DECENTRALIZED CONTROL OF
TIME-DELAY SYSTEMS

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Master of Science Thesis

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Enstitü Müdürü

ABSTRACT

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Supervisor : Prof. Dr. Altuğ İftar
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In this thesis, decentralized controller design for time-delay systems is considered. A necessary and sufficient condition, in terms of decentralized fixed modes, for stabilizability of linear time-invariant (LTI) time-delay systems by LTI time-delay controllers is given. Also, it is presented that a LTI time-delay system can be stabilized by a LTI decentralized time-delay controller if and only if it can be stabilized by a LTI decentralized finite-dimensional controller. This condition extends a previously known fact for centralized control to decentralized control. Although stabilizability problem is considered in a large perspective which includes neutral and retarded systems with commensurate- and incommensurate-time-delays, decentralized controller design problem is considered only for retarded commensurate-time-delay systems. At first, to be used as a centralized controller design algorithm, the continuous pole assignment algorithm is extended to design dynamic output feedback controllers. Then, based on the decentralized pole placement algorithm, two decentralized controller design algorithms are proposed. One of these algorithms is a dynamic output feedback and the other algorithm is an observer based state vector feedback design algorithm.

Keywords: Decentralized control; time-delay systems; dynamic output feedback; decentralized fixed modes; pole placement.

ÖZET

Yüksek Lisans Tezi

ZAMAN GECİKMELİ SİSTEMLERİN MERKEZİ OLMAYAN DENETİMİ

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Bu tezde, zaman gecikmeli sistemler için merkezi olmayan denetleyici tasarımı ele alınmıştır. Doğrusal zamandan bağımsız (DZB) zaman gecikmeli bir sistemin merkezi olmayan zaman gecikmeli denetleyiciler ile kararlı kılınabilmesi için merkezi olmayan sabit modlar cinsinden bir gerek ve yeter koşul verilmiştir. Ayrıca, DZB zaman gecikmeli bir sistemin merkezi olmayan DZB zaman gecikmeli denetleyiciler ile kararlılaştırılabilmesi için gerek ve yeter bir koşulun sistemin DZB merkezi olmayan sonlu boyutlu bir denetleyici ile kararlı kılınabilmesi olduğu gösterilmiştir. Bu koşul, merkezi denetlemede önceden bilinen bir koşulun merkezi olmayan denetlemeye genişletilmesidir. Kararlılaştırılabilirlik problemi daha geniş bir perspektifte ele alınmasına karşın, merkezi olmayan denetleyici tasarımında sadece orantılı zaman gecikmeli geri-tipli sistemler ele alınmıştır. Öncelikle, merkezi denetleyici tasarımında kullanılmak üzere, sürekli kutup atama algoritması dinamik çıktı geri beslemeli denetleyiciler tasarlanacak şekilde genişletilmiştir. Sonrasında, merkezi olmayan kutup yerleştirme algoritması temel alınarak iki merkezi olmayan denetleyici tasarım algoritması önerilmiştir. Bu algoritmalardan biri dinamik çıktı geri besleme, diğeri ise gözlemci tabanlı durum vektörü geri besleme tasarım algoritmasıdır.

Anahtar Kelimeler: Merkezi olmayan denetleme; zaman gecikmeli sistemler; dinamik çıktı geri beslemesi; merkezi olmayan sabit modlar; kutup atama.

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NOTATION

$\mathbb{R}$	Real numbers
$\mathbb{C}$	Complex numbers
$\mathbb{N}$	Natural numbers with zero
$\mathbb{R}^n$	Space of n-dimensional real vectors
$\mathbb{C}^n$	Space of n-dimensional complex vectors
$\mathbb{R}^{k\times l}$	Space of $k \times l$-dimensional real matrices
$\mathbb{C}^{k\times l}$	Space of $k \times l$-dimensional complex matrices
$\mathbb{R}[\cdot]$	Ring of polynomials in $\cdot$ with real coefficients
$\mathbb{R}^{k\times l}[\cdot]$	Ring of $k \times l$ dimensional polynomial matrices in $\cdot$ with real coefficients
$\mathrm{Re}(s)$	Real part of $s \in \mathbb{C}$
$\mathrm{Im}(s)$	Imaginary part of $s \in \mathbb{C}$
$\bar{\Omega}$	Closure of a set Ω
$\mathbb{C}_\mu^-$	$\{s \in \mathbb{C} \mid \mathrm{Re}(s) < \mu\}$ for $\mu \in \mathbb{R}$
$\mathbb{C}_\mu^+$	$\{s \in \mathbb{C} \mid \mathrm{Re}(s) > \mu\}$ for $\mu \in \mathbb{R}$
$\bar{\mathbb{C}}_\mu^+$	$\{s \in \mathbb{C} \mid \mathrm{Re}(s) \geq \mu\}$ for $\mu \in \mathbb{R}$
i	Imaginary unit, $\mathrm{i}^2 = -1$
I	Identity matrix of appropriate dimensions
I_k	$k \times k$ identity matrix
0	Zero matrix of appropriate dimensions
0_k	$k \times k$ zero matrix
$0_{k\times l}$	$k \times l$ zero matrix
$\mathrm{rank}(\Gamma)$	Rank of a matrix Γ
$\det(\Gamma)$	Determinant of a matrix Γ
Γ^T	Transpose of a matrix or vector Γ
$\|\|\Gamma\|\|$	2-norm of a vector Γ
$\|\|\Gamma\|\|$	Induced 2-norm of a matrix Γ
$\\|\cdot\\|_s$	Supremum norm
Γ^*	Complex-conjugate transpose of a matrix or vector Γ

Γ^{-1}	Inverse of a matrix Γ
$\bar{\nu}$	$\{1, \ldots, \nu\}$ for a positive integer ν
$\mathrm{bdiag}[\cdots]$	A block diagonal matrix with blocks $\cdots$ on its diagonal
$\mathcal{C}([a,b], \mathbb{R}^n)$	Banach space of continuous functions mapping the interval $[a,b]$ into $\mathbb{R}^n$ and equipped with $\lVert\cdot\rVert_s$

ACRONYMS

CFM	Centralized Fixed Mode
DFM	Decentralized Fixed Mode
LTI	Linear Time-Invariant

1. INTRODUCTION

1.1. Overview and Motivation

Many control systems may include time-delays due to the required time to acquire information needed for decision making, to create and execute control decisions, etc. [1–3]. Such systems, which involve time-delays in their dynamics, inputs, and/or outputs, are generally called *time-delay systems*.

It is well known that the presence of time-delays may be detrimental to the stability of systems, for instance, communication systems, biological systems, mechanical systems, etc. (see [3] and reference therein). On the other hand, the presence of time-delays may also be beneficial to the stability of some unstable systems [4, 5]. Therefore, in the controller design for a given system, the existing time-delays, whether small or not, should be taken into account, otherwise, the designed controller may not be successful in the stabilization of the actual system or may exhibit poor performance. However, the controller design problem for time-delay systems is a difficult task since these systems are infinite-dimensional [1, 6, 7].

Some practical control problems may not be solved by conventional methods in control theory, since the systems to be controlled may become too large and the problems to be solved may become too complex [8]. Hence, the notion of large-scale systems was required to be introduced. Since the notion of *large-scale* is very subjective, more pragmatic views have been adopted instead of formal definitions. From one point of view, a system is considered large-scale if it is necessary to partition the given analysis or synthesis into manageable subproblems for either computational or practical reasons [9]. From another point of view, simply, a system is large when it requires more than one controller [10]. As a result of both views, for many large-scale systems, decentralized control is either preferable or necessary [11–15]. Decentralized controller design problems for finite-dimensional systems have been studied in the past four decades [16–29]. In decentralized control, the overall plant is controlled by several local control stations, i.e., *control agents*, which all together represent a decentralized controller. In many applications, there are some constraints on transfer of information between control agents and a full information access is rarely possible for all the agents. In some cases, a total decentralization is assumed where every control agent observes only local system outputs and controls only local inputs [12]. Also, the effects of the time-delays are quite noticeable in the control of large-scale systems where multiple time-delays can be introduced by multiple sensors, actuators and controllers. Therefore, alongside of the studies on decentralized control of finite-dimensional systems, problem of decentralized control of large-scale time-delay systems is attracting attention recently [8, 30–39].

In the stabilization and mode placement of control systems the notion of *fixed modes* plays a central role. A mode of a LTI dynamic system which remains fixed in the complex plane for all LTI static output feedback controllers under given information constraints, is called as a *fixed mode*.

In centralized case, it was shown by Kamen et. al. [40] that a LTI retarded time-delay system can be stabilized by a LTI dynamic controller (with sufficiently large dimension) if and only if the system does not have any CFMs in $\bar{\mathbb{C}}_0^+$. In decentralized case, DFM notion was first introduced in [16] and according to this notion, it was established that a necessary and sufficient condition for stabilizability of a LTI decentralized finite-dimensional dynamic system by LTI decentralized finite-dimensional dynamic controllers is that it should not have any DFMs in $\bar{\mathbb{C}}_0^+$. Furthermore, the notion of μ-DFM, which corresponds to a DFM with real part greater than or equal to μ, was introduced in [37] where μ defines the border of the relative stability region $\mathbb{C}_\mu^-$. According to the relative stability region defined by μ, a LTI dynamic system is said to be *stable* or *μ-stable* if all of its modes are in $\mathbb{C}_\mu^-$. Considering the μ-DFM notion, it was established in [37] that a LTI decentralized retarded time-delay system with commensurate-time-delays can be μ-stabilized by LTI decentralized finite-dimensional dynamic controllers if and only if the system does not have any μ-DFMs in $\bar{\mathbb{C}}_\mu^+$. This result was a generalization of the result in [16] to retarded commensurate-time-delay systems. The same result was generalized to retarded systems with incommensurate-time-delays in [38]. In [39], it was shown that the same condition is also necessary and sufficient for μ-stabilization of retarded time-delay systems by decentralized dynamic time-delay controllers and it is also a necessary condition for the μ-stabilization of neutral time-delay systems. Furthermore, in the same work, it was shown that sufficiency also continues to hold under some additional conditions. As a well known result of Kamen et. al. [40], a LTI centralized retarded time-delay system can be stabilized by a LTI time-delay controller if and only if it can be stabilized by a LTI finite-dimensional controller. Extension of this result to the decentralized case was given in [39]. In this thesis, based on the given results in [39], studies will be carried on stabilizability of neutral incommensurate-time-delay systems.

As a result of many studies on the control of time-delay systems, there are many stabilization methods for centralized time-delay systems in the literature (see [5] and references therein for a wide survey). However, stabilization methods for decentralized time-delay systems are relatively few. In [22], the *decentralized pole assignment algorithm* was proposed for decentralized finite-dimensional systems. In this algorithm, control synthesis proceeds in the control agent order and a centralized controller is designed for each control agent sequentially. In this thesis, by utilizing this algorithm, decentralized controller synthesis procedures for LTI retarded commensurate-time-delay systems will be introduced. In these procedures, in order to design a centralized controller for each control agent two different methods, which are based on *continuous pole placement algorithm* [41], will be used. One of these methods is the extension of the continuous pole placement algorithm, which was proposed for static state vector feedback controllers, to the dynamic output feedback controllers. The other method is an implementation of the observer based continuous pole placement algorithm.

1.2. Thesis Outline

In Chapter 2, important background material and definitions are presented. In Section 2.1, an overview of the centralized neutral and retarded time-delay systems is provided. In Section 2.2, the continuous pole placement algorithm of Michiels et. al. [41], which is a centralized controller design procedure, is introduced. In Section 2.3, the finite-dimensional decentralized system and controller structures are presented and some required definitions are given. Also, in this section, the decentralized pole assignment algorithm of Davison and Chang [22], which is a decentralized controller synthesis algorithm for finite-dimensional systems, is introduced.

In Chapter 3, representations and important properties of decentralized neutral and retarded time-delay systems are given. Then the DFMs of these time-delay systems with respect to different controller classes are characterized and stabilizability of LTI decentralized time-delay systems by means of decentralized LTI output feedback controllers is considered and the results obtained in this regard are given.

In Chapter 4, decentralized controller design problem for LTI retarded commensurate-time-delay systems is considered. In Section 4.1, a centralized dynamic output feedback controller synthesis procedure is introduced. In Section 4.2, a decentralized controller synthesis procedure, based on decentralized pole assignment algorithm and the centralized controller synthesis procedure introduced in Section 4.1, for LTI retarded commensurate-time-delay systems is introduced. In addition to these procedures, in Section 4.3, an observer based decentralized controller synthesis procedure, based on decentralized pole assignment algorithm and the continuous pole placement algorithm, for LTI retarded commensurate-time-delay systems is proposed.

In Chapter 5, two examples are presented in order to illustrate the design approaches proposed in Sections 4.2 and 4.3.

In Chapter 6, the concluding remarks are given.

2. BACKGROUND

In this chapter, background information and definitions to be used in the subsequent chapters are presented. In Section 2.1, representation and important properties of neutral and retarded time-delay systems are given. In Section 2.2, the *continuous pole placement algorithm* of Michiels et. al. [41], which is a centralized controller design procedure, is introduced. In Section 2.3, finite-dimensional decentralized controller structures are defined, some required definitions are given, and, at the end of the section, the *decentralized pole assignment algorithm* of Davison and Chang [22], which is a decentralized finite-dimensional controller synthesis algorithm for finite-dimensional systems, is introduced.

2.1. Time-Delay Systems

Consider a LTI time-delay system Σ, which can be defined as

$$\begin{aligned} \dot{x}(t) + \sum_{i=1}^{\sigma} E_i \dot{x}(t-h_i) &= \sum_{i=0}^{\sigma} \Big(A_i x(t-h_i) + B_i u(t-h_i) \Big) \\ y(t) &= \sum_{i=0}^{\sigma} C_i x(t-h_i) \end{aligned}, \tag{2.1}$$

where $x(t) \in \mathbb{R}^n$, $u(t) \in \mathbb{R}^p$ and $y(t) \in \mathbb{R}^q$ are, respectively, the state vector, the input and the output vectors at time t, $0 = h_0 < h_1 < \cdots < h_\sigma$ are time-delays where σ is the number of distinct time-delays involved, and $E_i \in \mathbb{R}^{n\times n}$ $(i \in \bar{\sigma})$, $A_i \in \mathbb{R}^{n\times n}$, $B_i \in \mathbb{R}^{n\times p}$ and $C_i \in \mathbb{R}^{q\times n}$ $(i \in \{0\} \cup \bar{\sigma})$ are constant matrices. When the time-delays are integer multiples of a common divisor $h > 0$, i.e., $h_i := ih$, $i \in \bar{\sigma}$, then the delays are said to be *commensurate*. Otherwise, they are called *incommensurate*. When there is no time-delay in the derivative of the state vector, i.e., when $E_i = 0$, $\forall i \in \bar{\sigma}$, the system (2.1) is said to be *retarded*. Otherwise, it is called *neutral*. Different from Σ, retarded time-delay systems will be indicated by Σ_r.

As in [1], let $x(t;\xi)$ be the unique forward solution of the system Σ, with initial condition $\xi \in \mathcal{C}([-h_\sigma, 0], \mathbb{R}^n)$, i.e., $x(\theta;\xi) = \xi(\theta)$ for all $\theta \in [-h_\sigma, 0]$.

Then, the state of the system Σ at time t is given by the function segment $x_t(\xi) \in \mathcal{C}([-h_\sigma, 0], \mathbb{R}^n)$ defined as $x_t(\xi)(\theta) = x(t+\theta; \xi)$, $\theta \in [-h_\sigma, 0]$.

Definition 2.1. For any given $\mu \in \mathbb{R}$, the set of *μ-modes* of the system Σ, described by (2.1), is defined as

$$\Omega_\mu(\Sigma) := \{s \in \mathbb{C} \mid \mathrm{Re}(s) \geq \mu \text{ and } \phi_\Sigma(\mathrm{s}) = 0\} \tag{2.2}$$

where $\phi_\Sigma(s) := \det\big(\Delta_N(s)\big)$ is the *characteristic function* of the system Σ, where $\Delta_N(s) := s\bar{E}(s) - \bar{A}(s)$ is the *characteristic matrix* of the system Σ and

$$\bar{E}(s) := I + \sum_{i=1}^{\sigma} E_i e^{-sh_i}, \quad \bar{A}(s) := \sum_{i=0}^{\sigma} A_i e^{-sh_i}\ . \tag{2.3}$$

For retarded time-delay systems, Σ_r, this characteristic equation can be written as $\phi_{\Sigma_r} := \det\big(\Delta_R(s)\big)$ where $\Delta_R(s) := sI - \bar{A}(s)$ is the characteristic matrix of the system Σ_r.

An important difference between Σ and Σ_r is that, for any given finite real μ, $\Omega_\mu(\Sigma_r)$ is a finite set, whereas $\Omega_\mu(\Sigma)$ may have infinitely many elements [2].

Now, we define μ-stability.

Definition 2.2. For any given $\mu \in \mathbb{R}$, the system Σ is said to be *μ-stable* if $\Omega_{\mu-\epsilon}(\Sigma) = \emptyset$ for some $\epsilon > 0$. Furthermore, a controller K is said to *μ-stabilize* the system Σ, if the closed-loop system obtained by applying the controller K to system Σ is μ-stable.

For $\mu \leq 0$, the μ-stability can be related with the exponential stability of the system Σ. System Σ is said to be exponentially stable if and only if there exist constants $\beta > 0$ and $\gamma < 0$ such that for all $\xi \in \mathcal{C}([-h_\sigma, 0], \mathbb{R}^n)$, $\|x_t(\xi)\|_s \leq \beta e^{\gamma t} \|\xi\|_s$ [1]. By using arguments similar to those in [1], it can be shown that the system Σ is μ-stable if and only if there exist constants $\beta > 0$ and $\gamma < \mu$ such that for all $\xi \in \mathcal{C}([-h_\sigma, 0], \mathbb{R}^n)$, $\|x_t(\xi)\|_s \leq \beta e^{\gamma t} \|\xi\|_s$.

Definition 2.3. *Spectral abscissa* for the system Σ is defined as follows

$$c_\Sigma := \sup \{\mathrm{Re}(s) \mid \phi_\Sigma(s) = 0\}\ . \tag{2.4}$$

For neutral and retarded time-delay systems, respectively, Σ and Σ_r, the spectral abscissa always exists and is finite. Furthermore, for retarded time-delay systems, there always exists (rightmost) modes such that the real parts of the modes are equal to c_{Σ_r} [1].

Now, define the associated delay-difference equation of the system Σ, as follows

$$x(t) + \sum_{i=1}^{\sigma} E_i x(t - h_i) = 0 \ . \tag{2.5}$$

The characteristic equation of the associated delay-difference equation is given by $\det(\Delta_D(s)) = 0$, where

$$\Delta_D(s) := I + \sum_{i=1}^{\sigma} E_i e^{-sh_i} \ . \tag{2.6}$$

The zeros of this equation are called the modes of the delay-difference equation (2.5) and if all the modes are in $\mathbb{C}_\mu^-$, the delay-difference equation (2.5) is said to be μ-stable. Then, define the collection of the real parts of all the modes of (2.5) as

$$\Omega_D := \{\mathrm{Re}(s) \mid \det(\Delta_D(s)) = 0\} \tag{2.7}$$

and let the spectral abscissa c_D be its supremum

$$c_D := \sup \{\mathrm{Re}(s) \mid \det(\Delta_D(s)) = 0\} \ . \tag{2.8}$$

The following result, obtained in [1], shows that the system Σ features chains of modes, whose position is determined by the associated delay-difference equation.

Lemma 2.1. *If $\alpha \in \overline{\Omega}_D$, with Ω_D defined by* (2.7)*, then there is a sequence of modes $\{s_n\}_{n\geq 1}$ of Σ satisfying*

$$\lim_{n\to\infty} \mathrm{Re}(s_n) = \alpha, \quad \lim_{n\to\infty} \mathrm{Im}(s_n) = \infty \ . \tag{2.9}$$

Proof. See Proposition 1.26 in [1]. □

As indicated in [1], according to Lemma 2.1, a necessary condition for the μ-stability of the system Σ, is the μ-stability of the delay-difference equation (2.5). In the half-plane $\mathrm{Re}(s) > c_D$, the set of the modes of the neutral systems has many properties similar to the retarded case. This can be seen in the following lemma.

Lemma 2.2. *For any $\epsilon > 0$, the system Σ has only a finite number of modes in $\bar{\mathbb{C}}^+_{c_D+\epsilon}$.*

Proof. See Proposition 1.27 in [1]. □

Using Lemmas 2.1 and 2.2, the following theorem can be concluded.

Theorem 2.1. *For any $a \in \mathbb{R}$, the system Σ has only a finite number of modes in $\bar{\mathbb{C}}^+_{a+\epsilon}$ for any $\epsilon > 0$, if and only if the associated delay-difference equation (2.5) does not have any modes in $\bar{\mathbb{C}}^+_a$.*

Proof. If part of the proof follows from Lemma 2.2. To show the only if part, assume that $s_0 \in \bar{\mathbb{C}}^+_a$ is a mode of the associated delay-difference equation (2.5). Then, by Lemma 2.1, there is a sequence of modes $\{s_n\}_{n\geq 1}$ of Σ satisfying (2.9) with $\alpha = \mathrm{Re}(s_0)$, which means that for any $\epsilon > 0$, there exists an N such that $s_n \in \bar{\mathbb{C}}^+_{a+\epsilon}$ for any $n \geq N$. This proves the only if part. □

Before ending this section, let us also present the following definition, which was given in [5], to be used in the sequel.

Definition 2.4. For any given $\mu \in \mathbb{R}$, $s_0 \in \Omega_\mu(\Sigma)$ is said to be *controllable* if

$$\mathrm{rank} \begin{bmatrix} s_0\bar{E}(s_0) - \bar{A}(s_0) & \bar{B}(s_0) \end{bmatrix} = n$$

and is said to be *observable* if

$$\mathrm{rank} \begin{bmatrix} s_0\bar{E}(s_0) - \bar{A}(s_0) \\ \bar{C}(s_0) \end{bmatrix} = n$$

where

$$\bar{B}(s) := \sum_{i=0}^{\sigma} B_i e^{-sh_i}, \qquad \bar{C}(s) := \sum_{i=0}^{\sigma} C_i e^{-sh_i} . \tag{2.10}$$

2.2. Continuous Pole Placement Algorithm

The main idea of continuous pole placement algorithm proposed by Michiels et. al. in [41] is to shift the unstable modes (in $\bar{\mathbb{C}}_0^+$) of a retarded time-delay system to $\mathbb{C}_0^-$ in a quasi-continuous way by applying small changes to the static state vector feedback controller parameters. Applied small changes to the controller parameters depend on the sensitivity of the rightmost modes, which are desired to be shifted, with respect to the controller parameters. Consider retarded time-delay system Σ_r which is defined in the previous section. At first assume that the full state vector $x(t)$ is available for measurement at time t.

The characteristic function of the closed-loop system, obtained by applying the static state vector feedback controller $u(t) = Kx(t)$, $K \in \mathbb{R}^{p\times n}$, to the system Σ_r, is obtained as

$$\phi_{\Sigma,K}(s) = \det\left[sI - \bar{A}(s) - \bar{B}(s)K\right]$$

where $\bar{A}(s)$ is defined in (2.3) and $\bar{B}(s)$ is defined in (2.10). Now let $s_i \in \mathbb{C}$ be a solution of the closed-loop characteristic equation $\phi_{\Sigma,K}(s) = 0$. Then,

$$\begin{aligned}\left(s_iI - \bar{A}(s_i) - \bar{B}(s_i)K\right)v_i &= 0\\ N(v_i) &= 1\end{aligned} \tag{2.11}$$

where $v_i \in \mathbb{C}^n$ is a non-zero vector and $N(\cdot)$ is a normalizing function, for example one can choose $N(v) = v^*v$. Let $\tilde{K} \in \mathbb{R}^{\hat{n}}$ be the vector of $\hat{n} := pn$ parameters, $\tilde{k}_1, \ldots, \tilde{k}_{\hat{n}}$, of the static state vector feedback controller K. Differentiating (2.11) with respect to ψ^{th} component $\tilde{k}_\psi$ $(\psi = 1, \ldots, \hat{n})$ of $\tilde{K}$, a linear system of $n+1$ equations is obtained in the $n+1$ unknowns, which are $\partial s_i/\partial k_\psi$ and the n components of $\partial v_i/\partial \tilde{k}_\psi$, as follows

$$\begin{bmatrix} s_iI - \bar{A}(s_i) - \bar{B}(s_i)K & \left(I - \dfrac{\partial \bar{A}(s_i)}{\partial s_i} - \dfrac{\partial \bar{B}(s_i)}{\partial s_i}K\right)v_i \\ \dfrac{\partial N(v_i)}{\partial v_i} & 0 \end{bmatrix} \begin{bmatrix} \dfrac{\partial v_i}{\partial \tilde{k}_\psi} \\ \dfrac{\partial s_i}{\partial \tilde{k}_\psi} \end{bmatrix}$$
$$= \begin{bmatrix} \bar{B}(s_i)\dfrac{\partial K}{\partial \tilde{k}_\psi}v_i \\ 0 \end{bmatrix}.$$

Assume that $k \leq \hat{n}$ modes, $s_1, \ldots, s_k$, which will be referred to as the controlled modes, are desired to be shifted towards $\mathbb{C}_0^-$. Let Θ_k be

$$\Theta_k := [\theta_{i,\psi}] \in \mathbb{C}^{k \times \hat{n}} \quad \text{where} \quad \theta_{i,\psi} := \frac{\partial s_i}{\partial \tilde{k}_\psi} \ ,$$

which is called *sensitivity matrix.* Also, let $\Delta S_k^d := \begin{bmatrix} \Delta s_1^d & \ldots & \Delta s_k^d \end{bmatrix}^T \in \mathbb{C}^k$ be the desired displacement of the k controlled modes. Assuming that ΔS_k^d is in the range space of Θ_k, the corresponding change ΔK for K can be computed such that

$$\Theta_k \Delta K = \Delta S_k^d \ . \tag{2.12}$$

All elements of ΔK become real by choosing ΔS_k^d such that complex-conjugate modes remain as complex-conjugate pairs or both become real and no real mode becomes a complex mode unless another real mode becomes its complex-conjugate. When $\operatorname{rank}(\Theta_k) = k$, a solution to (2.12), with minimal $\|\Delta K\|$, can be given by

$$\Delta K = \Theta_k^{\dagger} \Delta S_k^d \ , \tag{2.13}$$

where $\Theta_k^{\dagger}$ is the Moore-Penrose inverse of Θ_k [42]. In [41], it is indicated that, with the new controller $K + \Delta K$, the displacement of the controlled modes will generically not be equal to ΔS_k^d, since (2.12) is obtained by linearization. However, since it is also desirable to have modes close to the desired modes, (2.13) is generally a good solution of (2.12) for a small ΔS_k^d. Furthermore, when stabilization is of main concern, only the real parts of the complex pairs need to be controlled. This leads to the modified formula of (2.13),

$$\Delta K = \operatorname{Re}\big(\Theta_k\big)^{\dagger} \operatorname{Re}\big(\Delta S_k^d\big) \ , \tag{2.14}$$

where $\operatorname{Re}\big(\Delta S_k^d\big)$ is the desired displacement of the real parts of the controlled modes.

Secondly, assume that not the full state vector $x(t)$, but only the output $y(t)$ of the system Σ_r is available for measurement and the μ-modes in $\Omega_\mu(\Sigma_r)$ are observable according to Definition 2.4. As indicated in [1], in order to apply the continuous pole placement method, an observer can be used. It can be constructed as follows

$$\dot{\hat{x}}(t) = \sum_{i=0}^{\sigma} \Big(A_i \hat{x}(t-h_i) + B_i u(t-h_i) \Big) + L^T \left(\sum_{i=0}^{\sigma} \Big(C_i \hat{x}(t-h_i) \Big) - y(t) \right)$$

where $L \in \mathbb{R}^{q \times n}$ is the observer gain. Then, by defining the observer error $e(t) := x(t) - \hat{x}(t)$ and by applying estimated state vector feedback $u(t) = K\hat{x}(t)$, $K \in \mathbb{R}^{p \times n}$, following equations can be obtained

$$\begin{aligned} \dot{x}(t) &= \sum_{i=0}^{\sigma} \Big(\big(A_i + B_i K\big) x(t-h_i) - B_i K e(t-h_i) \Big) \\ \dot{e}(t) &= \sum_{i=0}^{\sigma} \Big(\big(A_i + L^T C_i\big) e(t-h_i) \Big) \end{aligned} . \tag{2.15}$$

Characteristic equation of the closed-loop system (2.15) is

$$\det \left(\begin{bmatrix} sI - \bar{A}(s) - \bar{B}(s)K & \bar{B}(s)K \\ 0 & sI - \bar{A}(s) - L^T \bar{C}(s) \end{bmatrix} \right) = 0 \ .$$

Because of the block-triangular structure, the separation principle is valid and the modes of the system (2.15) consist of the controller modes which are the solutions of

$$\det \Big(sI - \bar{A}(s) - \bar{B}(s)K \Big) = 0 \ , \tag{2.16}$$

and the observer modes which are the solutions of

$$\det \Big(sI - \bar{A}(s) - L^T \bar{C}(s) \Big) = \det \Big(sI - \bar{A}^T(s) - \bar{C}^T(s) L \Big) = 0 \ . \tag{2.17}$$

Thus the continuous pole placement method can be applied once to (2.16) to obtain the controller gain K and once to (2.17) to obtain the observer gain L.

2.3. Decentralized Control

Contrary to a centralized control system, a decentralized control system consists of several independent control agents, each of which measures only a subset of all the outputs and decides only a subset of all the inputs, so that the closed-loop system is μ-stable, for some $\mu \in \mathbb{R}$ (normally $\mu \leq 0$). Background information and definitions in the area of finite-dimensional systems decentralized control are stated here.

Consider stabilizability of a decentralized LTI finite-dimensional system Σ with ν control agents which is described as,

$$\begin{aligned} \dot{x}(t) =& Ax(t) + \sum_{j=1}^{\nu} B_j u_j(t) \\ y_j(t) =& C_j x(t), \qquad j \in \bar{\nu} \end{aligned} \tag{2.18}$$

where $x(t) \in \mathbb{R}^n$ is the state vector at time t and $u_j(t) \in \mathbb{R}^{p_j}$ and $y_j(t) \in \mathbb{R}^{q_j}$ are, respectively, the input and the output vectors at time t, accessible by the j^{th} control agent ($j \in \bar{\nu}$).

Definition 2.5. Consider the decentralized finite-dimensional system Σ. Finite-dimensional controller classes of interest are as follows:

1) $\mathbf{K_c}$: *the class of centralized static LTI controllers* is all the controllers of the form:

$$u(t) = Ky(t) \;, \tag{2.19}$$

where $K \in \mathbb{R}^{p \times q}$ for $p := \sum_{j=1}^{\nu} p_j$, $q := \sum_{j=1}^{\nu} q_j$, and

$$\begin{aligned} u(t) :=& \begin{bmatrix} u_1^T(t) & \cdots & u_\nu^T(t) \end{bmatrix}^T \in \mathbb{R}^p \\ y(t) :=& \begin{bmatrix} y_1^T(t) & \cdots & y_\nu^T(t) \end{bmatrix}^T \in \mathbb{R}^q \end{aligned} . \tag{2.20}$$

2) $\mathbf{K_s}$: *the class of decentralized static LTI controllers* is all the controllers of the form:

$$u_j(t) = K_j y_j(t) \;, \qquad j \in \bar{\nu} \;, \tag{2.21}$$

where $K_j \in \mathbb{R}^{p_j \times q_j}$.

3) $\mathbf{K_f}$: *the class of decentralized finite-dimensional dynamic LTI controllers* is all the controllers of the form:

$$\begin{aligned} \dot{z}_j(t) &= F_j z_j(t) + G_j y_j(t) \\ u_j(t) &= H_j z_j(t) + K_j y_j(t) \end{aligned} \;, \qquad j \in \bar{\nu} \;, \tag{2.22}$$

where $z_j(t) \in \mathbb{R}^{m_j}$ is the state vector of the j^{th} controller at time t and F_j, G_j, H_j, and K_j are appropriately sized real constant matrices. When $m_j = 0$, for all $j \in \bar{\nu}$, such a controller reduces to a decentralized static LTI controller; thus, $\mathbf{K_s} \subset \mathbf{K_f}$.

In the stabilization and mode placement, an important notion, that of *fixed modes*, play an important role. Set of the fixed modes of the system Σ with respect to a given controller class is defined as follows.

Definition 2.6. For any given $\mu \in \mathbb{R}$, the set of μ-fixed modes of the system Σ with respect to the class of controllers $\mathbf{K}$ (where $\mathbf{K}$ may be $\mathbf{K_c}$, $\mathbf{K_s}$ or $\mathbf{K_f}$) is defined as

$$\Lambda_\mu\left(\Sigma, \mathbf{K}\right) := \{s \in \mathbb{C} \mid \mathrm{Re}(s) \geq \mu \text{ and } \phi_{\Sigma,K}(s) = 0,\ \forall K \in \mathbf{K}\} \tag{2.23}$$

where $\phi_{\Sigma,K}(\cdot)$ is the characteristic function of the closed-loop system obtained by applying controller K to system Σ defined in (2.18).

For any class of controllers $\mathbf{K}$ which includes the *zero controller*, $\Lambda_\mu(\Sigma, \mathbf{K}) \subset \Omega_\mu(\Sigma)$ for any $\mu \in \mathbb{R}$. Here the zero controller is the controller which applies $u_j(t) = 0$ $(j \in \bar{\nu})$, for all t, independently of $y_i(t)$ $(i \in \bar{\nu})$. It can be seen that each of the controller classes, $\mathbf{K_c}$, $\mathbf{K_s}$ and $\mathbf{K_f}$, includes the zero controller. When considering the controller class $\mathbf{K_c}$, these modes are called μ-centralized fixed modes (μ-CFMs). As for controller classes $\mathbf{K_s}$ and $\mathbf{K_f}$, they are called μ-decentralized fixed modes (μ-DFMs). It was shown in [16] that $\Lambda_\mu(\Sigma, \mathbf{K_s}) = \Lambda_\mu(\Sigma, \mathbf{K_f})$. In addition, a necessary and sufficient condition for μ-stabilizability of the system Σ, is given by the following theorem, which was presented in [16].

Theorem 2.2. *For any $\mu \in \mathbb{R}$, there exists a decentralized finite-dimensional LTI dynamic controller, i.e., a controller in class $\mathbf{K_f}$, for the system Σ such that all the modes of the closed-loop system are contained in $\mathbb{C}_\mu^-$ if and only if $\Lambda_\mu(\Sigma, \mathbf{K_s}) \subset \mathbb{C}_\mu^-$.*

Decentralized controller design problem is based on separating the overall problem into subproblems, where each problem is designing a controller where only feedback from y_j to u_j is allowed, $j \in \bar{\nu}$. Under this approach, in the decentralized pole assignment algorithm given in [22], a centralized controller is designed for each control agent sequentially. Before giving this algorithm, which utilizes Theorem 2.2, let us define the following closed-loop system structure.

Consider a LTI finite-dimensional decentralized system Σ, which has ν control agents, $S_1, \ldots, S_\nu$. Suppose that decentralized controllers of the form (2.22) has been designed for the first k control agents, where $k < \nu$. Define now the resultant system Σ_k for the local control agent S_{k+1} with input u_{k+1} and output y_{k+1} as follows

$$\begin{aligned} \dot{\eta}_k(t) &= \bar{\Delta}_k \eta_k(t) + \bar{B}_{k+1} u_{k+1}(t) \\ y_{k+1}(t) &= \bar{C}_{k+1} \eta_k(t) \end{aligned} \tag{2.24}$$

where $\eta_k(t) = \begin{bmatrix} x^T(t) & z_1^T(t) & \ldots & z_k^T(t) \end{bmatrix}^T \in \mathbb{R}^{n+m^k}$ with $m^k = \sum_{j=1}^k m_j$,

$$\bar{\Delta}_k := \begin{bmatrix} A + \sum\limits_{j=1}^{k} B_j K_j C_j & B_1 H_1 & \ldots & B_k H_k \\ G_1 C_1 & F_1 & \ldots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ G_k C_k & 0 & \ldots & F_k \end{bmatrix},$$

and

$$\bar{B}_{k+1} := \begin{bmatrix} B_{k+1} \\ 0_{m^k \times p_{k+1}} \end{bmatrix}, \quad \bar{C}_{k+1} := \begin{bmatrix} C_{k+1} & 0_{q_{k+1} \times m^k} \end{bmatrix}. \tag{2.25}$$

Main idea of the decentralized pole assignment algorithm is to design stabilizing agents sequentially for assigning as many modes to $\mathbb{C}_\mu^-$ as possible [22]. Assume that the DFMs of the system Σ are contained in $\mathbb{C}_\mu^-$. Decentralized pole assignment algorithm in [22] is as follows:

Algorithm 2.1. *Assume that the control synthesis proceeds in the control agent order* $1, 2, \ldots, \nu$.

1) *Using centralized synthesis, design the stabilizing agent* S_1 *for* Σ_0 *so that all modes of the closed-loop system matrix* $\bar{\Delta}_1$*, except the CFMs of* Σ_0*, are in* $\mathbb{C}_\mu^-$.

2) *Using centralized synthesis, design the stabilizing agent* S_2 *for* Σ_1 *so that all modes of the closed-loop system matrix* $\bar{\Delta}_2$*, except the CFMs of* Σ_1*, are in* $\mathbb{C}_\mu^-$.

$\vdots$

κ) *Using centralized synthesis, design the stabilizing agent S_κ for $\Sigma_{\kappa-1}$ so that all modes of the closed-loop system matrix $\bar{\Delta}_\kappa$, except the CFMs of $\Sigma_{\kappa-1}$, are in $\mathbb{C}_\mu^-$.*

$\vdots$

ν) *Using centralized synthesis, design the stabilizing agent S_ν for $\Sigma_{\nu-1}$ so that all modes of the closed-loop system matrix $\bar{\Delta}_\nu$, except the CFMs of $\Sigma_{\nu-1}$, are in $\mathbb{C}_\mu^-$.*

It was proved in [22] that, for almost all control agents $S_1, \ldots, S_{\nu-1}$, it is possible to design an agent S_ν in step ν, so that the overall system Σ_ν is μ-stable if and only if the original system Σ does not have any μ-DFMs. In addition, as remarked in [22], whenever $\Sigma_{\kappa-1}$ is μ-stable at step κ, it is essential for the progress of the algorithm that a stabilizing static LTI controller $u_\kappa(t) = K_\kappa y_\kappa(t)$, still be applied, where $K_\kappa \in \mathbb{R}^{p_\kappa \times q_\kappa}$. The reason for applying a static output feedback controller is to make sure that any μ-modes of Σ, which is not a μ-DFM, is a controllable and observable mode of Σ_s, for some $s > \kappa$. If such a feedback loop is not closed, some μ-modes may appear as a CFM in all the remaining steps so that they can not be moved towards $\mathbb{C}_\mu^-$. Also, in general, the required order of the stabilizing agent S_κ increases as κ increases [22].

Based on this algorithm, decentralized controller synthesis procedures for LTI retarded commensurate-time-delay systems will be proposed in Chapter 4.

3. CHARACTERIZATION OF DFMs FOR TIME-DELAY SYSTEMS

In this chapter, representation and important properties of decentralized neutral and retarded time-delay systems are given. Then, the DFMs of considered time-delay systems with respect to different controller classes are characterized and a necessary and sufficient condition for stabilizability of decentralized time-delay systems is presented. Then, the extension of the main result of [40] to the decentralized case is given.

Consider the stabilization problem of a decentralized LTI neutral time-delay system Σ^d with ν control agents described as,

$$\begin{aligned} \dot{x}(t) + \sum_{i=1}^{\sigma} E_i \dot{x}(t-h_i) &= \sum_{i=0}^{\sigma} \left(A_i x(t-h_i) + \sum_{j=1}^{\nu} B_{j,i} u_j(t-h_i) \right) \\ y_j(t) &= \sum_{i=0}^{\sigma} C_{j,i} x(t-h_i), \qquad j = 1, \ldots, \nu \end{aligned} \tag{3.1}$$

where $x(t) \in \mathbb{R}^n$ is the state vector at time t,, $u_j(t) \in \mathbb{R}^{p_j}$ and $y_j(t) \in \mathbb{R}^{q_j}$ are, respectively, the input and the output vectors at time t, accessible by the j^{th} control agent ($j \in \bar{\nu}$). Also, E_i ($i \in \bar{\sigma}$), A_i, $B_{j,i}$ and $C_{j,i}$ ($i \in \{0\} \cup \bar{\sigma}$, $j \in \bar{\nu}$) are appropriately sized constant real matrices and $0 = h_0 < h_1 < \cdots < h_\sigma$ are time-delays, where σ is the number of distinct time-delays involved. Some of these delays may be commensurate, while others are incommensurate. When $E_i = 0$, for all $i \in \bar{\sigma}$, the resulting retarded time-delay system will be denoted by Σ_r^d.

Definition 3.1. Consider the decentralized neutral time-delay system Σ^d. Define the *class of decentralized LTI time-delay controllers*, $\mathbf{K_d}$, which is all the controllers of the form:

$$\begin{aligned} \dot{z}_j(t) &= \sum_{i=0}^{\rho_j} \left(F_{j,i} z_j(t-\tilde{h}_{j,i}) + G_{j,i} y_j(t-\tilde{h}_{j,i}) \right) \\ u_j(t) &= \sum_{i=0}^{\rho_j} \left(H_{j,i} z_j(t-\tilde{h}_{j,i}) + K_{j,i} y_j(t-\tilde{h}_{j,i}) \right) \end{aligned} , \quad j \in \bar{\nu}, \tag{3.2}$$

where $z_j(t) \in \mathbb{R}^{m_j}$ is the state vector at time t and $0 = \tilde{h}_{j,0} < \tilde{h}_{j,1} < \ldots < \tilde{h}_{j,\rho_j}$ are the time-delays of the j^{th} controller where $\rho_j \in \mathbb{N}$ is the number of distinct time-delays involved in the j^{th} controller. Also, $F_{j,i}$, $G_{j,i}$, $H_{j,i}$, and $K_{j,i}$ are appropriately sized real constant matrices. When $\rho_j = 0$, for all $j \in \bar{\nu}$, such a controller reduces to a decentralized finite-dimensional dynamic LTI controller; thus, $\mathbf{K_f} \subset \mathbf{K_d}$ where $\mathbf{K_f}$ is defined in Definition 2.5.

Similar to the finite-dimensional control classes $\mathbf{K_s}$ and $\mathbf{K_f}$ which are defined in Definition 2.5, for any given $\mu \in \mathbb{R}$, the set of μ-DFMs of the system Σ^d with respect to $\mathbf{K_d}$ is defined as

$$\Lambda_\mu\left(\Sigma^d, \mathbf{K_d}\right) := \left\{s \in \mathbb{C} \mid \mathrm{Re}(s) \geq \mu \text{ and } \phi_{\Sigma^d,K}(s) = 0,\ \forall K \in \mathbf{K_d}\right\} \ . \tag{3.3}$$

Also, for the controller class $\mathbf{K_d}$, $\Lambda_\mu(\Sigma^d, \mathbf{K_d}) \subset \Omega_\mu(\Sigma^d)$ for any $\mu \in \mathbb{R}$.

For a retarded time-delay system Σ_r^d, it is indicated in [38] that $s_0 \in \Omega_\mu(\Sigma_r^d)$ is not a μ-CFM if and only if it is both controllable and observable. Although the proof of this result was presented only for retarded time-delay systems, it is also valid for neutral time-delay systems as Σ^d. In order to determine $\Lambda_\mu(\Sigma_r^d, \mathbf{K_s})$, a numerical procedure, which was originally proposed in [16] for finite-dimensional systems and presented for retarded time-delay systems in [38], may be used. This procedure gives the desired set with probability 1. As an alternative, an algebraic test, which was originally proposed in [19] for finite-dimensional systems and presented for retarded time-delay systems in [38], may also be used to determine $\Lambda_\mu(\Sigma_r^d, \mathbf{K_s})$. Advantage of this test over the numerical procedure is the certainty of the given set. Also, in [39], it is stated that this test can also be used for neutral time-delay systems at least in the case when $\Omega_\mu(\Sigma^d)$ is a finite set. The algebraic test is given by the following lemma.

Lemma 3.1. *Let* $\mathrm{Re}(s_0) \geq \mu$. $s_0 \in \Lambda_\mu\left(\Sigma^d, \mathbf{K_s}\right)$ *if and only if there exists* $k \in \{0\} \cup \bar{\nu}$ *and* $\{i_1, \ldots, i_k\} \subset \bar{\nu}$, *where* $i_1, \ldots, i_k$ *are distinct, such that*

$$\mathrm{rank}\begin{bmatrix} \bar{A}(s_0) - s_0\bar{E}(s_0) & \bar{B}_{i_1}(s_0) & \cdots & \bar{B}_{i_k}(s_0) \\ \bar{C}_{i_{k+1}}(s_0) & 0 & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ \bar{C}_{i_\nu}(s_0) & 0 & \cdots & 0 \end{bmatrix} < n \tag{3.4}$$

where $\{i_{k+1},\ldots,i_\nu\} := \bar\nu \setminus \{i_1,\ldots,i_k\}$ *and*

$$\bar B_j(s) := \sum_{i=0}^{\sigma} B_{j,i}e^{-sh_i}, \qquad \bar C_j(s) := \sum_{i=0}^{\sigma} C_{j,i}e^{-sh_i}\,. \tag{3.5}$$

It should be noted that, when $k=\nu$ (in which case $\{i_1,\ldots,i_k\}=\bar\nu$ and $\{i_{k+1},\ldots,i_\nu\}=\emptyset$) this test becomes a test for controllability and when $k=0$ (in which case $\{i_1,\ldots,i_k\}=\emptyset$ and $\{i_{k+1},\ldots,i_\nu\}=\bar\nu$), it becomes a test for observability of a mode which were given in Definition 2.4. It is also worth to note that, since $\Lambda_\mu(\Sigma^d,\mathbf{K_s})\subset\Omega_\mu(\Sigma^d)$, this test need to be applied only for $s_0\in\Omega_\mu(\Sigma^d)$. For Σ_r^d, $\Omega_\mu(\Sigma_r^d)$ is a finite set and this test need to be applied for finitely many s_0. However, for Σ^d, if $\Omega_\mu(\Sigma^d)$ is not a finite set, this test need to be applied for infinitely many s_0 to determine $\Lambda_\mu(\Sigma^d,\mathbf{K_s})$.

Now some important lemmas are presented below.

Lemma 3.2. *For any given* $\mu\in\mathbb{R}$, $\Lambda_\mu\left(\Sigma^d,\mathbf{K_s}\right)=\Lambda_\mu\left(\Sigma^d,\mathbf{K_f}\right)$.

Proof. $\Lambda_\mu\left(\Sigma_r^d,\mathbf{K_s}\right)\subset\Lambda_\mu\left(\Sigma_r^d,\mathbf{K_f}\right)$ was proved in [38] and the proof for the system Σ^d is similar and it is as follows.

Let $s_0\in\Lambda_\mu(\Sigma^d,\mathbf{K_s})$. Then, from Lemma 3.1, there exists $k\in\{0\}\cup\bar\nu$ and $\{i_1,\ldots,i_k\}\subset\bar\nu$ such that

$$\operatorname{rank}\begin{bmatrix} \bar A(s_0)-s_0\bar E(s_0) & \bar B_{i_1}(s_0) & \cdots & \bar B_{i_k}(s_0)\\ \bar C_{i_{k+1}}(s_0) & 0 & \cdots & 0\\ \vdots & \vdots & \ddots & \vdots\\ \bar C_{i_\nu}(s_0) & 0 & \cdots & 0 \end{bmatrix} < n \tag{3.6}$$

where $\{i_{k+1},\ldots,i_\nu\}:=\bar\nu\setminus\{i_1,\ldots,i_k\}$. Then, consider a controller $\tilde K\in\mathbf{K_f}$, described as in (2.22). Characteristic function of the closed-loop system is given by $\phi_{\Sigma^d,\tilde K}(s)=\det(\Phi(s))=0$ where $\Phi(s)$ is the the characteristic matrix of the closed-loop system given as

$$\Phi(s)=\begin{bmatrix} s\bar E(s)-\bar A(s)-\sum\limits_{j=1}^{\nu}\bar B_j(s)K_j\bar C_j(s) & -\bar B_1(s)H_1 & \ldots & -\bar B_\nu(s)H_\nu\\ -G_1\bar C_1(s) & sI_{m_1}-F_1 & & \\ \vdots & & \ddots & \\ -G_\nu\bar C_\nu(s) & & & sI_{m_\nu}-F_\nu \end{bmatrix}.$$

Now, assume that $s_0 \notin \Lambda_\mu(\Sigma^d, \mathbf{K_f})$ which means that, $\phi_{\Sigma^d,\tilde{K}}(s_0) \neq 0$ for some $\tilde{K} \in \mathbf{K_f}$. By arranging the rows and columns of $\Phi(s)$, this means that, for some $\tilde{K} \in \mathbf{K_f}$,

$$\operatorname{rank}\Big(\Phi(s_0)\Big) = \operatorname{rank}\left(\begin{bmatrix} \bar{M}(s_0) & -\bar{B}_{i_1}(s_0)H_{i_1} & \cdots & -\bar{B}_{i_k}(s_0)H_{i_k} & -\bar{B}_{i_{k+1}}(s_0)H_{i_{k+1}} & \ldots & -\bar{B}_{i_\nu}(s_0)H_{i_\nu} \\ -G_{i_{k+1}}\bar{C}_{i_{k+1}}(s_0) & 0 & \cdots & 0 & sI_{m_{i_{k+1}}} - F_{i_{k+1}} & & 0 \\ \vdots & \vdots & \ddots & \vdots & & \ddots & \\ -G_{i_\nu}\bar{C}_{i_\nu}(s_0) & 0 & \cdots & 0 & 0 & & sI_{m_{i_\nu}} - F_{i_\nu} \\ -G_{i_1}\bar{C}_{i_1}(s_0) & sI_{m_{i_1}} - F_{i_1} & & 0 & 0 & \ldots & 0 \\ \vdots & & \ddots & & \vdots & \ddots & \vdots \\ -G_{i_k}\bar{C}_{i_k}(s_0) & 0 & & sI_{m_{i_k}} - F_{i_k} & 0 & \ldots & 0 \end{bmatrix}\right) = n + m$$

where $m = \sum_{j=1}^{\nu} m_{i_j}$ and $\bar{M}(s) := s\bar{E}(s) - \bar{A}(s) - \sum_{j=1}^{\nu} \bar{B}_j(s)K_j\bar{C}_j(s)$. By deleting last $m_{i_1} + \cdots + m_{i_k}$ rows and last $m_{i_{k+1}} + \cdots + m_{i_\nu}$ columns, following inequality can be written

$$\operatorname{rank}\Big(\Phi(s_0)\Big) \leq m + \operatorname{rank}\left(\begin{bmatrix} \bar{M}(s_0) & -\bar{B}_{i_1}(s_0)H_{i_1} & \cdots & -\bar{B}_{i_k}(s_0)H_{i_k} \\ -G_{i_{k+1}}\bar{C}_{i_{k+1}}(s_0) & 0 & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ -G_{i_\nu}\bar{C}_{i_\nu}(s_0) & 0 & \cdots & 0 \end{bmatrix}\right).$$

This inequality can also be written as follows

$$\operatorname{rank}\Big(\Phi(s_0)\Big) \leq m + \operatorname{rank}\left(\begin{bmatrix} I_n & -\bar{B}_{i_{k+1}}K_{i_{k+1}} & \ldots & -\bar{B}_{i_\nu}K_{i_\nu} \\ 0 & -G_{i_{k+1}} & & 0 \\ \vdots & & \ddots & \\ 0 & 0 & & -G_{i_\nu} \end{bmatrix}\right.$$

$$
\left.\left.\begin{bmatrix} s_0\bar{E}(s_0)-\bar{A}(s_0) & \bar{B}_{i_1}(s_0) & \cdots & \bar{B}_{i_k}(s_0) \\ \bar{C}_{i_{k+1}}(s_0) & 0 & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ \bar{C}_{i_\nu}(s_0) & 0 & \cdots & 0 \end{bmatrix}\begin{bmatrix} I_n & 0 & \dots & 0 \\ -K_{i_1}\bar{C}_{i_1}(s_0) & -H_{i_1} & & 0 \\ \vdots & & \ddots & \\ -K_{i_k}\bar{C}_{i_k}(s_0) & 0 & & -H_{i_k} \end{bmatrix}\right)\right)
$$

and this yields

$$
\operatorname{rank}\Big(\Phi(s_0)\Big) \leq m + \operatorname{rank}\left(\begin{bmatrix} s_0\bar{E}(s_0)-\bar{A}(s_0) & \bar{B}_{i_1}(s_0) & \cdots & \bar{B}_{i_k}(s_0) \\ \bar{C}_{i_{k+1}}(s_0) & 0 & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ \bar{C}_{i_\nu}(s_0) & 0 & \cdots & 0 \end{bmatrix}\right) .
$$

Then, from (3.6), for all $\tilde{K} \in \mathbf{K_f}$, $\operatorname{rank}(\Phi(s_0)) < n+m$. This result is in contradiction with (3.7). So, by contradiction, $\Lambda_\mu(\Sigma^d, \mathbf{K_s}) \subset \Lambda_\mu(\Sigma^d, \mathbf{K_f})$.

Also, since $\mathbf{K_s} \subset \mathbf{K_f}$, a mode which can not be moved by any controller in $\mathbf{K_f}$, can not be moved by any controller in $\mathbf{K_s}$. Thus, $\Lambda_\mu\left(\Sigma^d, \mathbf{K_s}\right) \supset \Lambda_\mu\left(\Sigma^d, \mathbf{K_f}\right)$. □

Lemma 3.3. *For any given* $\mu \in \mathbb{R}$, $\Lambda_\mu\left(\Sigma^d, \mathbf{K_f}\right) = \Lambda_\mu\left(\Sigma^d, \mathbf{K_d}\right)$.

Proof. Consider the system Σ^d and a controller $K \in \mathbf{K_d}$, described as in (3.2). Let $m = \sum_{j=1}^{\nu} m_j$ and define

$$
\{\hat{h}_0, \dots, \hat{h}_\rho\} := \bigcup_{j=1}^{\nu} \{\tilde{h}_{j,0}, \dots, \tilde{h}_{j,\rho_j}\} ,
$$

where $\hat{h}_0 = 0$ and $\hat{h}_1, \dots, \hat{h}_\rho$ are all non-zero and distinct, and ρ is the number of distinct time-delays involved in all control agents. Next define

$$
\hat{F}_{j,i} := \begin{cases} F_{j,k} \ , & \text{if } \hat{h}_i = \tilde{h}_{j,k} \\ 0_{m_j} \ , & \text{if } \hat{h}_i \notin \{\tilde{h}_{j,0}, \dots, \tilde{h}_{j,\rho_j}\} \end{cases}
$$

and

$$
\hat{F}_i = \operatorname{bdiag}\left[\hat{F}_{1,i}, \dots, \hat{F}_{\nu,i}\right] , \qquad i \in \{0\} \cup \bar{\rho} .
$$

$\hat{G}_i$, $\hat{H}_i$, and $\hat{K}_i$ can be defined similarly. Then, let

$$
\hat{K}_i^e := \begin{bmatrix} \hat{K}_i & \hat{H}_i \\ \hat{G}_i & \hat{F}_i \end{bmatrix}
$$

and $\hat{K}^e(s) := \sum_{i=0}^{\rho} e^{-s\hat{h}_i}\hat{K}_i^e$. Also, define

$$\hat{E}^e(s) := \begin{bmatrix} \bar{E}(s) & 0 \\ 0 & 0_m \end{bmatrix}, \qquad \hat{A}^e(s) := \begin{bmatrix} \bar{A}(s) & 0 \\ 0 & 0_m \end{bmatrix},$$

and

$$\hat{C}^e(s) := \begin{bmatrix} \bar{C}(s) & 0 \\ 0 & I_m \end{bmatrix}, \qquad \hat{B}^e(s) := \begin{bmatrix} \bar{B}(s) & 0 \\ 0 & I_m \end{bmatrix},$$

where $\bar{E}(s)$, $\bar{A}(s)$ are defined in (2.3) and

$$\bar{B}(s) := \begin{bmatrix} \bar{B}_1(s) & \dots & \bar{B}_\nu(s) \end{bmatrix}, \; \bar{C}(s) := \begin{bmatrix} \bar{C}_1^T(s) & \dots & \bar{C}_\nu^T(s) \end{bmatrix}^T$$

where $\bar{B}_j(s)$ and $\bar{C}_j(s)$ $(j \in \bar{\nu})$ are defined in (3.5). Then, the characteristic function of the closed-loop system is given by

$$\phi_{\Sigma^d,K}(s) = \det\left[s\hat{E}^e(s) - \hat{A}^e(s) - \hat{B}^e(s)\hat{K}^e(s)\hat{C}^e(s)\right] .$$

Now, consider $s_0 \in \Lambda_\mu\left(\Sigma^d, \mathbf{K_f}\right)$. Then,

$$\det\left[s_0\hat{E}^e(s_0) - \hat{A}^e(s_0) - \hat{B}^e(s_0)\hat{K}_0^e\hat{C}^e(s_0)\right] = 0 \tag{3.7}$$

for any $\hat{K}_0^e \in \mathbb{R}^{(m+p)\times(m+q)}$ with the above defined structure. However, since s_0 is fixed, $\hat{K}^e(s_0)$ is also a fixed matrix which has the same structure as $\hat{K}_0$, except that $\hat{K}_0^e$ is assumed to be real, whereas $\hat{K}^e(s_0)$ may be non-real for a non-real s_0. However, if (3.7) holds for all real $\hat{K}_0$ with a given structure, then it should also hold for all complex $\hat{K}_0$ with the same structure. This implies that $\phi_{\Sigma^d,K}(s_0) = 0$. Thus $\Lambda_\mu\left(\Sigma^d, \mathbf{K_f}\right) \subset \Lambda_\mu\left(\Sigma^d, \mathbf{K_d}\right)$. On the other hand, since $\mathbf{K_f} \subset \mathbf{K_d}$, a mode which can not be moved by any controller in $\mathbf{K_d}$, can not be moved by any controller in $\mathbf{K_f}$. Thus, $\Lambda_\mu\left(\Sigma^d, \mathbf{K_f}\right) \supset \Lambda_\mu\left(\Sigma^d, \mathbf{K_d}\right)$. □

Lemma 3.4. *For any given $\mu \in \mathbb{R}$, $\Lambda_\mu\left(\Sigma^d, \mathbf{K_s}\right) = \Lambda_\mu\left(\Sigma^d, \mathbf{K_f}\right) = \Lambda_\mu\left(\Sigma^d, \mathbf{K_d}\right)$.*

Proof. Follows from Lemmas 3.2 and 3.3. □

It was established in [16] that a necessary and sufficient condition for stabilizability of a LTI decentralized finite-dimensional dynamic system by controllers in $\mathbf{K_f}$ is that the system should not have any DFMs in $\bar{\mathbb{C}}_0^+$.

Also, it was shown in [38] that a necessary and sufficient condition for μ-stabilizability of a LTI retarded time-delay system Σ_r^d, by controllers in $\mathbf{K_f}$ is that the system should not have μ-DFMs with respect to the controller class $\mathbf{K_s}$. This condition was generalized for time-delay controllers characterized by $\mathbf{K_d}$ in [39] and it was shown that the necessity of this condition also continues to hold for LTI neutral time-delay systems as Σ^d. For the sufficiency of this condition, the proposed border of the stability region on the complex plane μ must be greater than or equal to $c_D + \epsilon$ for some $\epsilon > 0$ where c_D is defined in (2.8). Under this assumption, the LTI neutral time-delay system has only a finite number of modes in $\bar{\mathbb{C}}_\mu^+$ and sufficiency also continuous to hold.

According to these results, the following theorem can be concluded.

Theorem 3.1. *For a given $\mu \in \mathbb{R}$, there exists a μ-stabilizing controller $K \in \mathbf{K_d}$ for the system Σ^d if and only if $\mu \geq c_D + \epsilon$ for some $\epsilon > 0$ and $\Lambda_\mu(\Sigma^d, \mathbf{K_s}) = \emptyset$.*

Proof. First consider the closed-loop system obtained by applying a controller in $\mathbf{K_d}$ to Σ^d. Characteristic equation of the associated delay-difference equation of the closed-loop system can be given as

$$\det\left(\begin{bmatrix} I_n + \sum_{i=1}^{\sigma} E_i e^{-sh_i} & 0 \\ 0 & I_m \end{bmatrix}\right) = \det\left(I_n + \sum_{i=1}^{\sigma} E_i e^{-sh_i}\right) = 0 \ . \tag{3.8}$$

Since the modes of the associated delay-difference equations of Σ^d and of the closed-loop system are equal, μ-stability of the associated delay-difference equation of the closed-loop system is equivalent to the μ-stability of the the associated delay-difference equation of Σ^d. Then, from Lemma 2.1, there exits a controller $K \in \mathbf{K_d}$ which μ-stabilizes the system Σ^d only if $\mu \geq c_D + \epsilon$ for some $\epsilon > 0$ where c_D is the spectral abscissa, i.e., supremum of of the real parts of the roots of (3.8). Given this is true, assume that $\Lambda_\mu\left(\Sigma^d, \mathbf{K_s}\right) \neq \emptyset$, so there exists $s_0 \in \Lambda_\mu\left(\Sigma^d, \mathbf{K_s}\right)$. Then, according to Lemma 3.4, s_0 is also in $\Lambda_\mu\left(\Sigma^d, \mathbf{K_d}\right)$. This implies that $\phi_{\Sigma^d,K}(s_0) = 0$ for all $K \in \mathbf{K_d}$. This means that there is no μ-stabilizing controller in $\mathbf{K_d}$ for the system Σ^d. Thus, $\Lambda_\mu\left(\Sigma^d, \mathbf{K_s}\right) = \emptyset$ is a necessary condition in order to find a μ-stabilizing controller $K \in \mathbf{K_d}$ for the system Σ^d.

To prove the if part, first note that it was proved by Momeni et. al. [38] that there exists a 0-stabilizing controller $K \in \mathbf{K_f}$ for the retarded time-delay systems as Σ_r^d, if and only if $\Lambda_\mu(\Sigma, \mathbf{K_s}) = \emptyset$. This result is based on the main result of Kamen et. al. [40] and also all the arguments there are also valid for μ-stability instead of 0-stability for any finite real μ. In [40], it has been shown that a LTI centralized retarded time-delay system can be stabilized by a LTI time-delay controller if and only if it can be stabilized by a LTI finite-dimensional controller. Using the results of Emre and Knowles [43] for neutral time-delay systems, it can be shown that the main result of Kamen et. al. [40] is also valid for neutral time-delay systems as Σ^d, which has finitely many modes in $\bar{\mathbb{C}}^+_{\mu-\epsilon}$ for some $\epsilon > 0$. Then, from Theorem 2.1, it can be said that Σ^d has finitely many modes in $\bar{\mathbb{C}}^+_{\mu-\epsilon}$ for some $\epsilon > 0$, if and only if the associated delay difference equation (3.8) is μ-stable. This condition can be achieved by choosing $\mu \geq c_D + \epsilon$ for some $\epsilon > 0$. Also, as it was mentioned above, the set of the modes of neutral systems has many properties similar to the retarded case. So under the assumption of $\mu \geq c_D + \epsilon$ for some $\epsilon > 0$, the if part of the present theorem then follows from the result of Momeni et. al. [38], since $\mathbf{K_f} \subset \mathbf{K_d}$. □

Based on these results, decentralized counterpart of the main result of [40] can be obtained as follows.

Corollary 3.1. *For a given $\mu \in \mathbb{R}$, there exists a controller $K \in \mathbf{K_d}$ which μ-stabilizes the system Σ^d, if and only if there exists a controller $K \in \mathbf{K_f}$ which μ-stabilizes Σ^d.*

4. DECENTRALIZED CONTROLLER DESIGN BY CONTINUOUS POLE PLACEMENT METHOD

The objective in this chapter is to introduce controller synthesis techniques to design decentralized controllers, so that the closed-loop system is μ-stable, for some given real μ. In these methods, to remove the fixed-modes stemming from structural reasons, which will be called as *structural fixed modes*, the stabilization algorithms will be applied only to structurally controllable and observable part of the systems. For this reason, all the methods which will be introduced in this chapter, are for systems with commensurate-time-delays because a decomposition which gives the structurally controllable and observable part of a system with incommensurate-time-delays has not been investigated in the literature. Also, only the stabilization of the retarded time-delay systems are considered in this chapter. The reason behind this is that the neutral time-delay systems may have infinitely many μ-modes and this causes difficulties in finding the μ-modes of these systems. In Section 4.1, a centralized dynamic output feedback controller synthesis procedure is proposed. In Section 4.2, a decentralized controller synthesis procedure, based on decentralized pole assignment algorithm and the centralized controller synthesis procedure introduced in Section 4.1, is proposed. In Section 4.3, an observer based decentralized controller synthesis procedure, based on decentralized pole assignment algorithm and the continuous pole placement algorithm, for LTI retarded commensurate-time-delay systems is proposed.

4.1. Centralized Dynamic Output Feedback Controller Design

As it was mentioned in Section 2.2, continuous pole placement algorithm was originally presented for static state vector feedback controllers.

However, in decentralized control, the whole state vector is not generally available to any control agent. Furthermore, using static feedback almost never produce useful results to control a decentralized time-delay system. The algorithm introduced in the present section is the extension of the continuous pole placement algorithm to the case of centralized dynamic output feedback controllers.

Consider a centralized retarded commensurate-time-delay system described as

$$\begin{aligned} \dot{x}(t) &= \sum_{i=0}^{\sigma} \Big(A_i x(t-h_i) + B_i u(t-h_i) \Big) \\ y(t) &= \sum_{i=0}^{\sigma} C_i x(t-h_i) \end{aligned} \quad , \tag{4.1}$$

where $x(t) \in \mathbb{R}^n$, $u(t) \in \mathbb{R}^p$ and $y(t) \in \mathbb{R}^q$ are, respectively, the state, the input and the output vectors at time t and, $h_i = ih$ ($i \in \{0\} \cup \bar{\sigma}$) are commensurate-time-delays with a common divisor $h > 0$. Also, $A_i \in \mathbb{R}^{n \times n}$, $B_i \in \mathbb{R}^{n \times p}$ and $C_i \in \mathbb{R}^{q \times n}$ are constant real matrices. In order to obtain a more compact representation of (4.1), a delay operator τ, can be defined as $(\tau f)(t) := f(t-h)$ for any real valued function f of time t. Then, the matrix operators with elements in $\mathbb{R}[\tau]$ can be defined as

$$A(\tau) := \sum_{i=0}^{\sigma} A_i \tau^i \ , \quad B(\tau) := \sum_{i=0}^{\sigma} B_i \tau^i \ , \quad C(\tau) := \sum_{i=0}^{\sigma} C_i \tau^i \ . \tag{4.2}$$

Then, using the delay operator τ, (4.1) can be compactly represented as

$$\begin{aligned} \dot{x}(t) &= A(\tau)x(t) + B(\tau)u(t) \\ y(t) &= C(\tau)x(t) \end{aligned} \quad . \tag{4.3}$$

Then, consider the LTI centralized finite-dimensional dynamic controllers of the following form:

$$\begin{aligned} \dot{z}(t) &= Fz(t) + Gy(t) \\ u(t) &= Hz(t) + Ky(t) \end{aligned} \tag{4.4}$$

where $z(t) \in \mathbb{R}^m$ is the state vector of the controller at time t, and $F \in \mathbb{R}^{m \times m}$, $G \in \mathbb{R}^{m \times q}$, $H \in \mathbb{R}^{p \times m}$, and $K \in \mathbb{R}^{p \times q}$ are the matrices of the controller which are structured in a canonical form (e.g., see Chapter 6 of [44]). Note that when the controller dimension $m = 0$, such a controller reduces to a centralized static output feedback controller defined in class $\mathbf{K_c}$. Also, by using the controller matrices, define

$$K^e := \begin{bmatrix} K & H \\ G & F \end{bmatrix} \in \mathbb{R}^{(m+p)\times(m+q)} .$$

In [40], it was shown that there exists a μ-stabilizing controller of the form (4.4) (with sufficiently large dimension) for a system of the form (4.1) if and only if the system does not have any μ-CFMs. However, in a decentralized framework, even though the overall system does not have any μ-DFMs, the system from a particular input channel to the corresponding output channel may have μ-CFMs. In order to remove μ-CFMs which may be stemming from structural reasons, this centralized stabilization algorithm should be applied only to structurally controllable and observable part of the given system. To identify the structurally controllable and observable part, it is required to present the following definition and lemma from [45].

Definition 4.1. The system (4.3), equivalently the pair $\big(A(\cdot), B(\cdot)\big)$, is said to be *structurally controllable* if the matrix

$$\begin{bmatrix} B(\tau) & A(\tau)B(\tau) & \dots & (A(\tau))^{n-1}\,B(\tau) \end{bmatrix}$$

is full rank over $\mathbb{R}[\tau]$. Also, the system (4.1), equivalently the pair $\big(C(\cdot), A(\cdot)\big)$, is said to be *structurally observable* if the matrix

$$\left[C^T(\tau) \;\; A^T(\tau)C^T(\tau) \;\; \dots \;\; \left(A^T(\tau)\right)^{n-1} C^T(\tau) \right]$$

is full rank over $\mathbb{R}[\tau]$. Furthermore, the triple $\big(C(\cdot), A(\cdot), B(\cdot)\big)$ is said to be *structurally controllable and observable* if the pair $\big(A(\cdot), B(\cdot)\big)$ is structurally controllable and the pair $\big(C(\cdot), A(\cdot)\big)$ is structurally observable.

Lemma 4.1. *Consider the system (4.3). There exist a unimodular transformation matrix $T(\tau) \in \mathbb{R}^{n\times n}[\tau]$ such that the transformed system has the* *following canonical form*

$$\begin{bmatrix} \dot{x}_{co}(t) \\ \dot{x}_{c\bar{o}}(t) \\ \dot{x}_{\bar{c}}(t) \end{bmatrix} = \begin{bmatrix} A_{co}(\tau) & 0 & A_{13}(\tau) \\ A_{21}(\tau) & A_{c\bar{o}}(\tau) & A_{23}(\tau) \\ 0 & 0 & A_{\bar{c}}(\tau) \end{bmatrix} \begin{bmatrix} x_{co}(t) \\ x_{c\bar{o}}(t) \\ x_{\bar{c}}(t) \end{bmatrix} + \begin{bmatrix} B_{co}(\tau) \\ B_{c\bar{o}}(\tau) \\ 0 \end{bmatrix} u(t)$$
$$y(t) = \begin{bmatrix} C_{co}(\tau) & 0 & C_{\bar{c}}(\tau) \end{bmatrix} \begin{bmatrix} x_{co}(t) \\ x_{c\bar{o}}(t) \\ x_{\bar{c}}(t) \end{bmatrix} \tag{4.5}$$

where the triple $\big(C_{co}(\cdot), A_{co}(\cdot), B_{co}(\cdot)\big)$ *is structurally controllable and observable. Furthermore, the system (4.3) is zero-state equivalent to the system*

$$\begin{aligned} \dot{x}_{co}(t) &= A_{co}(\tau)x_{co}(t) + B_{co}(\tau)u(t) \\ y(t) &= C_{co}(\tau)x_{co}(t) \end{aligned} \tag{4.6}$$

which is both structurally controllable and observable.

Proof. See [45]. □

The system (4.6) will be referred as the *structurally controllable and observable part* of the system (4.3). The modes of the rest of the system, i.e., the roots of

$$\det\left(\begin{bmatrix} sI - \bar{A}_{c\bar{o}}(s) & -\bar{A}_{23}(s) \\ 0 & sI - \bar{A}_{\bar{c}}(s) \end{bmatrix}\right) = 0 \ , \tag{4.7}$$

will be called *structural fixed modes* of the system where $\bar{A}_{c\bar{o}}(s)$, $\bar{A}_{23}(s)$ and $\bar{A}_{\bar{c}}(s)$ are respectively obtained from the operator matrices $A_{c\bar{o}}(\tau)$, $A_{23}(\tau)$, and $A_{\bar{c}}(\tau)$, appearing in (4.5), by replacing the operator τ by the function e^{-hs}. Note that, any structural fixed mode is a CFM of the original system (4.1). Unlike, finite-dimensional systems, the structurally controllable and observable part may still have fixed modes, which are also CFMs of the original system (4.1). These modes will be called *unstructural fixed modes*.

During the continuous pole placement algorithm, when a controlled mode approaches to a fixed mode, the sensitivity of the controlled mode with respect to changes in the controller parameters become considerably small and this causes very large changes in the controller parameters even for very small desired displacements for the controlled modes. This is one of the major reasons for the failure of the continuous pole placement algorithm. By using the transformation given in Lemma 4.1, structural CFMs can be separated from the system. Thus, this problem will be avoided for structural CFMs by using only the structurally controllable and observable part in the stabilization algorithm instead of the whole system. However, even if such a decomposition is done, there may exist unstructural μ-CFMs in the structurally controllable and observable part of the system. It should be noted that, presence of a real unstructural μ-CFM on the left side of any real controlled mode may result in the failure of the stabilization algorithm. Because, when only the real parts of modes are controlled, approach of a real mode to an unstructural μ-CFM may cause sensitivity matrices with considerably small norms which result in very large changes in the controller parameters. Also, a similar situation may occur for any real transmission zero in $\bar{\mathbb{C}}_\mu^+$, which are located on the left side of any real controlled mode. Similar to the previous case, when only the real parts of modes are controlled, approach of a real mode to a real transmission zero in $\bar{\mathbb{C}}_\mu^+$ may also cause sensitivity matrices with considerably small norms. In order to avoid these situations, define a set Ψ_μ which contains all the real unstructural μ-CFMs and real transmission zeros in $\bar{\mathbb{C}}_\mu^+$. If Ψ_μ is not an empty set, while moving the controlled modes towards $\mathbb{C}_\mu^-$, they must be forced to go around any member of Ψ_μ. That is, any real controlled mode on the right side of any member of Ψ_μ, must first be combined with another real controlled mode to form a complex-conjugate pair. Then, one of these modes must first be moved upwards, and the other one downwards on the complex plane, before moving to the left, one passing from above, and the other one from below the member of Ψ_μ. For this, however, there must exist an even number of real modes between any two members of Ψ_μ and to the right of the rightmost member of Ψ_μ. If the given system does not satisfy this condition, then controller parameters must be initialized such that real controller modes are added wherever needed.

Before presenting our algorithm, following definitions are required to define the closed loop system structure. First define

$$\hat{A}^e_{co}(s) := \begin{bmatrix} \bar{A}_{co}(s) & 0 \\ 0 & 0_m \end{bmatrix}, \quad \hat{B}^e_{co}(s) := \begin{bmatrix} \bar{B}_{co}(s) & 0 \\ 0 & I_m \end{bmatrix},$$

and

$$\hat{C}^e_{co}(s) := \begin{bmatrix} \bar{C}_{co}(s) & 0 \\ 0 & I_m \end{bmatrix},$$

where $\bar{A}_{co}(s)$, $\bar{B}_{co}(s)$, and $\bar{C}_{co}(s)$ are respectively obtained from the operator matrices $A_{co}(\tau)$, $B_{co}(\tau)$, and $C_{co}(\tau)$, appearing in (4.6), by replacing the operator τ by the function e^{-hs}.

Now consider the controller (4.4) with $\hat{m} := m(p+q)+pq$ free parameters. Let $\tilde{K}^e \in \mathbb{R}^{\hat{m}}$ be the vector of the free parameters, $\tilde{k}^e_1, \ldots, \tilde{k}^e_{\hat{m}}$, of the controller (4.4). Then, the characteristic function of the closed-loop system, obtained by applying the controller (4.4) to the system (4.6), is then obtained as

$$\phi_{\Sigma,K}(s) = \det\left[sI - \hat{A}^e_{co}(s) - \hat{B}^e_{co}(s)K^e\hat{C}^e_{co}(s)\right]. \tag{4.8}$$

Therefore, finding a μ-stabilizing controller (4.4) for (4.6) is equivalent to finding a parameter set $\tilde{K}^e \in \mathbb{R}^{\hat{m}}$ such that all roots of $\phi_{\Sigma,K}(s) = 0$ are in $\mathbb{C}^-_\mu$.

Now, let $s_i \in \mathbb{C}$ be a mode of the closed-loop system, i.e., $\phi_{\Sigma,K}(s_i) = 0$. Then,

$$\begin{aligned} \left(s_iI - \hat{A}^e_{co}(s_i) - \hat{B}^e_{co}(s_i)K^e\hat{C}^e_{co}(s_i)\right)v_i &= 0 \\ N(v_i) &= 1 \end{aligned} \tag{4.9}$$

where $v_i \in \mathbb{C}^{n_{co}+m}$ is a non-zero vector, where n_{co} is the dimension of x_{co} in (4.6), and $N(\cdot)$ is a normalizing function, for example, one can choose $N(v) = v^*v$. Differentiating (4.9) with respect to a controller parameter $\tilde{k}^e_\psi$ $\big(\psi = 1, \ldots, \hat{m}\big)$, a linear system of $n_{co}+m+1$ equations is obtained as follows

$$\begin{aligned} &\left(s_iI - \hat{A}^e_{co}(s_i) - \hat{B}^e_{co}(s_i)K^e\hat{C}^e_{co}(s_i)\right)\frac{\partial v_i}{\partial \tilde{k}^e_\psi} \\ +&\left(I - \frac{\partial \hat{A}^e_{co}(s_i)}{\partial s_i} - \frac{\partial \hat{B}^e_{co}(s_i)}{\partial s_i}K^e\hat{C}^e_{co}(s_i) - \hat{B}^e_{co}(s_i)K^e\frac{\partial \hat{C}^e_{co}(s_i)}{\partial s_i}\right)v_i\frac{\partial s_i}{\partial \tilde{k}^e_\psi} \\ &- \left(\hat{B}^e_{co}(s_i)\frac{\partial K^e}{\partial \tilde{k}^e_\psi}\hat{C}^e_{co}(s_i)\right)v_i = 0 \end{aligned}$$

and

$$\left(\frac{\partial N(v_i)}{\partial v_i}\right)\frac{\partial v_i}{\partial \tilde{k}^e_\psi} = 0 \ ,$$

with $n_{co}+m+1$ unknowns, where the unknowns are $\partial s_i/\partial \tilde{k}^e_\psi$ and the $n_{co}+m$ components of $\partial v_i/\partial \tilde{k}^e_\psi$.

Assume that the modes $s_1,\ldots,s_k$ ($k \le \hat{m}$) are desired to be shifted towards $\mathbb{C}^-_\mu$. In the sequel these modes will be referred to as the *controlled modes*. Now define the *sensitivity matrix* Θ_k as follows

$$\Theta_k := [\theta_{i,\psi}] \in \mathbb{C}^{k\times\hat{m}} \ \text{ where } \ \theta_{i,\psi} := \frac{\partial s_i}{\partial \tilde{k}^e_\psi} \ . \tag{4.10}$$

Let $\Delta\tilde{S}^d_k := \left[\Delta s^d_1 \ \ldots \ \Delta s^d_k\right]^T \in \mathbb{C}^k$ be the desired displacement of the k controlled modes. Assuming that $\Delta\tilde{S}^d_k$ is in the range space of Θ_k, the corresponding change $\Delta\tilde{K}^e$ for $\tilde{K}^e$ can be computed from

$$\Theta_k\Delta\tilde{K}^e = \Delta\tilde{S}^d_k \ . \tag{4.11}$$

We note that $\Delta\tilde{S}^d_k$ must be chosen such that all elements of $\Delta\tilde{K}^e$ are real. This is achieved by choosing $\Delta\tilde{S}^d_k$ such that complex-conjugate modes remain as complex-conjugate or both become real and no real mode becomes a complex mode unless another real mode becomes its complex-conjugate. As in [41], when $\text{rank}(\Theta_k) = k$, a solution to (4.11), with minimal $\|\Delta\tilde{K}^e\|$, is given by

$$\Delta\tilde{K}^e = \Theta^\dagger_k\Delta\tilde{S}^d_k \ , \tag{4.12}$$

where $\Theta^\dagger_k$ is the Moore-Penrose inverse of Θ_k (see [42]).

Now the basic algorithm for designing a centralized dynamic output feedback controller, is as follows.

Algorithm 4.1. *Centralized dynamic output feedback controller design algorithm by continuous pole placement method*

1) *Initialize controller dimension m (see Remark 4.1 below).*

2) *Initialize $\tilde{K}^e \in \mathbb{R}^{\hat{m}}$, where $\hat{m} = m(p+q)+pq$ (see Remark 4.1 below).*

3) Compute (e.g., by the method proposed in [46]) the roots of (4.8) with real parts greater than or equal to $(\mu - \varepsilon)$ *for some* $\varepsilon > 0$. *If there are no roots with real parts greater than or equal* μ, *stop:* μ*-stability is achieved with the current* $\tilde{K}^e$. *Otherwise, let* η *be the real part of the rightmost root and* k *be the number of roots with real part greater than or equal to* $\eta - \varepsilon$ *(note that* $k \geq 1$*). If* $k > \hat{m}$, *increase* m *so that* $k \leq \hat{m}$ *and go to step 2. Otherwise, define the rightmost* k *roots as the controlled modes and continue with step 4.*

4) Compute the sensitivity matrix, Θ_k, *defined in (4.10). Let* $\rho := \operatorname{rank}(\Theta_k)$.

5) If $\rho = k$, *choose the desired displacement of the* k *controlled modes,* $\Delta\tilde{S}_k^d$, *such that all* k *controlled modes move towards* $\mathbb{C}_\mu^-$ *(see Remark 4.2 below). Compute* $\Delta\tilde{K}^e$ *by (4.12) and go to step 7.*

6) If $\rho < k$, *check if a* $\Delta\tilde{S}_k^d$ *in the range space of* Θ_k *can be chosen so that all* k *controlled modes move towards* $\mathbb{C}_\mu^-$ *(see Remark 4.2 below). If so, using this* $\Delta\tilde{S}_k^d$, *compute a suitable* $\Delta\tilde{K}^e$ *which satisfies (4.11) and go to step 7. Otherwise, increase the controller dimension* m *by one and go to step 2.*

7) Update $\tilde{K}^e$ *as* $\tilde{K}^e + \Delta\tilde{K}^e$ *and go to step 3.*

Remark 4.1. If the set Ψ_μ is empty, initialize the controller dimension as $m = 0$ and the vector of the free controller parameters as $\tilde{K}^e = 0_{\hat{m}\times 1}$. Otherwise, initialize appropriate controller dimension and parameters to make sure that there exists an even number of real modes on the right side of the rightmost member of Ψ_μ and between any two members of Ψ_μ.

Remark 4.2. If the set Ψ_μ is not empty, desired displacements of the controlled modes must first be chosen to form complex-conjugate pairs by combining the real controlled modes on the right side of any member of Ψ_μ. Then, for each complex-conjugate pair, by choosing the desired displacements in complex-conjugate pairs, one of the modes forming the pair must first be moved upwards, and the other one downwards on the complex plane, before moving towards $\mathbb{C}_\mu^-$, one passing from above, and the other one from below any member of Ψ_μ.

4.2. Decentralized Controller Design

The objective in this section is to design μ-stabilizing decentralized controllers for the system described as

$$\begin{aligned} \dot{x}(t) &= \sum_{i=0}^{\sigma}\left(A_i x(t-h_i) + \sum_{j=1}^{\nu} B_{j,i} u_j(t-h_i)\right) \\ y_j(t) &= \sum_{i=0}^{\sigma} C_{j,i} x(t-h_i), \quad j \in \bar{\nu} \end{aligned} \tag{4.13}$$

where $x(t) \in \mathbb{R}^n$ is the state vector at time t, $u_j(t) \in \mathbb{R}^{p_j}$ and $y_j(t) \in \mathbb{R}^{q_j}$ are, respectively, the input and the output vectors at time t, accessible by the j^{th} control agent ($j \in \bar{\nu}$). The matrices $A_i \in \mathbb{R}^{n\times n}$, $B_{j,i} \in \mathbb{R}^{n\times p_j}$ and $C_{j,i} \in \mathbb{R}^{q_j\times n}$ are constant real matrices and $h_i = ih$, ($i \in \{0\} \cup \bar{\sigma}$), are commensurate-time-delays with a common divisor $h > 0$. Similar to (4.3), using the delay operator τ, this system can be compactly represented as

$$\begin{aligned} \dot{x}(t) =& A(\tau)x(t) + \sum_{j=1}^{\nu} B_j(\tau)u_j(t) \\ y_j(t) =& C_j(\tau)x(t), \quad j \in \bar{\nu} \end{aligned}, \tag{4.14}$$

where

$$B_j(\tau) := B_{j,0} + \sum_{i=1}^{\sigma} B_{j,i}\tau^i, \;\; C_j(\tau) := C_{j,0} + \sum_{i=1}^{\sigma} C_{j,i}\tau^i,$$

for $j \in \bar{\nu}$, and $A(\tau)$ is defined in (4.2).

The controllers which are considered in this section are in class $\mathbf{K_f}$, described as (2.22). Now suppose that decentralized controllers of the form (2.22) has been designed for the first k control agents, where $k < \nu$. Let $m^k = \sum_{j=1}^{k} m_j$, $p^k = \sum_{j=1}^{k} p_j$, $q^k = \sum_{j=1}^{k} q_j$ and define

$$\hat{B}_k(\tau) := \begin{bmatrix} B_1(\tau) & \dots & B_k(\tau) \end{bmatrix}, \;\; \hat{C}_k(\tau) := \begin{bmatrix} C_1^T(\tau) & \dots & C_k^T(\tau) \end{bmatrix}^T,$$

and

$$\hat{A}_k^e(\tau) := \begin{bmatrix} A(\tau) & 0 \\ 0 & 0_{m^k} \end{bmatrix}, \hat{B}_k^e(\tau) := \begin{bmatrix} \hat{B}_k(\tau) & 0 \\ 0 & I_{m^k} \end{bmatrix}, \hat{C}_k^e(\tau) := \begin{bmatrix} \hat{C}_k(\tau) & 0 \\ 0 & I_{m^k} \end{bmatrix}.$$

Also, define the controller matrix

$$\hat{K}_k^e := \begin{bmatrix} \hat{K}_k & \hat{H}_k \\ \hat{G}_k & \hat{F}_k \end{bmatrix} \in \mathbb{R}^{(m^k+p^k)\times(m^k+q^k)} , \tag{4.15}$$

where

$$\begin{aligned} &\hat{F}_k := \text{bdiag}\left(F_1, \ldots, F_k\right) \ , \quad \hat{G}_k := \text{bdiag}\left(G_1, \ldots, G_k\right) \ , \\ &\hat{H}_k := \text{bdiag}\left(H_1, \ldots, H_k\right) \ , \quad \hat{K}_k := \text{bdiag}\left(K_1, \ldots, K_k\right) \ . \end{aligned} \tag{4.16}$$

Then, the resultant system, Σ_k, for the $(k+1)^{th}$ local control agent, with input u_{k+1} and output y_{k+1}, is described by

$$\begin{aligned} \dot{\xi}_k(t) &= \left(\hat{A}_k^e(\tau) + \hat{B}_k^e(\tau)\hat{K}_k^e\hat{C}_k^e(\tau)\right)\xi_k(t) + \begin{bmatrix} B_{k+1}(\tau) \\ 0_{m^k\times p_{k+1}} \end{bmatrix} u_{k+1}(t) \\ y_{k+1}(t) &= \begin{bmatrix} C_{k+1}(\tau) & 0_{q_{k+1}\times m^k} \end{bmatrix} \xi_k(t) \end{aligned} \tag{4.17}$$

where $\xi_k(t) := \begin{bmatrix} x^T(t) & z_1^T(t) & \ldots & z_k^T(t) \end{bmatrix}^T \in \mathbb{R}^{n+m^k}$. Since Σ_k is a centralized control system for $k = 0, \ldots, \nu - 1$, as in Lemma 4.1, Σ_k can be decomposed. Structurally controllable and observable part of Σ_k will be denoted by Σ_k^{co}.

Now the basic algorithm for designing a μ-stabilizing overall decentralized controller for the system (4.14), is as follows.

Algorithm 4.2. *Decentralized dynamic output feedback controller design algorithm by output continuous pole placement method*

1) *Fix upper limits, $\tilde{m}_1, \ldots, \tilde{m}_\nu$, on the dimensions of the decentralized controllers.*

2) *Let $k = 0$.*

3) *If Σ_k^{co} is μ-stable, let $m_{k+1} = 0$ and choose a random non-zero $K_{k+1} \in \mathbb{R}^{p_{k+1}\times q_{k+1}}$ such that the closed-loop system obtained by applying the static output feedback $u_{k+1}(t) = K_{k+1}y_{k+1}(t)$ to Σ_k^{co} is μ-stable (by the continuity of the modes with respect to the feedback gains, there exists such a K_{k+1} - see [47]) and go to step 5. Otherwise, continue with step 4.*

4) Apply Algorithm 4.1 to Σ_k^{co} to design a controller of the form (2.22) with $j = k+1$ of dimension not greater than $\tilde{m}_{k+1}$ to μ-stabilize it. If such a controller can not be designed, use the last controller with dimension $\tilde{m}_{k+1}$ which moves as many controlled modes as possible towards $\mathbb{C}_\mu^-$.

5) If $k = \nu - 1$, go to step 6. Otherwise, set $k = k+1$ and go to step 3.

6) If the overall closed-loop system Σ_ν is μ-stable, stop: the desired decentralized controller has been obtained. Otherwise, increase the upper limits, $\tilde{m}_1, \ldots, \tilde{m}_\nu$, and go to step 2.

This algorithm is an extension of the decentralized pole assignment algorithm of Davison and Chang [22] to the time-delay case. In the algorithm, the extension of the continuous pole placement algorithm, which is given by Algorithm 4.1, is used as the centralized controller design algorithm at each step. To avoid using unnecessarily high-dimensional controllers for the lower indexed control agents, in the first step, upper limits on the dimensions of the decentralized controllers are defined. In the third step, even if the resultant system is stable, it is essential to apply a μ-stabilizing controller. It is indicated in [22] that if at least a static output feedback loop is not closed at each step, some μ-modes of the overall system may not appear as the modes of Σ_k^{co} in the remaining steps, even if they are not μ-DFMs. So the reason for applying a static output feedback controller in the third step, whenever Σ_k^{co} is μ-stable, is to make sure that any μ-mode of the system (4.13), which is not a μ-DFM, appears as a mode of Σ_s^{co} for some $s > k$, so that, it can be moved towards $\mathbb{C}_\mu^-$ in a later step.

4.3. Observer Based Decentralized Controller Design

As it was mentioned above, in decentralized control, the whole state vector is not generally available to any control agent and each agent measures only a subset of all the outputs. Therefore, in Section 4.1, an extension of the continuous pole placement algorithm to the case of dynamic output feedback controllers was introduced to be used as a centralized controller synthesis procedure in the decentralized pole assignment algorithm. On the other hand, instead of this algorithm, static state vector feedback controller based continuous pole placement method can also be used in the decentralized pole assignment algorithm by constructing observers at each step of the algorithm. As discussed in Section 4.1, when only the real parts of the modes are controlled in the continuous pole placement algorithm, approach of a real mode to a member of Ψ_μ, which contains all the real unstructural μ-CFMs and real transmission zeros in $\bar{\mathbb{C}}_\mu^+$, may cause sensitivity matrices with considerably small norms which may result in very large changes in controller parameters. Unlike the algorithm introduced in the previous section, the algorithm in this section is not able to overcome this problem without adding some additional dynamics to the system. Therefore, for this algorithm it is assumed that if Ψ_μ is not an empty set, all the modes on the right of any member are formed as complex-conjugate pairs or there are an even number of real modes, which can be formed as complex-conjugate pairs, on the right of the rightmost member of Ψ_μ or between any two members of Ψ_μ. Also, it should be noted that, as indicated in the previous section, in some steps of the decentralized pole assignment algorithm, systems seen by the control agents can be stable. In such a case, because of the reasons mentioned in the previous section, a feedback loop should be closed. But instead of an observer based state vector feedback controller, a stabilizing static output feedback controller, $u_j(t) = \tilde{K}_j y_j(t)$, $\tilde{K}_j \in \mathbb{R}^{p_j \times q_j}$, can be designed. Also, as it was indicated in the previous section, even though the overall system does not have any μ-DFMs, the system from a particular input channel to the corresponding output channel may have μ-CFMs. Thus, firstly at each step, system should be decomposed as in Lemma 4.1 to remove the structural CFMs. Then, only structurally controllable and observable part of the system will be taken in the controller and observer synthesis at each step.

As in Section 4.2, assume that the first k control agents has been designed, where $k < \nu$. Then, by applying these k control agents to the system Σ, the resultant system Σ_k for the $(k+1)^{th}$ local control agent with input

$u_{k+1}(t)$ and output $y_{k+1}(t)$ is obtained. As in Lemma 4.1, Σ_k can be decomposed and its structurally controllable and observable part can be obtained. Let the structurally controllable and observable part, Σ_k^{co}, of Σ_k be as follows

$$\begin{aligned}\dot{x}_k^{co}(t) &= \Delta_k^{co}(\tau)x_k^{co}(t) + B_{k+1}^{co}(\tau)u_{k+1}(t)\\ y_{k+1}(t) &= C_{k+1}^{co}(\tau)x_k^{co}(t)\end{aligned}$$

where $x_k^{co} \in \mathbb{R}^{n_k^{co}}$ is the structurally controllable and observable part of the state vector of Σ_k, with dimension n_k^{co}, and $\Delta_k^{co}(\tau)$ is the structurally controllable and observable part of the dynamics matrix $\Delta_k(\tau)$ of the system Σ_k. An observer based state vector feedback for Σ_k^{co} can be obtained as

$$\begin{aligned}\dot{\hat{x}}_k^{co}(t) &= \Delta_k^{co}(\tau)\hat{x}_k^{co}(t) + B_{k+1}^{co}(\tau)u_{k+1}(t)\\ &\quad + L_{k+1}^T\left(C_{k+1}^{co}(\tau)\hat{x}_k^{co}(t) - y_{k+1}(t)\right)\\ u_{k+1}(t) &= K_{k+1}\hat{x}_k^{co}(t)\end{aligned} \tag{4.18}$$

where $\hat{x}_k^{co}(t) \in \mathbb{R}^{n_k^{co}}$ is the estimated state vector at time t, $K_{k+1} \in \mathbb{R}^{p_{k+1}\times n_k^{co}}$ is the controller gain and $L_{k+1} \in \mathbb{R}^{q_{k+1}\times n_k^{co}}$ is the observer gain matrices. Then, by defining the state vector estimation error for this observer as $e_k(t) := x_k^{co}(t) - \hat{x}_k^{co}(t)$ and applying the estimated state vector feedback, following equations can be obtained

$$\begin{aligned}\dot{x}_k^{co}(t) &= \left(\Delta_k^{co}(\tau) + B_{k+1}^{co}(\tau)K_{k+1}\right)x_k^{co}(t) - B_{k+1}^{co}(\tau)K_{k+1}e_k(t)\\ \dot{e}_k(t) &= \Big(\Delta_k^{co}(\tau) + L_{k+1}^T C_{k+1}^{co}(\tau)\Big)e_k(t)\end{aligned}\,. \tag{4.19}$$

Characteristic equation of (4.19) can be written as

$$\det\left(\begin{bmatrix} sI_{n_k^{co}} - \bar{\Delta}_k^{co}(s) - \bar{B}_{k+1}^{co}(s)K_{k+1} & \bar{B}_{k+1}^{co}(s)K_{k+1}\\ 0_{n_k^{co}} & sI_{n_k^{co}} - \bar{\Delta}_k^{co}(s) - L_{k+1}^T\bar{C}_{k+1}^{co}(s)\end{bmatrix}\right) = 0$$

where the Laplace transform of the operator matrices can be obtained by replacing the operator τ by the function e^{-hs}. Because of the block-triangular structure, the modes of (4.19) consist of the controller modes which are the solutions of

$$\det\left(sI_{n_k^{co}} - \bar{\Delta}_k^{co}(s) - \bar{B}_{k+1}^{co}(s)K_{k+1}\right) = 0\ , \tag{4.20}$$

and the observer modes which are the solutions of

$$\begin{aligned} &\det\left(sI_{n_k^{co}} - \bar{\Delta}_k^{co}(s) - L_{k+1}^T \bar{C}_{k+1}^{co}(s)\right) \\ &= \det\left(sI_{n_k^{co}} - \left(\bar{\Delta}_k^{co}(s)\right)^T - \left(\bar{C}_{k+1}^{co}(s)\right)^T L_{k+1}\right) = 0 \end{aligned} . \tag{4.21}$$

Thus the continuous pole placement method need to be applied once to (4.20) to obtain the controller gain K_{k+1} and once to (4.21) to obtain the observer gain L_{k+1}.

Now the basic algorithm, for designing μ-stabilizing decentralized controllers for the system (4.14), is as follows.

Algorithm 4.3. *Observer based decentralized controller design algorithm*

1) *Let $k = 0$.*

2) *If Σ_k^{co} is μ-stable, choose a random non-zero $\tilde{K}_{k+1} \in \mathbb{R}^{p_{k+1} \times q_{k+1}}$ such that the closed-loop system obtained by applying the static output feedback $u_{k+1}(t) = \tilde{K}_{k+1} y_{k+1}(t)$ to Σ_k^{co} is μ-stable and go to step 4. Otherwise, continue with step 3.*

3) *Apply the continuous pole placement method once to (4.20) to obtain the controller gain K_{k+1} and once to (4.21) to obtain the observer gain L_{k+1}. If stabilizing gain matrices can not be found, use the controller and observer gains which move as many controlled modes as possible towards $\mathbb{C}_\mu^-$. Then, apply the resulting observer based controller (4.18) to Σ_k^{co}.*

4) *If $k = \nu - 1$, go to step 5. Otherwise, set $k = k + 1$ and go to step 2.*

5) *If the overall closed-loop system Σ_ν is μ-stable, stop: the desired decentralized observer based controller has been obtained. Otherwise repeat the algorithm by trying to design different observer based control agents or control agents which apply dynamic output feedback in step 3.*

Similar to Algorithm 4.2, this algorithm is also an extension of the decentralized pole assignment algorithm to the time-delay case. In the algorithm, depending on the stability of the system at each step, controller agents which apply observer based state vector feedback or static output feedback, are designed by a centralized controller design algorithm. Similar to Algorithm 4.2, the reason behind applying static output feedback controllers even if the resultant system is stable is to make sure that any μ-mode of the system (4.13), which is not a μ-DFM, appears as a mode of Σ_s^{co} for some $s > k$, so that, it can be moved towards $\mathbb{C}_\mu^-$ in a later step. A μ-stabilizing decentralized controller can be designed by this algorithm if a μ-stabilizing control agent, which applies observer based state vector feedback, can be designed at each step that requires observer based state vector feedback. Otherwise, designing a μ-stabilizing decentralized controller by this algorithm can not be guaranteed.

5. EXAMPLES

In this chapter, we will present two examples in order to illustrate the design approaches proposed in Section 4.2 and Section 4.3. In the following example, a decentralized controller is designed by using the design approach proposed in Section 4.2.

Example 5.1. *Consider a LTI retarded time-delay system Σ_0 described as in (4.13) with $\nu = 2$, $\sigma = 1$, $h = h_1 = 1$,*

$$A_0 = \begin{bmatrix} 7 & 9 & 7 & 9 \\ 0 & -1 & 4 & -2 \\ -11 & -6 & -7 & -11 \\ -22 & -12 & -4 & -27 \end{bmatrix}, \quad A_1 = \begin{bmatrix} -4 & 6 & -8 & -1 \\ 0 & 4 & 0 & 0 \\ 5 & -3 & 9 & 1 \\ 10 & -6 & 6 & 8 \end{bmatrix},$$

$$B_{1,0} = \begin{bmatrix} -4 \\ -3 \\ 2 \\ 4 \end{bmatrix}, \quad B_{1,1} = \begin{bmatrix} 2 \\ 1 \\ -1 \\ -2 \end{bmatrix}, \quad B_{2,0} = \begin{bmatrix} 3 \\ 0 \\ -3 \\ -5 \end{bmatrix}, \quad B_{2,1} = \begin{bmatrix} 1 \\ 0 \\ -1 \\ -1 \end{bmatrix},$$

$$C_{1,0} = \begin{bmatrix} 0 & 1 & 4 & -2 \end{bmatrix}, \quad C_{1,1} = \begin{bmatrix} 1 & -1 & 1 & 0 \end{bmatrix},$$

$$C_{2,0} = \begin{bmatrix} 1 & -1 & 2 & -0.5 \end{bmatrix}, \quad C_{2,1} = \begin{bmatrix} 1 & 0 & 0 & 1 \end{bmatrix}.$$

By using the programs of [46], for $\epsilon = 1$, $-\epsilon$-modes of the system can be found as

$$\begin{aligned} \Omega_{-\epsilon}(\Sigma) = \{&0.7990, 0.1523, -0.1904 \pm 5.4367\mathrm{i}, \\ &-0.2104 \pm 4.8730\mathrm{i}, -0.7049 + 11.3571\mathrm{i}\}, \end{aligned} \tag{5.1}$$

which are plotted in Figure 5.1., using '$*$'. Since the system has two modes, $s_1 = 0.7990$ and $s_2 = 0.1523$, with non-negative real parts, the system is not μ-stable for $\mu = 0$. Furthermore, s_1 is a CFM for control agent 2 and s_2 is a CFM for control agent 1. Hence, the system is not stabilizable by any one of the control agents alone. However, neither s_1, nor s_2 is a DFM, hence, it is possible to μ-stabilize the system by decentralized feedback. To obtain a stabilizing decentralized controller, Algorithm 4.2 is applied.

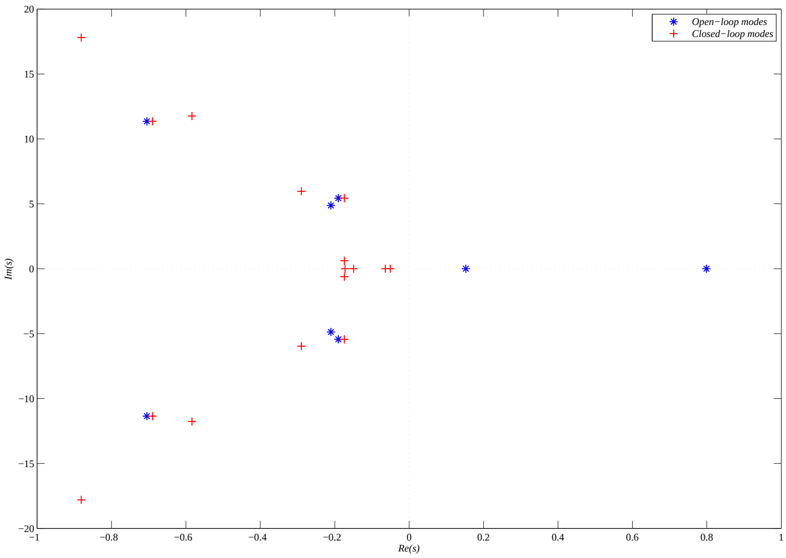

Figure 5.1.: Open-loop and closed-loop modes of the system in Example 5.1.

In the first phase, structurally controllable and observable part, Σ_0^{co}, of the system seen by the first control agent is obtained as follows

$$\begin{aligned}
\dot{x}_1^{co}(t) &= \begin{bmatrix} -1 & -9 \\ 0 & -4 \end{bmatrix} x_1^{co}(t) + \begin{bmatrix} 4 & -6 \\ 0 & 1 \end{bmatrix} x_1^{co}(t-1) \\
&+ \begin{bmatrix} 0 & 3 \\ 0 & 0 \end{bmatrix} x_1^{co}(t-2) + \begin{bmatrix} 0 \\ 1 \end{bmatrix} u_1(t) \qquad . \\
y_1(t) &= \begin{bmatrix} 1 & -3 \end{bmatrix} x_1^{co}(t) + \begin{bmatrix} 0 & 2 \end{bmatrix} x_1^{co}(t-1)
\end{aligned} \tag{5.2}$$

Then, Algorithm 4.1 is applied to Σ_0^{co} to design a stabilizing controller and for $m = 0$, a stabilizing controller can be designed as

$$u_1(t) = 2.60718\ y_1(t) \tag{5.3}$$

The progress of the algorithm, i.e., the real parts of the rightmost modes and the controller parameters as a function of the iterations, is shown in Figure 5.2..

In the second phase, structurally controllable and observable part of the system seen by the second control agent, Σ_1^{co}, is obtained as follows

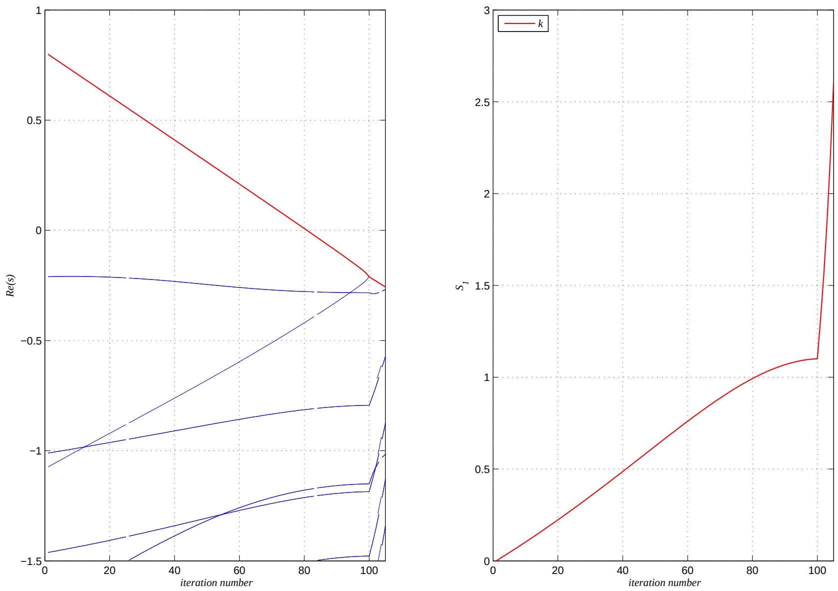

Figure 5.2.: Real parts of the rightmost modes (left) and the controller parameters (right), as a function of the iterations, for the first control agent of Example 5.1.

$$
\begin{aligned}
\dot{x}_2^{co} &= \begin{bmatrix} 7 & -1.4287 & -34.7149 & 29.8574 \\ 0 & -8.8215 & -27.2862 & 13.6431 \\ -11 & -0.7856 & 13.8574 & -21.4287 \\ -22 & -1.5713 & 37.7149 & -47.8574 \end{bmatrix} x_2^{co}(t) \\
&+ \begin{bmatrix} -14.4287 & 21.6431 & 2.4287 & -11.4287 \\ -7.8215 & 14.4287 & 2.6072 & -5.2144 \\ 10.2144 & -10.8215 & 3.7856 & 6.2144 \\ 20.4287 & -21.6431 & -4.4287 & 18.4287 \end{bmatrix} x_2^{co}(t-1) \\
&+ \begin{bmatrix} 5.2144 & -5.2144 & 5.2144 & 0 \\ 2.6072 & -2.6072 & 2.6072 & 0 \\ -2.6072 & 2.6072 & -2.6072 & 0 \\ -5.2144 & 5.2144 & -5.2144 & 0 \end{bmatrix} x_2^{co}(t-2) \\
&+ \begin{bmatrix} 3 \\ 0 \\ -3 \\ -5 \end{bmatrix} u_2(t) + \begin{bmatrix} 1 \\ 0 \\ -1 \\ -1 \end{bmatrix} u_2(t-1) \\
y_2(t) &= \begin{bmatrix} 1 & -1 & 2 & -0.5 \end{bmatrix} x_2^{co}(t) + \begin{bmatrix} 1 & 0 & 0 & 1 \end{bmatrix} x_2^{co}(t-1)
\end{aligned}
$$

For this system, Algorithm 4.1 fails to find a stabilizing controller with dimension $m = 0$. Also based on the controllable canonical form

$$\dot{z}_2(t) = \begin{bmatrix} 0 & 1 & 0 & \dots & 0 \\ 0 & 0 & 1 & \dots & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & 0 & \dots & 1 \\ f_1 & f_2 & f_3 & \dots & f_m \end{bmatrix} z_2(t) + \begin{bmatrix} 0 \\ 0 \\ \vdots \\ 0 \\ 1 \end{bmatrix} y_2(t),$$
$$u_2(t) = \begin{bmatrix} h_1 & h_2 & h_3 & \dots & h_m \end{bmatrix} z_2(t) + \begin{bmatrix} k \end{bmatrix} y_2(t)$$

Algorithm 4.1 fails to find a stabilizing controller with dimension $m = 1$, and $m = 2$. However, with $m = 3$, the controller

$$\dot{z}_2(t) = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ -0.24403 & -0.17958 & -0.59405 \end{bmatrix} z_2(t) + \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} y_2(t) \tag{5.4}$$
$$u_2(t) = \begin{bmatrix} -0.19224 & -0.22153 & -0.28122 \end{bmatrix} z_2(t) - 0.0949\, y_2(t)$$

is found to stabilize Σ_1^{co}. The progress of the algorithm is shown in Figure 5.3..

The overall closed-loop system, Σ_2, following the application of the decentralized controllers (5.3) and (5.4) to the original system Σ is then obtained. By using the programs of [46], for $\epsilon = 1$, the $-\epsilon$-modes of this system are computed as

$$\Omega_{-\epsilon}(\Sigma_2) = \big\{ -0.0508, -0.06406, -0.14947, -0.17236, -0.17413 \pm 0.61987\text{i}, \\ -0.17426 \pm 5.43642\text{i}, -0.28966 \pm 5.96136\text{i}, -0.58366 \pm 11.76667\text{i}, \\ -0.6897 \pm 11.35471\text{i} - 0.88144 \pm 17.80495\text{i}\big\},$$

which are plotted in Figure 5.1. using '+'. It is seen that the closed-loop system does not have any modes with non-negative real parts. Hence, the decentralized controllers (5.3) and (5.4) stabilize the given system.

Following example is provided here to illustrate the design approach proposed in Section 4.3.

Example 5.2. *Consider the same LTI retarded time-delay system Σ_0 described in Example 5.1. For $\epsilon = 1$, $-\epsilon$-modes of the system Σ_0 is given in (5.1)*

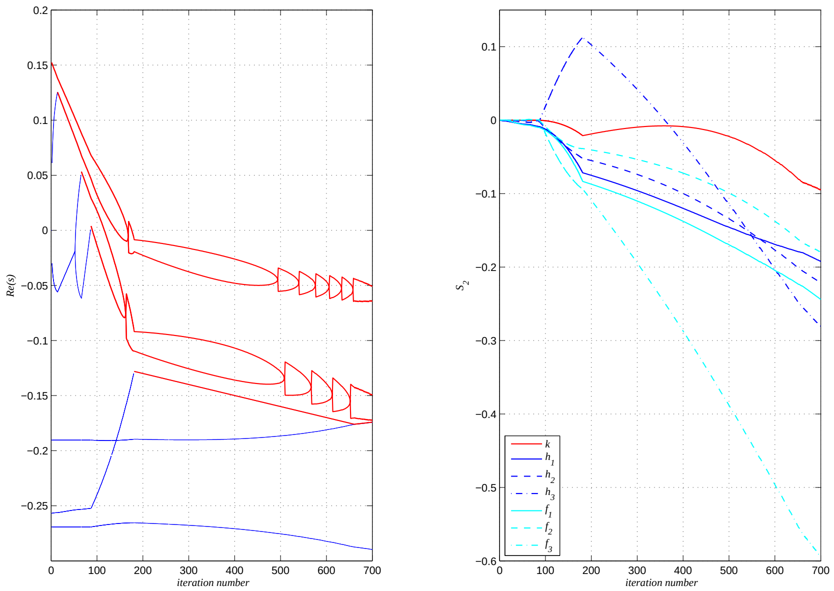

Figure 5.3.: Real parts of the rightmost modes (left) and the controller parameters (right), as a function of the iterations, for the second control agent of Example 5.1.

and they are plotted in Figure 5.4., using ' $*$*'. As mentioned in the previous example, it can be verified that the system* Σ_0 *is not stabilizable by any one of the control agents alone. However, the system does not have any unstable DFMs, hence it can be possible to find a stabilizing decentralized feedback. To obtain a decentralized controller, the procedure in Algorithm 4.3 is used.*

In the first phase, structurally controllable and observable part, Σ_0^{co}*, of the system seen by the first control agent is obtained as in* (5.2)*. Then, the continuous pole placement algorithm is applied once to obtain the controller gain* K_1 *and once to obtain the observer gain* L_1*. At the end of the procedure, controller gain* K_1 *is obtained as*

$$K_1 = \begin{bmatrix} 20.5113 & -28.3535 \end{bmatrix} ,$$

and the observer gain L_1 *is obtained as*

$$L_1 = \begin{bmatrix} -43.1485 & 0.5327 \end{bmatrix} .$$

The observer based controller for Σ_0^{co} *is obtained as follows*

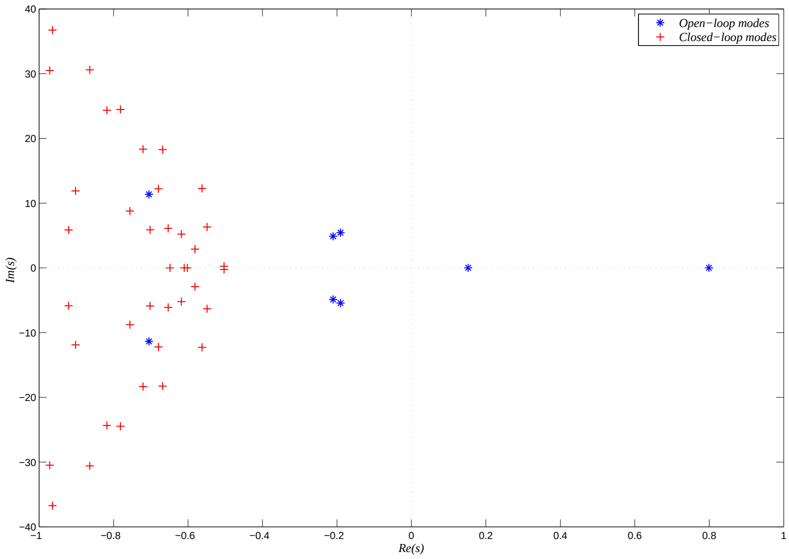

Figure 5.4.: Open-loop and closed-loop modes of the system in Example 5.2.

$$\begin{aligned}
\dot{\hat{x}}_1^{co}(t) &= \begin{bmatrix} -1 & -9 \\ 0 & -4 \end{bmatrix} \hat{x}_1^{co}(t) + \begin{bmatrix} 4 & -6 \\ 0 & 1 \end{bmatrix} \hat{x}_1^{co}(t-1) \\
&+ \begin{bmatrix} 0 & 3 \\ 0 & 0 \end{bmatrix} \hat{x}_1^{co}(t-2) + \begin{bmatrix} 0 \\ 1 \end{bmatrix} u_1(t) \\
&+ \begin{bmatrix} -43.1485 \\ 0.5327 \end{bmatrix} \left(\begin{bmatrix} 1 & -3 \end{bmatrix} \hat{x}_1^{co}(t) + \begin{bmatrix} 0 & 2 \end{bmatrix} \hat{x}_1^{co}(t-1) - y_1(t) \right) \\
u_1(t) &= \begin{bmatrix} 20.5113 & -28.3535 \end{bmatrix} \hat{x}_1^{co}(t)
\end{aligned} \tag{5.5}$$

The real parts of the rightmost modes during the continuous pole placement algorithms in the first phase with controller and observer parameters as a function of the iterations are respectively shown in Figure 5.5. and Figure 5.6..

In the second phase, structurally controllable and observable part Σ_1^{co} *of the system* Σ_1*, which is obtained by applying the observer based controller* (5.5) *to the system* Σ_0*, is obtained as follows*

$$\dot{x}_2^{co}(t) = \begin{bmatrix} -125.7 & 59.7 & 202.5 & -155.1 \\ -99.5 & 37 & 150.6 & -125.1 \\ 55.4 & -31.3 & -104.7 & 71 \\ 110.7 & -62.7 & -199.5 & 137.1 \end{bmatrix} x_2^{co}(t)$$

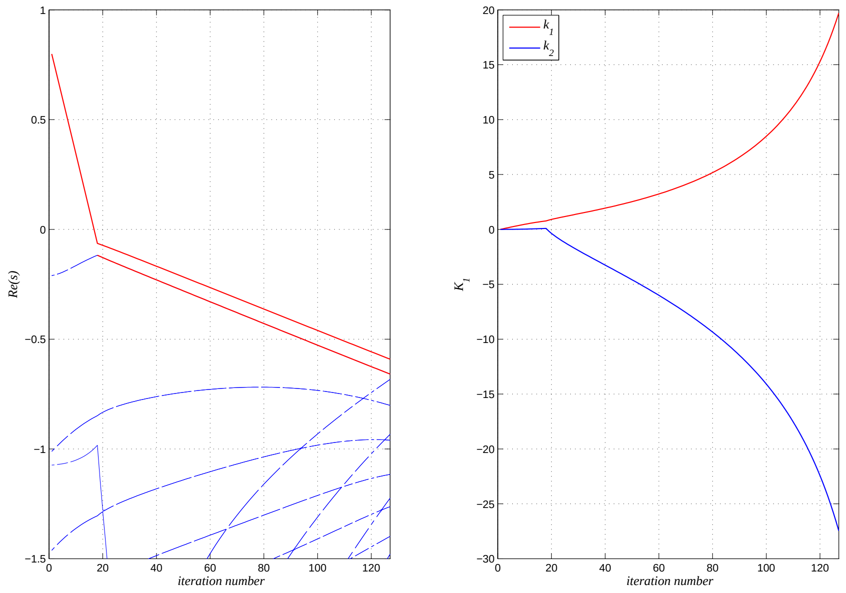

Figure 5.5.: Real parts of the rightmost modes (left) and the parameters of the controller gain $K_1 = [k_1\ k_2]$ (right) as a function of the iterations of the continuous pole placement algorithm in Example 5.2.

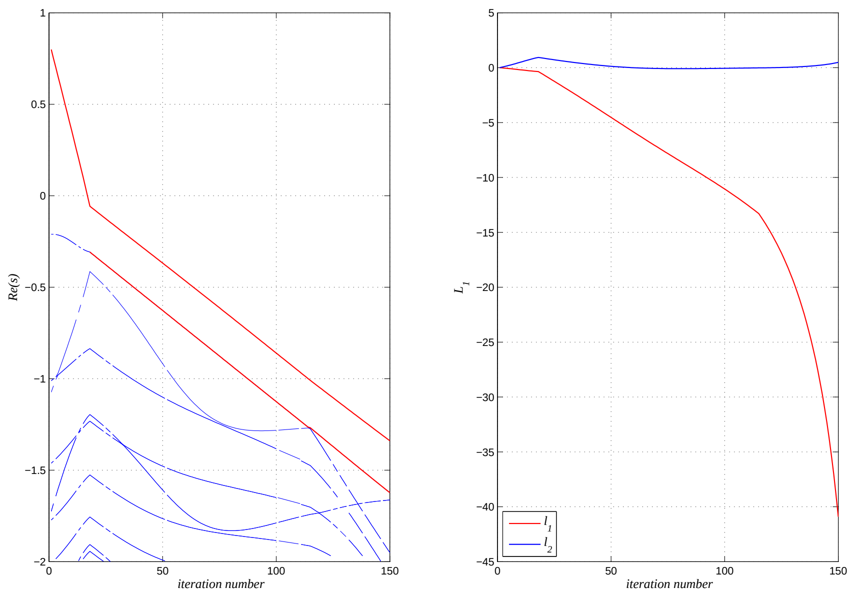

Figure 5.6.: Real parts of the rightmost modes (left) and the parameters of the observer gain $L_1 = [l_1\ l_2]$ (right) as a function of the iterations of the continuous pole placement algorithm in Example 5.2.

$$
+\begin{bmatrix} 144.4 & -101.4 & -23.7 & 81 \\ 94.7 & -70.2 & 12.7 & 41 \\ -69.2 & 50.7 & 16.8 & -40 \\ -138.4 & 101.4 & 21.7 & -74 \end{bmatrix} x_2^{co}(t-1)
$$

$$
+\begin{bmatrix} -41 & 41 & -41 & 0 \\ -20.5 & 20.5 & -20.5 & 0 \\ 20.5 & -20.5 & 20.5 & 0 \\ 41 & -41 & 41 & 0 \end{bmatrix} x_2^{co}(t-2)
$$

$$
+\begin{bmatrix} 3 \\ 0 \\ -3 \\ -5 \end{bmatrix} u_2(t) + \begin{bmatrix} 1 \\ 0 \\ -1 \\ -1 \end{bmatrix} u_2(t-1)
$$

$$
y_2(t) = \begin{bmatrix} 1 & -1 & 2 & -0.5 \end{bmatrix} x_2^{co}(t) + \begin{bmatrix} 1 & 0 & 0 & 1 \end{bmatrix} x_2^{co}(t-1)
$$

Then, continuous pole placement algorithm is applied once to obtain the controller gain K_2 and once to obtain the observer gain L_2. At the end of the procedure, controller gain K_2 is obtained as

$$
K_2 = \begin{bmatrix} -0.893 & 0.3704 & 0.77 & -1.474 \end{bmatrix} ,
$$

and the observer gain L_2 is obtained as

$$
L_2 = \begin{bmatrix} -147.5 & -71.04 & 55.58 & 147.85 \end{bmatrix} .
$$

The observer based controller for Σ_1^{co} is obtained as follows

$$
\dot{\hat{x}}_2^{co}(t) = \begin{bmatrix} -125.7 & 59.7 & 202.5 & -155.1 \\ -99.5 & 37 & 150.6 & -125.1 \\ 55.4 & -31.3 & -104.7 & 71 \\ 110.7 & -62.7 & -199.5 & 137.1 \end{bmatrix} \hat{x}_2^{co}(t)
$$

$$
+\begin{bmatrix} 144.4 & -101.4 & -23.7 & 81 \\ 94.7 & -70.2 & 12.7 & 41 \\ -69.2 & 50.7 & 16.8 & -40 \\ -138.4 & 101.4 & 21.7 & -74 \end{bmatrix} \hat{x}_2^{co}(t-1)
$$

$$
\begin{aligned}
&+\begin{bmatrix} -41 & 41 & -41 & 0 \\ -20.5 & 20.5 & -20.5 & 0 \\ 20.5 & -20.5 & 20.5 & 0 \\ 41 & -41 & 41 & 0 \end{bmatrix} \hat{x}_2^{co}(t-2) \\
&+\begin{bmatrix} 3 \\ 0 \\ -3 \\ -5 \end{bmatrix} u_2(t) + \begin{bmatrix} 1 \\ 0 \\ -1 \\ -1 \end{bmatrix} u_2(t-1) \\
&+\begin{bmatrix} -147.5 \\ -71.04 \\ 55.58 \\ 147.85 \end{bmatrix} \Big(\begin{bmatrix} 1 & -1 & 2 & -0.5 \end{bmatrix} \hat{x}_2^{co}(t) + \\
&+\begin{bmatrix} 1 & 0 & 0 & 1 \end{bmatrix} \hat{x}_2^{co}(t-1) - y_2(t) \Big) \\
u_2(t) &= \begin{bmatrix} -0.893 & 0.3704 & 0.77 & -1.474 \end{bmatrix} \hat{x}_2^{co}(t)
\end{aligned}
\quad . \qquad (5.6)
$$

The real parts of the rightmost modes during the continuous pole placement algorithms in the second phase with controller and observer parameters as a function of the iterations are respectively shown in Figure 5.7. and Figure 5.8..

By using the programs of [46], for $\epsilon = 1$*, the* $-\epsilon$*-modes of the closed-loop system, obtained by applying observer based controller* (5.6) *to* Σ_1*, are computed as*

$$
\begin{aligned}
\Omega_{-\epsilon}(\Sigma_2) = \{ & -0.5031 \pm 0.2534i,\ -0.5488 \pm 6.3059i,\ -0.5624 \pm 12.2662i, \\
& -0.5813 \pm 2.8987i,\ -0.6018,\ -0.61,\ -0.618 \pm 5.222i,\ -0.6486, \\
& -0.6532 \pm 6.1082i,\ -0.6683 \pm 18.2687i,\ -0.6794 \pm 12.2224i, \\
& -0.702 \pm 5.8868i,\ -0.7207 \pm 18.3383i,\ -0.7562 \pm 8.776i, \\
& -0.7815 \pm 24.4575i,\ -0.818 \pm 24.348i,\ -0.8638 \pm 30.5868i, \\
& -0.9022 \pm 11.8936i,\ -0.921 \pm 5.8564i,\ -0.964 \pm 36.7338i, \\
& -0.9716 \pm 30.4884i \}
\end{aligned}
$$

which are plotted in Figure 5.4. using '+'. It is seen that the closed-loop system does not have any modes with non-negative real parts. Hence, the decentralized observer based controllers stabilize the given system.

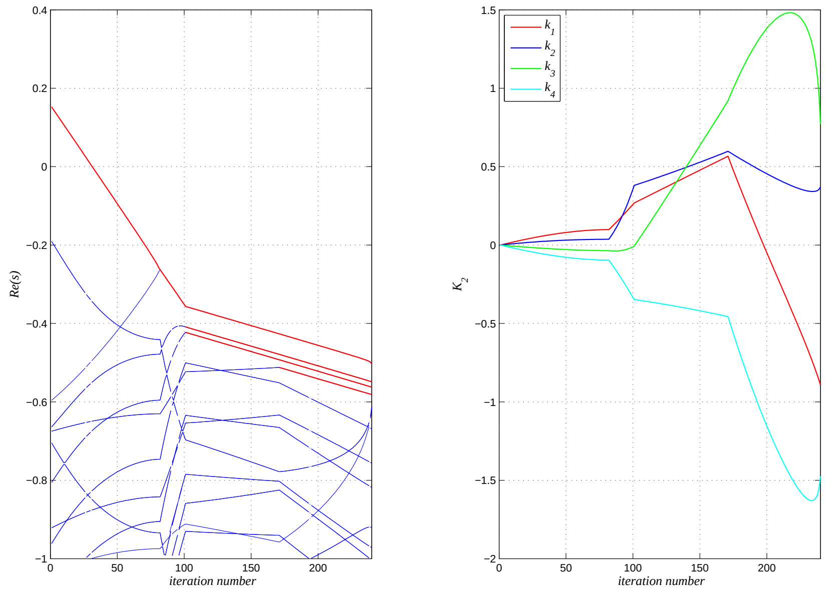

Figure 5.7.: Real parts of the rightmost modes (left) and the parameters of the controller gain $K_2 = [k_1\ k_2\ k_3\ k_4]$ (right) as a function of the iterations of the continuous pole placement algorithm in Example 5.2.

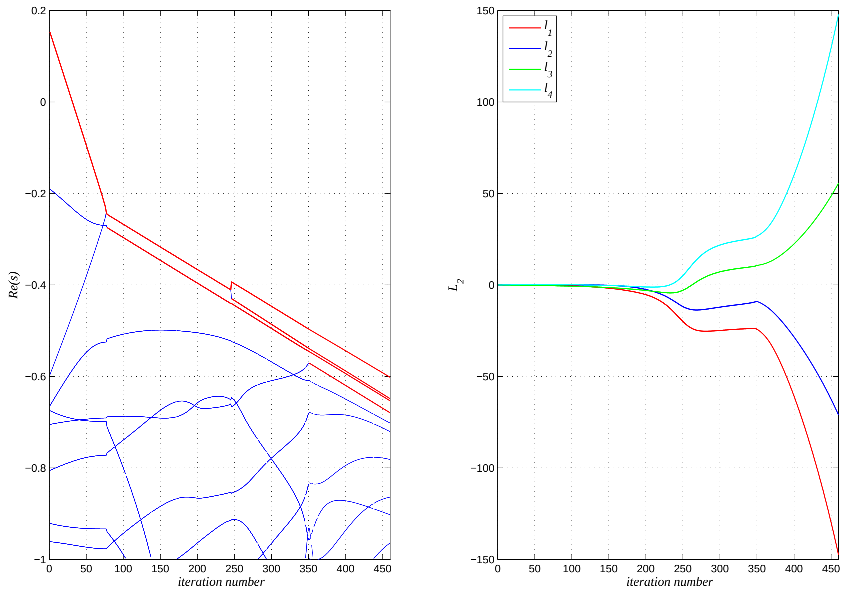

Figure 5.8.: Real parts of the rightmost modes (left) and the parameters of the observer gain $L_2 = [l_1\ l_2\ l_3\ l_4]$ (right) as a function of the iterations of the continuous pole placement algorithm in Example 5.2.

In this chapter, stabilizing decentralized controllers for an unstable retarded commensurate-time-delay system was designed by using Algorithms 4.2 and 4.3. In Example 5.1, the first control agent of the designed decentralized controller applies static output feedback and the second control agent applies a dynamic output feedback. In Example 5.2, the first and the second control agents apply observer based state feedback and, as can be seen, the observer poles were placed slightly farther to the left than the dominant poles of each resultant system. It should be noted that the spectral abscissas of the closed-loop systems obtained in the examples are considerably different. The spectral abscissa of the closed-loop system obtained at the end of Example 5.1 is -0.0508 and the spectral abscissa of the closed-loop system obtained at the end of Example 5.2 is -0.5031. Compared to Example 5.1, the growth in the dimension of the designed controllers in Example 5.2 has allowed to get a better spectral abscissa.

6. CONCLUSION

In this thesis, stabilization of LTI time-delay systems by decentralized controllers has been considered. In Chapter 3, in terms of μ-DFMs, necessary and sufficient conditions for the decentralized stabilizability of LTI time-delay systems was obtained. To obtain these conditions, it was first shown that the sets of μ-DFMs of a LTI time-delay system under decentralized static, dynamic finite-dimensional or dynamic time-delay output feedback controllers are equivalent. Then it was proved that there exists a μ-stabilizing time-delay dynamic output feedback controller for a neutral time-delay system if and only if the proposed border of the stability region on the complex plane μ is greater than or equal to $c_D + \epsilon$ for some $\epsilon > 0$ (where c_D is the spectral abscissa of the associated delay difference equation) and the system does not have any μ-DFMs with respect to LTI decentralized static output feedback controllers. In the light of the obtained results, the decentralized counterpart of the main result of Kamen et. al. [40] was also obtained. According to this result, a LTI neutral time-delay system can be μ-stabilized by a decentralized time-delay dynamic output feedback controller if and only if there exists a μ-stabilizing decentralized finite-dimensional dynamic output feedback controller for this system. However, in some cases, there may exist only very high dimensional stabilizing controllers which require a large number of integrators. In such a case, it may be possible to design time-delay controllers which require a smaller number of integrators. Such a time-delay controller may be implemented by using delay elements besides integrators.

In Chapter 4, decentralized controller design problem for LTI retarded commensurate-time-delay systems was considered. The reason behind considering only commensurate-time-delay systems, rather than more general case of incommensurate-time-delay systems, is that a decomposition such as the one presented in Lemma 4.1 is required in order to leave the structural fixed modes out of the controller design problem to avoid a possible failure of the continuous pole placement algorithm. On the other hand, even if the considered system does not have any structural fixed modes, there may still exist unstructural fixed modes or transmission zeros in $\bar{\mathbb{C}}_\mu^+$ that may cause the failure of the continuous pole placement algorithm. Among other reasons, to overcome this problem, in Section 4.1, the extension of the continuous pole placement algorithm to dynamic output feedback controller design was given. Then, using this centralized controller design algorithm, in Section 4.2, a decentralized controller design algorithm, which is based on the decentralized pole assignment algorithm, was introduced. Furthermore, in Section 4.3, a decentralized controller design algorithm, which is based on the decentralized pole assignment algorithm and observer based continuous pole placement algorithm, was introduced. The problems that might be caused by the unstructural fixed modes and transmission zeros in $\bar{\mathbb{C}}_\mu^+$, however, were not considered in Section 4.3.

As noted, in order to leave the structural fixed modes out of the controller design problem, a decomposition such as the one presented in Lemma 4.1 is needed. Working on another approach which do not require such a decomposition can be one of the further research lines. Another further research line is to develop techniques to overcome the problems that might be caused by the unstructural fixed modes and transmission zeros in $\bar{\mathbb{C}}_\mu^+$ during the observer based design proposed in Section 4.3. As another possible further research line, in the decentralized controller design problem, time-delay controllers can be considered instead of finite-dimensional controllers. Furthermore, decentralized controller design problem for LTI neutral time-delay systems is also not discussed in this thesis. The reason behind this is that the neutral time-delay systems may have infinitely many μ-modes and this causes difficulties in finding the μ-modes of these systems. However, in the existence of an approach which is also capable of finding the μ-modes of neutral time-delay systems, similar to the algorithms presented here, a decentralized controller design algorithm for neutral incommensurate-time-delay systems, which has finitely many unstable modes, may also be obtained.

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