcation may be experimentally implemented by means of spatial light modulators. It was stated that a simpler approximate approach would involve the combination of Gaussian beams. Further standard manipulations may be combined with the proposed technique, in particular a differential phase among the two final parts may be imprinted by illuminating one of them with a detuned laser.

Recently, there is a very much surge of interest to cut down the time it takes to change the state of a quantum system adiabatically. Up to now we have focused on theoretical studies on shortcut to adiabticity and the methods used in this area, in the following we will continue with experimental realization of shortcuts to adiabaticity. Researchers pay attention to the experimental studies [13-15, 20] as well as theoretical studies. In the following, we will overview the experimental realization of shortcuts to adiabaticity for a non-interacting and an interacting cold atoms.

3.3. Experimental realization of shortcuts to adiabaticity

In this subsection, firstly we will describe the experimental schemes and then investigate how the decompression is controlled and monitored.

Control of the trapping frequencies

In [81], the trap was well approximated by a 3D harmonic potential for sufficiently low temperatures. This temperature was approximately 100 μK for typical bias of 1.5 G. In the initial compressed trap, the frequencies were measured to be $\omega_x(0)/2\pi = 228.1\ Hz$, $\omega_y(0)/2\pi = 22.2\ Hz$, $\omega_z(0)/2\pi = 235.8\ Hz$.

Obtaining shortcuts to adiabaticity requires a precise control of the trapping frequencies, in a dynamical fashion. In [81] quadrupole-Ioffe-configuration trap (QUIC) was used, this can be achieved by varying the current i_Q running through the three coils, and the current i_{B_0} running through an additional pair of Helmholtz coils positioned along the axial dimension of the trap (compensation coils). The resulting potential is

$$U(x,y,z) = \mu|B| \cong \mu\left[B_0 + \frac{1}{2}\frac{B'^2}{B_0}(x^2+z^2) + \frac{1}{2}B''y^2\right], \qquad (3.60)$$

where $\mu/h = 1.4 MHz/G$ for atoms in $\left|5^2S_{1/2}, F=2, m_F=+2\right\rangle$, B' is the radial magnetic field gradient while B'' corresponds to its curvature along *y*. The radial and axial angular frequencies are recalled as,

$$\omega_\perp \equiv \omega_z \cong \omega_x \cong \sqrt{\frac{\mu}{m}}\frac{B'(i_Q)}{\sqrt{B_0(i_Q, i_{B_0})}},$$

$$\omega_{II} \equiv \omega_y = \sqrt{\frac{\mu}{m}}\sqrt{B''(i_Q)}\,. \qquad (3.61)$$

These expressions show that the radial and axial frequencies can be controlled independently to some extent. The experimental realization of the shortcut trajectories requires a careful preliminary calibration of the frequencies versus currents, which was achieved by monitoring the cloud's center-of-mass oscillations after a small excitation.

Shortcut to adiabaticity for a non-interacting gas

In order to produce an ultracold thermal cloud sufficiently dilute for collisions to be negligible, the loading time of second magneto-optical trap (MOT2) was reduced [81]. Then an evaporation ramp similar to the one used to obtain BECs was applied. This produced a dilute thermal gas, with a low elastic collision rate. It contained $N \cong 10^5$ atoms at a temperature of $T_0 = 1.6\ \mu K$. This corresponds to an average elastic collision rate per particle of $\gamma_{el} \cong 8\ Hz$, and a collision time of 125 ms. This is 30 times the oscillation period, and more than 3 times the decompression time, which justifies the non-interacting approximation. The three dimensions of the trap are thus not coupled and the system is equivalent to *N* simultaneous realizations of three independent harmonic oscillators.

In theory, starting from a gas at equilibrium and temperature T_0 in the compressed trap, a shortcut to adiabaticity should lead to an equilibrium state in the final trap, with a temperature $T_f = T_0 \omega(t_f)/\omega(0)$. This corresponds to a situation where entropy is not increased. On the contrary, for a non-optimal decompression, one expects to observe oscillations of the cloud's size and center of mass in the decompressed trap, once the decompression is completed. To evaluate the efficiency of shortcut, Schaff et al. [20] thus performed the fast decompression, and hold the cloud in the decompressed trap for a variable amount of time. The trap was then abruptly switched off, and an absorption image was taken after a constant time of free expansion (6 ms). The amplitude of the dipole (oscillation of the center of mass) and breathing modes (oscillation of the size) give access to the excess energy provided to the cloud, as compared to an adiabatic modification of the potential. If the cloud is reasonably at equilibrium after decompression, one can also directly measure the final temperature by measuring the evolution of the size during a free expansion (see [20] for detailed experimental results).

In the following, we will overview four decompression trajectories which were compared in [81]:

i-) The shortcut (given in Figs. 3.2d and 3.6 in [81]),

ii-) A linear decompression of the same duration (35 ms),

iii-) An abrupt decompression, which, somehow, corresponds to a worst case scenario (in practice, the decompression time is 0.1ms and $\omega(t)$ is not controlled, and is imposed by the response of the magnetic trap control electronics),

iv-) A 6-s-long linear decompression, which can be considered nearly adiabatic.

In [81], linear decompression was referred to condition corresponds to both control currents being varied linearly with time. The corresponding frequency trajectory is not linear.

In the case of the 6-s-long linear ramp, very little residual excitation was observed (although the residual dipole mode is still measurable), and the temperature directly measured by time of flight was found close to the expected value for an adiabatic decompression. More detailed experimental results were summarized on Fig. 3.9 in [81]. In the shortcut case, clear oscillations of the cloud width and center-of-mass position were seen, but they were much reduced compared to the fast linear ramp and abrupt decompression.

Compared to the linear decompression in 35 ms, the shortcut reduces the amplitude of the dipole mode by a factor of 7.2 and the amplitude of the breathing mode by a factor of 3. The excess energy, which was dominated by the center-of-mass energy, was thus reduced by a factor of ≈ 52. In the case of the 6-s-long ramp, a final temperature of the cloud of 130 nK was measured, a factor 12.5 below the initial one. This is consistent with the expected value of 15. The small difference may arise from a small heating rate due to the fluctuations of the magnetic trap.

The fact that the shortcut decompression still produces sizeable excitations is due to experimental imperfections. Several possible causes can be called. Since the shortcut trajectory was designed only for the radial dimensions, the resulting axial breathing mode is of the same magnitude as for the linear decompression.

The results of the shortcut decompression to linear ramps of various durations were compared. Fulfilling the adiabaticity criterion was found easier for the breathing mode (size oscillation) than for the dipole mode (center-of-mass

oscillation): the oscillation amplitude was reduced by a factor of 2 for t_f =20 ms for the earlier, and for $t_f \approx 150$ ms for the latter. Using the amplitude of the dipole mode as a criterion to compare the linear and shortcut schemes, one sees that the decompression time was reduced by a factor of 37.

Shortcut to adiabaticity for an interacting condensate

For best understanding the experimental procedure of the shortcut to adiabaticity for an interacting condensate, let us examine the experiment of [20]. In that experiment, Schaff et al. study with the scheme which the radial frequency was decreased by a factor of 9, while the axial frequency was adjusted to maintain the axial size of the BEC fixed during the whole trajectory. Accordingly, the axial frequency was decreased by a factor of 3. In [20], the starting point is an initial BEC containing 1.3×10^5 atoms in the condensed fraction, and 7×10^4 non-condensed atoms at a temperature of 130 nK. The experimental scheme is similar to that employed for the thermal cloud. A longer time of flight of 28 ms was used to characterize the various excitations generated by rapid decompressions. Three decompression schemes were compared in [81]:

i-) The shortcut to adiabaticity in 30 ms,

ii-) The linear decompression in 30 ms,

iii-) An abrupt decompression.

The BEC can not be held for more than 150 ms in the compressed magnetic trap because of a relatively high heating rate. Thus, comparison of the scheme given in [81] to the adiabatic limit corresponding to a slow linear decompression can not be done in this case. The absorption images were taken in the (y, z) plane, after a certain holding time in the decompressed trap plus a 28-ms-long time of flight. In the linear case the BEC undergoes large deformations and oscillations of its aspect ratio, whereas in the shortcut case it remains nearly perfectly stationary. Surprisingly, it was found that in the case of the linear decompression the BEC also oscillates angularly. This was attributed to an uncontrolled tilt of the trap axes during the decompression. It was shown that, the nearly isotropic aspect of the BEC after the shortcut decompression is due to the value of the time of flight,

which is close to the critical time of inversion of the aspect ratio. The thermal component surrounding the BEC was also visible. All measurements were performed after a 28 ms time of flight. As in the case of the non-interacting cloud, the shortcut scheme reduces the amplitude of the dipole mode compared to a standard linear decompression, in interacting condensate case by a factor of 4.3. For relatively long time of flight, the measured positions reflect the atomic velocities. Thus, using the shortcut scheme reduces the kinetic energy associated with the dipole mode by a factor of 18.5 compared to the linear one (and 36 compared to the abrupt decompression). The residual energy after the shortcut decompression is 580 nK. Both non-optimal schemes induce very large oscillations of the BEC's aspect ratio, with a rather complicated dynamics. A Fourier analysis reveals a main oscillation frequency of 47 Hz, consistent with a radial breathing mode at $2\omega_{\perp}$. A smaller contribution at 12.5 Hz corresponds to the axial breathing mode at $\sqrt{5/2}\omega_{\prime\prime}$. The shortcut scheme suppresses strikingly these breathing oscillations, yielding a BEC close to the targeted equilibrium state.

After comparing the decompression schemes we will continue with the results of the experiment in [20]. A fast decompression and displacement of both a non-interacting gas and an interacting Bose-Einstein condensate which were initially at equilibrium were especially demonstrated experimentally. The decompression parameters were engineered such that the final state is identical to that obtained after a perfectly adiabatic transformation despite the fact that the fast decompression is performed in the strongly non-adiabatic regime. During the transfer the atomic sample goes through strongly out-of-equilibrium states while the external confinement is modified until the system reaches the desired stationary state. The transfer was achieved by engineering specific trajectories of the external trapping frequencies. This scheme was successfully applied to both a thermal gas with negligible interactions and an interacting Bose-Einstein condensate. The scheme used is flexible enough to be adapted to both situations even though, in the thermal gas, interactions do not play a significant role while the Bose-Einstein condensate is strongly affected by the *s*-wave repulsion between atoms. Theoretically, the design of the transfer process was based on the invariant

of motion and scaling equations techniques which turned out to be possible thanks to the harmonic shape of the external potential violate the criteria for adiabaticity. In the scheme [20] the invariant of motion technique (for non-interacting particles) and the scaling equations technique (valid for both the non-interacting and the interacting gas) are tightly connected. The invariant of motion used in [20] is a time-independent harmonic oscillator Hamiltonian that can be obtained by a time-dependent canonical transformation of the initial Hamiltonian. In the scaling equations technique, a transformation involving both a scaling and a displacement of the coordinates that allowed the equations of motion of the system to be time-independent was investigated. In experiments with ultracold gases, samples are often prepared by transferring atoms from some confinement to another, e.g., from a magneto-optical trap to a magnetic quadrupole, from a quadrupole trap to a Ioffe-Pritchard trap, from a harmonic confinement to an optical lattice, etc. the major limitation being that, for short transfer times, parasitic excitations may show up. The main application of the scheme is thus to guide this transfer in order to prepare a very cold sample in a very short time with the desired geometry and without undesired excitations. It was stated that the shortcut-to-adiabaticity scheme proposed in [20] could be applied to non-interacting particles such as cold gases or ultracold spin-polarized fermions, to normal or superfluid (bosonic or fermionic as well) gases in the hydrodynamic regimes, and to strongly correlated systems such as the Tonks gas.

In order to understand the experimental procedure of the shortcuts to adiabaticity, above we prefer to examine the study of [20] which is reviewed form of [13, 14]. After detailed investigation of [20] which depicted the experimental realization of shortcuts to adiabaticity very well, we will continue with the other experimental articles studied in this area [13-15]. In [13], the first experimental realization of the faster-than-adiabatic displacement and cooling of an ensemble of magnetically trapped ultracold atoms were presented using an optimal decompression sequence based on invariants of motion. Using this formalism, optimal trap frequency trajectories were derived in the case of a time-dependent harmonic potential plus a time-independent linear term accounting for gravity. The solution also applies to the simpler case of a purely harmonic potential such

as that treated in Ref. [8]. The validity of the scheme by applying a fast (35ms) 15-fold frequency decompression to the trap in the vertical dimension was also demonstrated, yielding a residual center-of-mass oscillation of the cloud equivalent to that of 1.3-s-long linear decompression (a reduction by a factor of 37). It was claimed that as a future prospect, one could apply this technique to more isotropic traps (such as crossed dipole traps) to obtain a faster and efficient cooling in 3D and produce very low temperatures. Optimal trajectories could also be searched for in other situations such as the moving quadrupole magnetic traps often used to transport cold atoms [104]. This method can also be readily applied to Bose-Einstein condensates with some restrictions on the dimensionality due to the scaling of the interaction term [60]. More generally, these optimal faster-than-adiabatic schemes could be adapted to many areas of physics where time-dependent Hamiltonians are employed. The method, which was studied in [13], is experimentally applied to the fast decompression of an ultracold cloud of ^{87}Rb atoms held in a harmonic magnetic trap in the presence of gravity. Let us briefly mention about the experimental procedure used in [13]. To experimentally investigate the shortcuts to adiabaticity, a sample of ultracold ^{87}Rb atoms held in a magnetic Ioffe-Pritchard trap were employed. This popular type of trap is harmonic (for cold enough atoms) and anisotropic with a typical ratio of 10 between the oscillation frequencies in the radial dimensions $\omega_{x,z}$ and the axial one ω_y (see Fig. 3.2.) yielding the well known cigar-shaped aspect of the trapped cloud. For shallow traps, gravity significantly affects the potential in the vertical dimension, yielding a displacement of the trap minimum $-g/\omega_z^2$ compared to a tight trap.

The magnetic trap used in [13] is of the quadrupole-Ioffe-configuration type (QUIC trap) introduced in [105], the three-coils setup sketched in Fig. 3.2. For sufficiently cold atoms $(k_B T << \mu B_0)$ the magnetic potential is harmonic of the form [106],

$$\mu B = \mu\left[B_0 + \frac{1}{2}\left(\frac{B'^2}{B_0} - \frac{B''}{2} \right)(x^2 + z^2) + \frac{1}{2} B'' y^2 \right] \qquad (3.62)$$

where $\mu / h \approx 1.4 MHz / G$ for the atoms in $\left|F = 2, m_F = +2\right\rangle$. B' is the radial gradient of the magnetic field while B'' represents its curvature along *y*. B_0 is the minimum of the magnetic field at the trap center, which can be adjusted using two independent parameters: the current i_Q running in the three QUIC coils or the current i_{B_0} in a pair of compensation coils providing a uniform field along *y* (see Fig. 3.2).

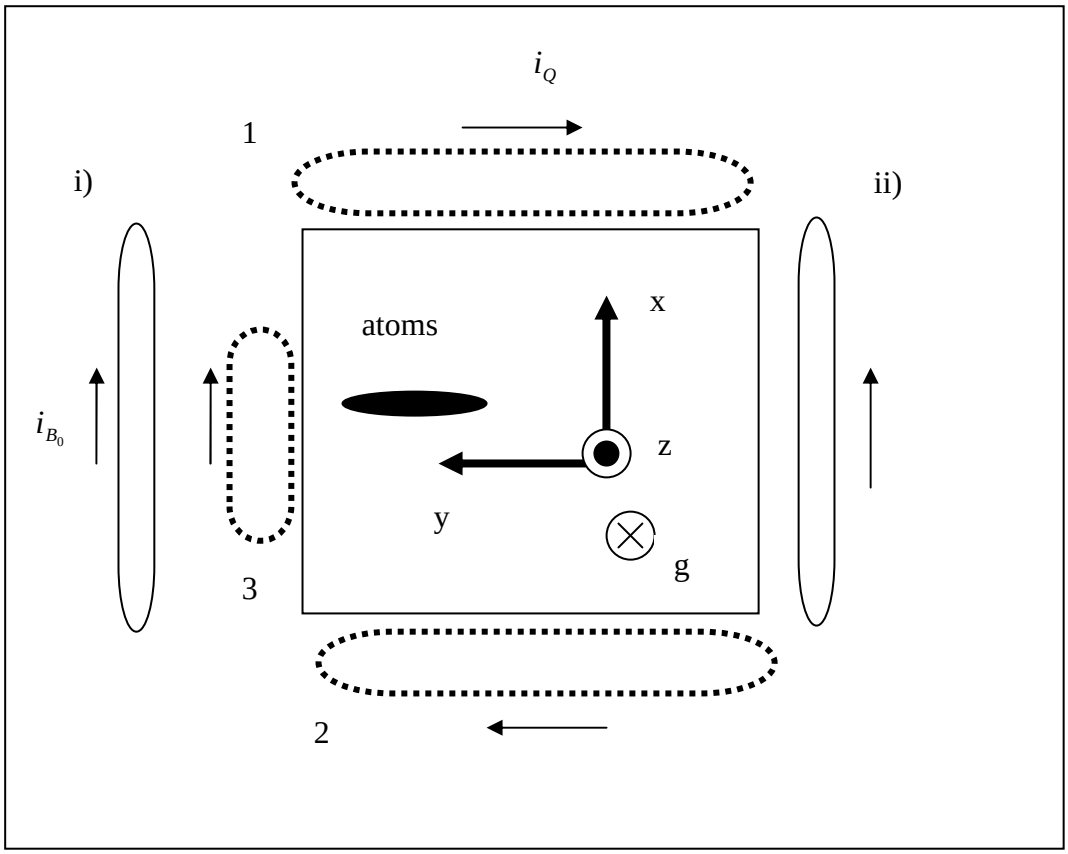

Fig. 3.2. Trapping geometry (figure in the horizontal plane). Ultracold ^{87}Rb atoms are trapped in an Ioffe-Pritchard-type magnetic trap created by current i_Q running through the three QUIC coils **1**, **2**, and **3**. An additional pair of coils (**i** and **ii**) produces a homogeneous field along *y*,which allows an independent tuning of the trapminimum field B_0 via the current i_{B_0}.

The initial sample was a small ($N = 10^5$ atoms) and cold ($T_0 = 1.63\ \mu$K) atomic cloud. The low temperature guarantees that the potential seen by the atoms remains harmonic even for large decompression factors. The small number of atoms was chosen to reduce the density and thus the elastic collision rate responsible for the energy transfer between dimensions and thermalization. In the compressed trap with the previously mentioned parameters, the typical time between two elastic collisions is ≈28 ms, quite larger than the radial oscillation period of 4 ms. It was illustrated that the efficiency of the shortcut method by realizing a fast (t_f =35ms) trap decompression optimized for the vertical dimension z, where gravity strongly affects the cloud's motion. The employed solution for the vertical trap frequency $\nu_z(t)$ was shown. Because of the finite time response of the trap electronic circuit, the measured trap field profile was found different from the computed one. Thus ν_z was monitored by interrupting the sequence at different times and adjusted the compensation field to obtain a measured $\nu_z(t)$ close to the theoretical one (deviation <5%). The time evolution of the cloud's center-of-mass position z_{cm} was also plotted once the decompression sequence was completed, and that of the cloud's size σ_z (for more detail see [13]).

Schaff et al. have another experimental study on shortcut to adiabaticity, in connection with an interacting Bose-Einstein condensate [14]. In that study, a method was presented to perform shortcut to adiabaticity transformations on a 3D interacting BEC, using a specifically designed parameter trajectory for the harmonic trapping potential [14]. A large trap decompression and displacement within a time comparable to the final radial trapping period were experimentally performed. By simultaneously monitoring the BEC and the non-condensed fraction, it was demonstrated that the specific trap trajectory is valid both for a quantum interacting many-body system and a classical ensemble of non-interacting particles. The rapid shortcut decompression of a 3D interacting BEC confined in an anisotropic harmonic trap was performed in [14]. The trap frequencies were decreased by a factor of 9 (radially) and 3 (axially) in a time comparable to the final radial trapping period, using a trajectory based on the scaling properties of the time-dependent Gross-Pitaevskii equation in the Thomas-

Fermi limit [61]. This shortcut trajectory leads to a final state identical (in theory) to the equilibrium state obtained via a perfectly adiabatic process. Experimentally, it was demonstrated that the collective excitations [107, 108] associated with the rapid trap decompression were strongly reduced by the shortcut scheme (see fig. 1 in [14]), the residual excitation being due to experimental imperfections. Additionally, it was showed that the trajectory is also valid for a classical ensemble of non-interacting particles, as demonstrated by monitoring the non-condensed fraction of the finite-temperature BEC. The system was described by a zero-temperature BEC plus a thermal cloud, assumed to behave independently. The ultracold ^{87}Rb atoms were trapped in a quadrupole-Ioffe-configuration magnetic trap [105]. In its compressed initial state this trap is anisotropic with radial and axial frequencies $\omega_{0_\perp}/2\pi$ =235.8 Hz and $\omega_{0_\parallel}/2\pi$ =22.2 Hz, respectively. In that experiment, firstly a BEC by RF (radio frequency) evaporation in the compressed trap was produced. The condensed fraction (N=1.3×10^5) represents 60% of the total number of atoms. The initial temperature, inferred from the size of the non-condensed fraction after time of flight, is T_0 =130 nK. Then a decompression sequence was applied, hold the ultracold cloud for a certain time t_h in the decompressed trap, then released it and monitored the cloud's parameters after a 28 ms time of flight via absorption imaging. This time of flight is close to the critical time (≈30 ms) where the aspect ratio of the decompressed BEC inverts, which explains its isotropic aspect. By varying t_h, the magnitude of the various modes excited by the decompression process were characterized. To extract quantitative estimates, the 2D column density profiles were fitted by a two-component distribution allowing for different angles for the BEC (Thomas-Fermi profile) and thermal fraction (Gaussian profile). The fit results were averaged over three different images taken in the same conditions. Throughout [14], three different decompression schemes were compared: an abrupt jump from the initial to final frequencies, a linear ramp of duration 30 ms, and the 30ms shortcut trajectory depicted. The abrupt decompression was used as a worst case to measure the magnitude of excitations associated with a strongly non-adiabatic transformation [14]. It was claimed that, the performances could be further improved using better-controlled potentials such as in optical traps or

lattices, where time-dependent manipulations are also easier and faster. Very short transition times could in principle be achieved by transiently applying negative (i.e. expelling) curvatures [8]. It was stated that the further work may include the direct comparison with other methods such as bang bang [87] or optimal control techniques. It was claimed that, more general shortcut solutions will also be searched for, and applied to other dimensionalities or non-harmonic potentials [109]. These fast-transition methods are not restricted to cold-atom manipulation, and can be readily adapted to topics as diverse as, e.g., macroscopic resonator cooling [22], temporal [10] and spatial [110] coherent population transfer, or quantum computation [111].

Completely controlling a quantum system is a basic requirement in quantum information processing and the coherent manipulation of molecular systems. The main goal in quantum control is to prepare a desired state with the highest fidelity allowed by the available resources and the experimental constraints. In [15], two optimal high-fidelity control protocols using a two-level quantum system comprising Bose–Einstein condensates in optical lattices was experimentally implemented. The first one is a shortcut protocol that reaches the maximum quantum-transformation speed compatible with the Heisenberg uncertainty principle. In the opposite limit, the recently proposed transitionless superadiabatic protocols were realized in which the system follows the instantaneous adiabatic ground state nearly perfect. It was demonstrated that superadiabatic protocols are extremely robust against control parameter variations, making them useful for practical applications. The evolution of a two-level system in a time *T*, as illustrated in Fig. 1a in [15], was investigated. Two states $|0\rangle$ and $|1\rangle$, the diabatic levels, are coupled via a time-dependent Landau-Zener Hamiltonian of the form

$$H = \Gamma(\tau)\sigma_z + \omega(\tau)\sigma_x \tag{3.63}$$

($\sigma_{z,x}$ being the Pauli operators with $\sigma_x|0\rangle = |1\rangle$) characterized by the instantaneous adiabatic levels of the system $|\psi(\tau)\rangle$, where ω is the coupling between the diabatic levels. The energy spectrum of the system for a constant ω

and a linear dependence of $\Gamma(\tau)$ on the rescaled time $\tau = t/T \in [0, 1]$ with $\Gamma(1) = -\Gamma(0)$ was shown in [15], which features an avoided crossing of the adiabatic levels $|\psi(\tau)\rangle$ at $\Gamma(\tau) = 0$ with an energy gap of 2ω. In the experiment, $\Gamma(\tau)$ and $\omega(\tau)$ can be controlled through the quasimomentum q and the depth V_0 of the optical lattice, respectively (for details see [15]). The system was initially prepared in the lowest energy band of the lattice with q=0 (corresponding to $|\psi_{initial}\rangle$), and the target state is to reach $|\psi_{final}\rangle$ after an evolution duration T.

The starting point is to consider a protocol with constant ω which, in principle, drives the system from $|\psi_{initial}\rangle$ to $|\psi_{final}\rangle$ with fidelity $F = |\langle\psi_{final}|\psi_{ground}(1)\rangle|^2 = 1$ in the shortest possible time T_{min}. By analogy with the equivalent classical case this kind of protocol has been called the "quantum brachistochrone". Imposing only the constraint that ω be constant (otherwise $T_{min} \to 0$ as $\omega \to \infty$), it was found that the protocol shown in Fig. 2c in [15] minimizes T. Different protocols were studied and fidelities for these protocols were also determined [15]. Finally, the speed of the superadiabatic tangent protocol with the composite pulse protocol as a function of ω', was compared. Where for the superadiabatic tangent protocol ω' is given as

$$\omega' = \omega\sqrt{1 + \frac{\arctan(\frac{2}{\omega})^2}{(T\omega)^2}}, \qquad (3.64)$$

and for the composite pulse protocol given as $\omega' = \omega$. Solving Eq. (3.63) for T gives a total time for the superadiabatic tangent protocol that depends on both ω and ω' (with $\omega' > \omega$). It is, therefore, possible to minimize T for a given ω' by choosing an appropriate value for ω. The detailed result of this minimization can be found in [15]. Let us summarize the results of the study given in [15]. It was shown that the superadiabatic transformations make it possible to readily implement protocols ensuring near-perfect adiabatic following in a variety of existing applications. In practice, of course, the choice of protocol will depend on

the boundary conditions and physical limitations of the system under consideration. It was stated that if both Γ and ω can be controlled, the superadiabatic protocols provide the possibility of state preparation with close to 100% fidelity, with high stability against parameter variations. That stability, in particular, should prove useful for improving existing adiabatic control protocols that already achieve high fidelities [112].

We have presented an overview of recent works on shorcuts to adiabaticity. Shortcuts to adiabaticity offer many promissing research and application avenues with practical and fundamental implications. In the next section, we will introduce our problem. In this thesis, we investigate fast frictionless expansion for growing Bose Einstein condensates described by the Gross-Pitaevskii gain equation [65]. Here we will use the method which is proposed to design the time-dependence of the trap frequency and achieve in a short time a frictionless evolution of Bose-Einstein condensates governed by the Gross-Pitaevskii equation [60].

4. SHORTCUTS TO ADIABATICITY FOR GROWING CONDENSATES

At first view, the fast adiabatic expansion in a short finite time looks like having the opposite meaning. However, in quantum mechanics, an adiabatic process is a slow process where the system follows at all times the instantaneous eigenvalues and eigenstates of the time-dependent Hamiltonian. This is in a sense maximally efficient as the populations do not change, i.e. there is no heating or friction, but the negative side is that the long times needed may make the process useless or even impossible to implement. Thus, a highly desirable goal is to prepare the same final states and energies of the adiabatic process in a given finite time t_f, without necessarily following the instantaneous eigenstates along the way. This has motivated researchers to find a way to speed up the process to reach the same final state as the state obtained by an adiabatic process. As we mentioned before, a new technique called fast frictionless process or shortcut to adiabaticity has been introduced [8, 10] and has attracted a lot of attention. In addition, fast frictionless expansion of harmonically trapped ultracold ^{87}Rb atoms was experimentally realized [13, 14].

Up to now, all studies on the subject of shortcut to adiabaticity have concentrate on condensates with fixed number of particles. So that, differently from the litterateur in this thesis, we will explore the answer of the following question: Is fast frictionless expansion possible for growing condensates? We will investigate the possibility of fast frictionless expansion for a condensate with variable number of atoms in this thesis. Several methods have been introduced theoretically to account for the growth of BEC [35–40]. In [40], an asymptotic analytic solution for the generic atom-laser system with gain in a *D*-dimensional trap is given and it is shown that this has a non-Thomas-Fermi behavior. The description of BEC growth [113-115] has become important for the physics of atom lasers [116]. These recently developed devices that emit coherent wavelike beams of atoms promise a new generation of precision measurements, applications in nanotechnology, and novel tests of fundamental concepts in quantum theory. In [40], an analytic asymptotic solution to the Gross-Pitaevskii equation describing the early stages of condensate growth in a trap is given. The

physical insight they obtain from this is that a growing non-equilibrium condensate has a non-uniform momentum distribution across the condensed region.

Let us roughly overview the Gross-Pitaevskii gain model [40]. We start by considering a commonly used model of a one-component trapped Bose-Einstein condensate—the Gross-Pitaevskii equation [48] modified by a linear gain term $\gamma(t)$ [117] , of the form

$$i\hbar\frac{\partial\psi(x,t)}{\partial t}=\left[i\hbar\gamma+\left(-\frac{\hbar^2}{2m}\nabla^2+V(x)+g(t)|\psi|^2\right)\right]\psi\,. \qquad (4.1)$$

Here $\psi(x,t)$ is the mean-field amplitude (so that $|\psi(x,t)|^2$ is the particle number density), *m* is the atomic mass, and *U* is the effective interaction potential. In the treatment of *D* equal to one, two, or three space dimensions, *g* is given by $g=4\pi\hbar^2a\Lambda^{3-D}/m$, where *a* is the scattering length and Λ is the confinement length. The potential term $V(\mathbf{x})$ is due to an optical or magnetic trap, which is assumed harmonic. In the simplest rotationally symmetric case, the trap potential is given by $V(x)=m\omega^2x^2/2$, where ω is the trap oscillation frequency. In [40], the field in terms of the amplitude and phase was expanded as, $\psi(x,t)=A(x,t)e^{-i\phi(x,t)}/\sqrt{g}$. And after some algebra, the coupled equations are obtained (for details see [40]). Next, the aim is to investigate the possible asymptotic solutions for long times, i.e., steadily growing solutions, valid some time after initial nucleation of the condensate, yet before any gain saturation has occurred. In brief, they have found an asymptotic solution to the Gross-Pitaevskii equation with gain, which has the advantage of yielding an explicit analytic result of great physical transparency. The solution shows that the nonequilibrium behavior of a growing Bose-Einstein condensate generally includes an outward momentum component and spatial oscillations.

The condensate growth by particle injection was experimentally realized with ^{23}Na [32], ^{7}Li [33] and ^{87}Rb [34]. By continuous evaporative cooling, Köhl et al. [34] directly control the thermal cloud from which the condensate grows. They

compare the experimental data with the results of a theoretical model based on quantum kinetic theory. Quantitative agreement with theory for the situation of strong cooling is found, whereas in the weak cooling regime a distinctly different behavior is found in the experiment.

In the following, we will use Gross-Pitaevskii gain equation to study fast frictionless expansion for both harmonically trapped condensate and an optical lattice. We will show that a correct choice of expansion trajectory allows us to get a final adiabatic state in a non-adiabatic way for the growing condensate

Gross-Pitaevskii Gain Equation

The Gross-Pitaevskii equation, also called the non-linear Schrödinger equation, describes zero-temperature Bose-Einstein condensates for which the scattering length between atoms, *a*, is smaller than the spacing between atoms. It defines the ground state of a quantum system of identical bosons using the Hartree–Fock approximation and the pseudopotential interaction model. It takes into account the trapping and the interactions in the atomic gas and also allows the study of the relationship between BEC and superfluidity. As well as the mean-field dynamics of the BEC can also be well described by the Gross-Pitaevskii equation. To account for the mechanism of loading atoms into the BEC by optically pumping them from the external cold source, a linear gain/loss term is phenomenologically added to the GP equation. In other words, the correction of ground state energy of one-dimensional Gross-Pitaevskii equation is done by adding a gain-loss term as a time-dependent external potential. The interesting purpose of this term is that it can be used to explain the experimental results. The GP equation with the complex linear gain or loss term, often called as GP gain equation, was used to model the dynamics of growing BEC. In this thesis, we start by considering GP equation modified by a complex term. The reduced 1-D GP gain equation for cigar-shaped traps reads

$$i\hbar\frac{\partial\Psi}{\partial t}=\left(-\frac{\hbar^2}{2m}\frac{\partial^2}{\partial x^2}+V(x,t)+g(t)\left|\Psi\right|^2+i\hbar\gamma(t)\right)\Psi \qquad (4.2)$$

where m is the atomic mass, $V(x, t)$ is the trap potential, $g(t)$ is the nonlinear interaction strength and $\gamma(t)$ is the gain/loss coefficient. The interaction strength can be varied by tuning of the s-wave scattering length due to the Feshbach resonance. With this experimental degree of freedom, it is possible to study the dynamics of BEC with variable particle number. The complex linear term with positive $\gamma(t)$ accounts for particle pumping. For the negative values of $\gamma(t)$, the above equation describes a BEC that is continuously depleted by loss. Note that the number of particles changes according to

$$N = N_0 e^{\int 2\gamma(t')dt'} \tag{4.3}$$

where N_0 is the initial number of particles. It is very difficult to obtain closed form solution to the GP gain Eq. (4.2). However, we can obtain conditions for fast frictionless expansion without getting the exact analytic solution. Suppose the system is initially in the ground state. To find the time evolution of the state, we introduce a transformation on the wave function as

$$\Psi(x,t) = e^{iA(t)x^2 + B(t)}\Phi(x,t) \tag{4.4}$$

where the unknown time-dependent functions $A(t)$ and $B(t)$ will be defined below. Adding particles into the system and time-dependent potential cause the condensate to expand. Hence, let us now introduce a scale transformation on the coordinate

$$q = \frac{x}{L(t)}, \tag{4.5}$$

where the time-dependent dimensionless function $L(t)$ accounts for the expansion of the condensate. Substituting Eq. (4.4) into Eq. (4.2) and using the scaling giving in Eq. (4.5) and redefined wavefunction $\Phi(q,t) = \Phi(x,t)$, we get

$$i\hbar\frac{\partial\Phi}{\partial t} = -\frac{\hbar^2}{2m}\frac{1}{L^2}\frac{\partial^2\Phi}{\partial q^2} + \left[\hbar\dot{A} + 2\frac{\hbar^2}{m}A^2\right]L^2q^2\Phi + g(t)e^{2B}|\Phi|^2\Phi$$
$$+ V\Phi + \left[-i\hbar\dot{B} + i\hbar\gamma - i\frac{\hbar^2}{m}A\right]\Phi + \left[i\frac{\dot{L}}{L} - 2i\frac{\hbar}{m}A\right]\hbar q\frac{\partial\Phi}{\partial q} \qquad (4.6)$$

where the dot means derivative with respect to time. Let us now impose that the coefficients in square brackets $[\cdots]$ of the last two terms vanish. Hence

$$A(t) = \frac{m}{2\hbar}\frac{\dot{L}}{L},$$
$$B(t) = \int\gamma(t')dt' - \frac{1}{2}\ln L \qquad (4.7)$$

$L(t)$ is to be defined below. The functional form of $L(t)$ depends on the time dependent parameters in the Hamiltonian. Here we are interested in some special forms of $L(t)$. To avoid vibrational excitation at the final time, we demand that some conditions on the scale function $L(t)$ are imposed. Suppose first that $A(t)$ in (4.4) vanishes at both initial and final times, $A(t = 0) = A(t = t_f) = 0$. Furthermore the acceleration, $\ddot{L}$, is set to zero at initial and final times. Hence, the conditions on $L(t)$ read

$$\dot{L}(0) = \dot{L}(t_f) = 0,$$
$$\ddot{L}(0) = \ddot{L}(t_f) = 0. \qquad (4.8)$$

There are infinitely many different solutions to (4.8). Here we are interested in two of them [1, 10]. The first one is given by

$$L_1(t) = 1 + (\kappa - 1)\left(10\frac{t^3}{t_f^3} - 15\frac{t^4}{t_f^4} + 6\frac{t^5}{t_f^5}\right) \qquad (4.9)$$

and the second one is given by

$$L_2(t) = 1 + (\kappa - 1)\left(\frac{t}{t_f} - \frac{1}{2n\pi} \sin(\frac{2n\pi\, t}{t_f}) \right) \tag{4.10}$$

where n is an an integer and the constant κ is the ratio of final spacing to the initial spacing. Although eliminating the phase-factor e^{iAx^2} at initial and final times from (4.4) is necessary, it is not sufficient for frictionless expansion. To get the sufficient conditions, let us substitute the Eq. (4.7) into Eq. (4.6). Then we get the following time-dependent equation,

$$i\hbar \frac{\partial \Phi}{\partial t} = -\frac{\hbar^2}{2mL^2} \frac{\partial^2 \Phi}{\partial q^2} + V(q,t)\Phi + \frac{mL\ddot{L}}{2} q^2 \Phi + g \frac{e^{\Gamma}}{L} |\Phi|^2 \Phi\,, \tag{4.11}$$

where we express the resulting wave equation in terms of a new scaled time,

$$\Gamma(t) = \int 2\gamma(t')dt'\,. \tag{4.12}$$

The transformations (4.4) and (4.5) allow us to understand the dynamics of the expanding system. The first term in Eq. (4.11) is the kinetic energy term and it changes with $1/L^2$. This is reasonable since kinetic energy scales as the inverse square of length. The second term is the trap potential in scaled coordinate. The next term tells us that variation of the lattice spacing induces an effective harmonic potential. Note that the induced effective potential vanishes at both initial and final times. Finally, it can be seen from the last term that the effective non-linear interaction strength changes with the gain/loss term γ and L. This is simply because the number of particles in the system increases with e^{Γ} while the density decreases with $1/L$ as the system expands.

After obtaining the general formalism, in the following subsections let us apply it specifically to harmonically trapped condensate and optical lattice. For this purpose we will use the Gross-Pitaevskii gain equation and we will show that

a correct choice of expansion trajectory allows us to obtain a final adiabatic state in a non-adiabatic way for the growing condensate.

4.1. Fast Frictionless expansion for the harmonic trap

In this subsection, we will look for the correct choice of expansion trajectory to get a final adiabatic state in a non-adiabatic way for the harmonically trap growing condensate.

Suppose first that the atoms are trapped by harmonic potential,

$$V(x,t) = \frac{m\omega^2(t)}{2}x^2, \tag{4.13}$$

where $\omega(t)$ is the time-dependent angular frequency.

In this case Eq. (4.11) becomes,

$$i\hbar\frac{\partial\Phi}{\partial t} = -\frac{\hbar^2}{2mL^2}\frac{\partial^2\Phi}{\partial q^2} + \frac{m(\omega^2L^2 + L\ddot{L})}{2}q^2\Phi + g\frac{e^{\Gamma}}{L}|\Phi|^2\Phi. \tag{4.14}$$

The harmonic potential is composed of external and induced ones. Now our aim is to find out the conditions for the fast transitionless expansion of the harmonically trapped condensate. Below we will consider two cases. First one is Thomas-Fermi approximation and the other is the exact treatment of the system. The latter case will be studied with no approximations, we will look for the conditions for the fast frictionless process of the harmonically trapped condensate without omitting the kinetic energy.

a-) Thomas-Fermi (TF) Approximation

The Eq. (4.14) can be solved analytically if we assume that the zero-point kinetic energy associated with the density variation becomes negligible in comparison to both the trap energy and the nonlinear interaction energy. If we use the TF

approximation, in which the kinetic energy operator is neglected in the equation (4.14), we obtain

$$i\hbar\frac{\partial\Phi}{\partial t}=\frac{m}{2}(\omega^2L^2+L\ddot{L})q^2\Phi+g\frac{e^{\Gamma}}{L}|\Phi|^2\Phi \tag{4.15}$$

Suppose that the following relation is satisfied by the control parameters $\omega(t)$, $g(t)$ and $\gamma(t)$

$$\omega^2L^2+L\ddot{L}=\omega_0^2\frac{g}{g_0}\frac{e^{\Gamma}}{L}, \tag{4.16}$$

where the constants ω_0 and g_0 are the initial trap frequency and interaction strength. Let us substitute (4.16) into (4.15). Then we get

$$i\hbar\frac{\partial\Phi(q,\tau)}{\partial\tau}=\left(\frac{m\omega_0^2}{2}q^2+g_0|\Phi(q,\tau)|^2\right)\Phi(q,\tau). \tag{4.17}$$

This equation is expressed in terms of a new scaled time,

$$\tau=\int\frac{ge^{\Gamma}}{g_0L}dt'. \tag{4.18}$$

Now we are left with the equation which has time-independent coefficients. We conclude that undesired excitations can be avoided if the trajectories of the control parameters, $\omega(t)$, $g(t)$ and $\gamma(t)$, are designed in such a way that the Eq. (4.16) holds. It is interesting to observe that fast frictionless expansion is in principle possible even if both the trap frequency and interaction strength are constants, $\omega(t)=\omega_0$, $g(t)=g_0$. In that case, the trajectory for $\gamma(t)$ must be designed according to the Eq. (4.16). In other words, the condensate can be expanded by adding particles to the system while the trap frequency and interaction strength are

left unchanged. Changing the gain parameter $\gamma(t)$ is not the only way to achieve frictionless expansion. One can also either change $\omega(t)$ or $g(t)$.

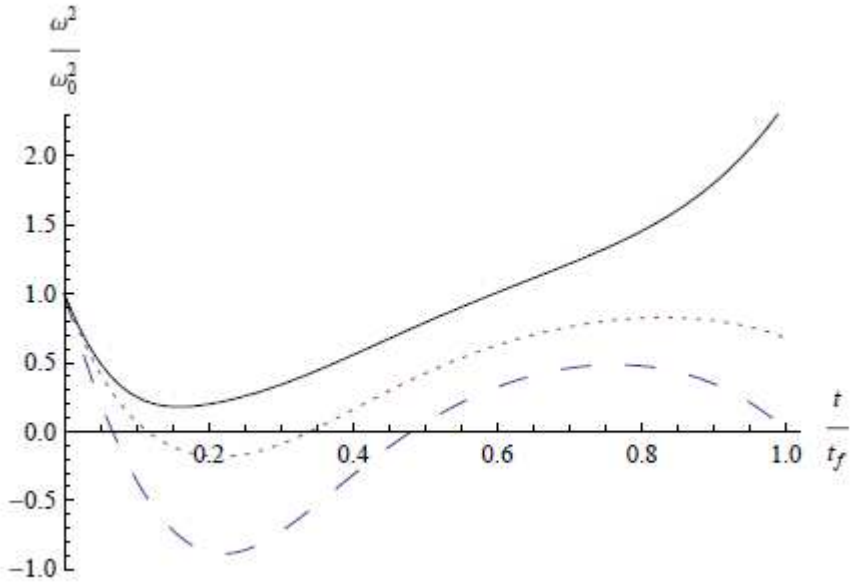

Fig. 4.1. $\omega^2(t)/\omega_0^2$ versus t/t_f for $L_1(t)$ with $\kappa = 4$ in the TF regime. The solid, dotted and dashed curves are for $\gamma = 0.4\omega_0$, $\gamma = 0.3\omega_0$ $\gamma = 0$, respectively.

In Fig.4.1, we plot the trajectory of $\omega^2(t)/\omega_0^2$ for $L_1(t)$, where $\omega_0 = 2\pi 100Hz$ and $g(t) = g_0$. The condensate is assumed to be decompressed by a factor of 4 in 5 milliseconds. This time is too short for an adiabatical expansion. However, the designed trajectory leads to the same final state, up to a global phase, as the state obtained by a slow adiabatic process for such an expansion. In the figure, the solid, dotted and dashed curves are for $\gamma = 0.4\omega_0$, $\gamma = 0.3\omega_0$ and $\gamma = 0$, respectively. For the system with constant particle number, $\gamma = 0$, ω^2 takes negative values on some interval and one may conclude that this accelerates the spreading. As can be seen from the solid curve, frictionless expansion is possible even if ω^2 is positive all the time. It is also surprising to see that the final trap frequency is bigger than the initial one for the solid curve, $\omega_f > \omega_0$. By inspecting the Eq. (4.16) it can be seen that the final and initial trap frequencies are equal to each other, $\omega_f = \omega_0$, at a critical value $\gamma_c = 3\ln(\kappa)$. This can be explained as

follows. The extension of the cloud in the TF regime depends strongly on the interaction strength. Adding particles into the system increase the effective interaction strength. This leads to the expansion of the condensate. For $\gamma > \gamma_c$, the ratio of final spacing to the initial one due to the nonlinear interaction is bigger than κ. So, ω must be increased to compensate the change.

Having studied the conditions, let us now get the analytical solution in the TF regime. It is given by,

$$\Phi(q,t) = e^{-i\mu\tau/\hbar}\Phi(q) \tag{4.19}$$

where $\Phi(q) = \left(\frac{1}{g_0}(\mu - \frac{m\omega_0^2}{2}q^2)\right)^{1/2}$ and μ is a constant. Note that μ does not coincide in general with the chemical potential since the parameters are time dependent for the original Hamiltonian. The particle density in terms of the original variables is given by

$$|\Psi(x,t)|^2 = \frac{e^{\Gamma}}{g_0 L}(\mu - \frac{m\omega_0^2}{2}\frac{x^2}{L^2}). \tag{4.20}$$

The normalization condition yields a relation between the constant μ and the total number of particles N_0,

$$\mu = (\frac{9}{32}m\omega_0^2 g_0^2 N_0^2)^{1/3}. \tag{4.21}$$

So far we have only investigated the problem in one dimension. It is straightforward to generalize our formalism to two and three dimensional traps with trap frequencies $\omega_i(t)$, where i = 1, 2, 3. Then the conditions (4.16) are modified

$$\omega_i^2 L^2 + L\ddot{L} = \omega_{0,i}^2 \frac{g}{g_0} \frac{e^{\Gamma}}{L^D}, \tag{4.22}$$

where g is the corresponding interaction strength, $\omega_{0,i}$ are constant angular frequencies in each directions and D is the dimension of the space.

b-) Exact Treatment

In this subsection, we will look for the conditions for fast frictionless process of the harmonically trapped condensate without omitting the kinetic energy term.

It is easy to see that fast transitionless expansion is possible if parabolic potential and nonlinear interaction terms in Eq. (4.14) scale as $1/L^2$,

$$\omega^2 L^2 + L\ddot{L} = \frac{\omega_0^2}{L^2},$$

$$g \frac{e^{\Gamma}}{L} = \frac{g_0}{L^2}, \tag{4.23}$$

where ω_0 and g_0 are constant.

Substitute these conditions into Eq. (4.14) and use the new scaled time,

$$\tau = \int L^{-2}(t')dt'. \tag{4.24}$$

Then we obtain,

$$i\hbar \frac{\partial \Phi}{\partial \tau} = \left(-\frac{\hbar^2}{2m} \frac{\partial^2}{\partial q^2} + \frac{m\omega_0^2}{2} q^2 + g_0 |\Phi|^2 \right) \Phi. \tag{4.25}$$

We are now left with a new time-independent Hamiltonian,

$$H' = \frac{p^2}{2m} + \frac{m\omega_0^2}{2} q^2 + g_0 |\Phi|^2 . \tag{4.26}$$

The final state of H' at t_f can be made identical up to a global phase factor to the final state of the adiabatic evolution with

$$H = \frac{p^2}{2m} + \frac{m\omega^2(t)}{2} q^2 + g(t)|\Psi|^2 + i\hbar\gamma(t) . \tag{4.27}$$

Note that at intermediate times the solution does not coincide with the instantaneous eigenstates.

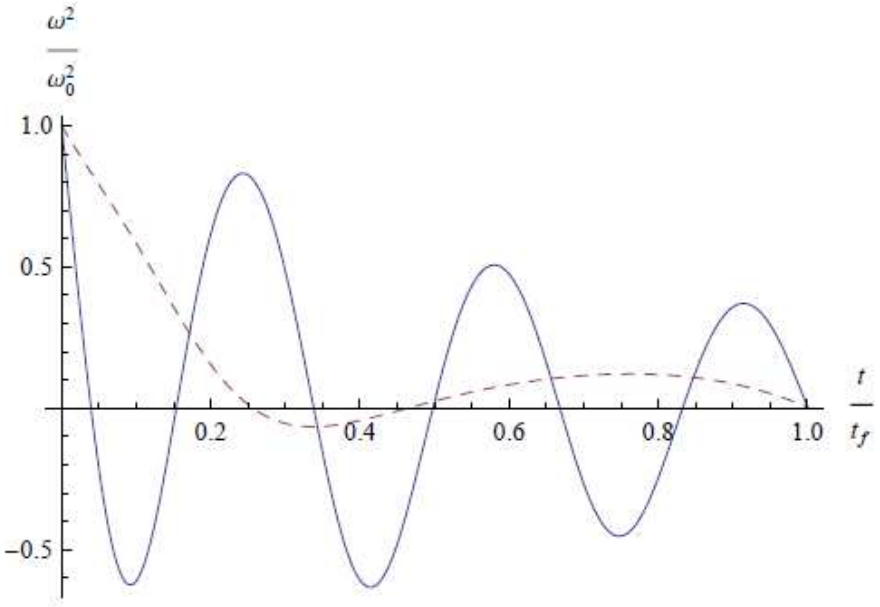

Fig. 4.2. $\omega^2(t)/\omega_0^2$ versus t/t_f. The system is decompressed by a factor of 4. The dashed (solid) curve is for $L_1(t)$ ($L_2(t)$ with $n = 3$).

The Eq. (4.23) are the conditions to avoid the undesired excitations. Compare them to the one in the TF limit (4.16). As opposed to the TF limit, the trajectory of $\omega(t)$ does not depend on $\gamma(t)$ and $g(t)$. In other words, shortcut to adiabacity is not possible without designing $\omega(t)$. The parameters $\gamma(t)$ and $g(t)$ can to be used

singly to design frictionless expansion as in TF regime. Adding particle into the system modulates interaction strength. To avoid undesired excitations, the interaction strength must be changed to oppose the change due to the atom injection. In Fig.4.2, we plot the trajectory of $\omega^2(t)/\omega_0^2$ (4.18) for $L_1(t)$ (dashed curves) and $L_2(t)$ (solid curves), respectively, where $\omega_0 = 2\pi 100Hz$. The condensate is assumed to be decompresses by a factor of 4 in 10 milliseconds. The final frequency is smaller than the initial one as expected.

Finally, let us write down the general conditions in two and three dimensional traps,

$$\omega_i^2 L^2 + L\ddot{L} = \frac{\omega_{0,i}^2}{L^2},$$

$$g\frac{e^{\Gamma}}{L^D} = \frac{g_0}{L^2}. \qquad (4.28)$$

where D is the dimension of the space. Atom injection into the expanding system leads to modulation of interaction strength. In two and three dimensions, interaction strength must be decreased according to $g(t) = g_0 e^{-\Gamma}$ and $g(t) = g_0 L e^{-\Gamma}$, respectively as atoms are injected into the system.

After examining fast frictionless expansion for the harmonically trapped growing condensate, let us investigate the frequency trajectories for an optical lattice.

4.2. Fast Frictionless expansion for an optical lattice

In this section, we will explore fast frictionless expansion for an optical lattice continuously replenished by pumping from the reservoir.

The physics of fast frictionless expansion for an optical lattice was explored in [17]. An expansion trajectory that yields a final state identical to the initial state up to an irrelevant phase factor was designed. Discussion of the effect of additional force and nonlinear interaction on the fast frictionless expansion were

also given. Their method of obtaining fast adiabatic transformation is to use a lattice with dynamically variable spacing (accordion lattice).

In an optical lattice, potential depth and lattice spacing can be changed experimentally by changing the power of the laser and the wavelength of the laser, respectively [81].

Consider ultracold atoms in an optical lattice with variable spacing and potential depth,

$$V(x,t) = V(t)\cos\left(2k_L \frac{x}{L(t)}\right) + \frac{m\omega^2(t)}{2}x^2, \tag{4.29}$$

where $V(t)$ is the time-dependent lattice depth, k_L is the optical lattice wave number, $\omega(t)$ is the time-dependent angular frequency and $L(t)$ is the scale parameter describing the expansion of the accordion lattice.

Initially, only the periodic potential is present in the system, $\omega(t=0)=0$. The external parabolic potential is applied just as the system starts to expand. The combined presence of the periodic and parabolic potentials enables us to get fast frictionless expansion. Finally, the parabolic potential is turned off at $t = t_f$ such that $\omega(t=t_f)=0$. To obtain frictionless expansion, we demand that no effective harmonic potential acts on the system. In other words, we choose $\omega^2(t)$ in such a way that it cancels the effective parabolic potential so that there is only periodic potential in the system from $t = 0$ to $t = t_f$. Secondly, we demand that effective nonlinear interaction strength and potential depth scale as $1/L^2$. Hence, we choose

$$\omega^2 L^2 + L\ddot{L} = 0,$$
$$g\frac{e^{\Gamma}}{L} = \frac{g_0}{L^2}. \tag{4.30}$$

This choice of the control parameters guarantees the fast frictionless expansion.

The above conditions reduce the equation (4.11) for the potential (4.29) to the following one

$$i\hbar\frac{\partial\Phi}{\partial\tau} = -\frac{\hbar^2}{2m}\frac{\partial^2\Phi}{\partial q^2} + V_0\cos(2k_L q)\Phi + g_0|\Phi|^2\Phi \qquad (4.31)$$

In the last step, we made another transformation on time,

$$\tau = \int L^{-2}(t')dt'. \qquad (4.32)$$

Compare (4.30) to the condition (4.23) for the harmonically trapped condensate. The right hand side of the first condition in (4.30) is equal to zero. This is because changing the wavelength of the laser is responsible for the expansion of the optical lattice. However, expansion is achieved by changing the trap frequency for the harmonically trapped system.

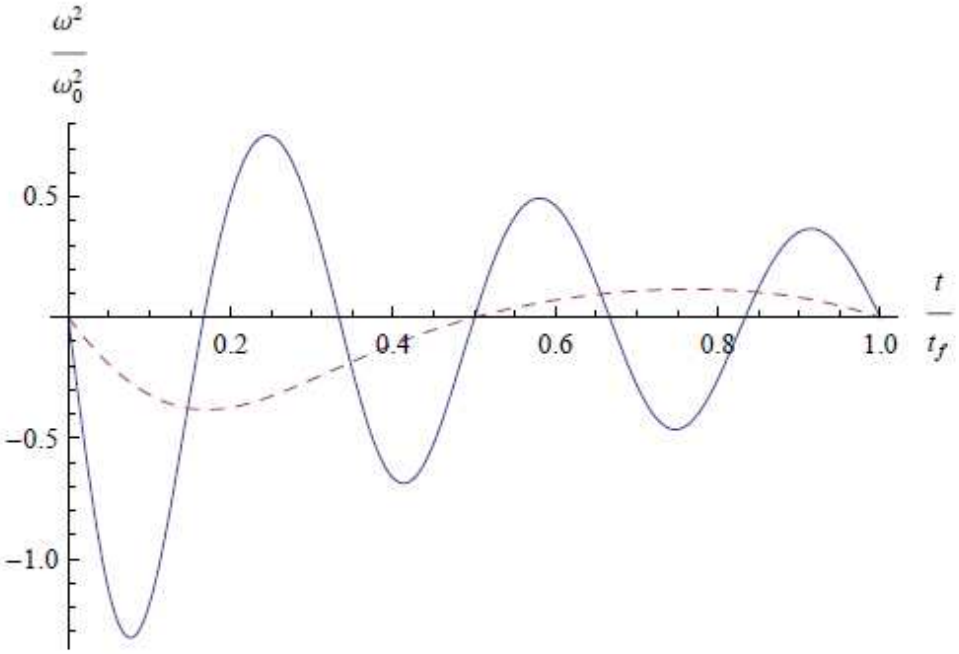

Fig. 4.3. $\omega^2(t)/\omega_0^2$ versus t/t_f. The system is decompressed by a factor of 4. The dashed (solid) curve is for $L_1(t)$ ($L_2(t)$ with $n = 3$).

In Fig.4.3, we plot $\omega^2(t)/\omega_0^2$ for $L_1(t)$ (dashed) and $L_2(t)$ (solid), respectively. The external frequency oscillates in positive and negative region. In fact there are infinitely many other solution for $L(t)$. All the trajectories oscillates since the initial and final accelerations, $\ddot{L}$, are zero and the acceleration takes both negative and positive values at intermediate times. Note that the trajectories of $\omega(t)$ are the same for linear and non-linear optical lattice. For non-linear lattice, interaction strength must satisfy the second condition in (4.30).

5. CONCLUSION

In this dissertation, fast frictionless expansion for growing Bose Einstein condensates described by the Gross-Pitaevskii gain equation has been investigated. So far, all the investigations have been focused on condensates with fixed number of particles. This dissertation has presented investigation of the possibility of fast frictionless expansion for a condensate with variable number of atoms. Gross-Pitaevskii gain equation has been used to study fast frictionless expansion for both harmonically trapped condensate and an optical lattice. It has been shown that a correct choice of expansion trajectory allows us to get a final adiabatic state in a non-adiabatic way for the growing condensate.

Firstly, our aim is to find out the conditions for the fast transitionless expansion of the harmonically trapped condensate. Two cases have been considered; Thomas-Fermi approximation and exact treatment.

For the Thomas-Fermi regim, the trajectory of $\omega^2(t)/\omega_0^2$ for $L_1(t)$, where $\omega_0 = 2\pi 100Hz$ and $g(t)=g_0$ has been plotted in Fig.4.1. The condensate has been assumed to be decompresses by a factor of 4 in 5 milliseconds. This time is too short for an adiabatical expansion. However, the designed trajectory leads to the same final state, up to a global phase, as the state obtained by a slow adiabatic process for such an expansion.

In the exact treatment, the conditions for fast frictionless process of the harmonically trapped condensate without omitting the kinetic energy have been explored. Differently from the TF limit, the trajectory of $\omega(t)$ does not depend on $\gamma(t)$ and $g(t)$. In other words, shortcut to adiabacity is not possible without designing $\omega(t)$. The parameters $\gamma(t)$ and $g(t)$ can be used singly to design frictionless expansion as in TF regime. The trajectory of $\omega^2(t)/\omega_0^2$ (4.18) for $L_1(t)$ (dashed curves) and $L_2(t)$ (solid curves), respectively, where $\omega_0 = 2\pi 100Hz$ has been plotted in Fig. 4.2. Here, the condensate is assumed to be decompresses by a factor of 4 in 10 milliseconds.

Secondly, fast frictionless expansion for an optical lattice continuously replenished by pumping from the reservoir has been investigated. Ultracold atoms

in an optical lattice with variable spacing and potential depth have been considered. The trajectory of the $\omega^2(t)/\omega_0^2$ for $L_1(t)$ (dashed) and $L_2(t)$ (solid), respectively has been plotted in Fig. 4.3.

As a result, it has been shown that fast frictionless expansion can be generalized to growing condensates. Our formalism has been applied to a harmonically trapped condensate and an optical lattice. It has been found that fast frictionless expansion is possible in the TF limit by changing the gain parameter while trap frequency and nonlinear interaction strength are left unchanged. It has been shown that the interaction strength must be changed to oppose the change due to the atom injection to avoid undesired excitation in an accordion optical lattice.

Finally, it has been shown that fast transitionless expansion is possible for harmonically trapped condensates and an optical lattice continuously replenished by pumping from the reservoir [65].

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