PORTFOLIO OPTIMIZATION WITH
OPTION IMPLIED INFORMATION:
AN APPLICATION IN BORSA ISTANBUL

Thea ANGURIDZE

(Ph.D Dissertation)

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ÖZET

OPSİYONLARDAN ELDE EDİLEN BİLGİLERLE PORTFÖY OPTİMİZASYONU: BORSA ISTANBUL'DA BİR UYGULAMA

Thea ANGURIDZE

İşletme Bölümü,

Anadolu Üniveristesi, Sosyal Bilimler Enstitüsü, Ağustos 2019

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Bu çalışmada, opsiyonlardan elde edilen bilgilere dayanan optimal portföylerin, tarihi bilgilere dayanan optimal portföylerden daha başarılı olup olmadığı araştırılmaktadır. Araştırmada, Borsa İstanbul Vadeli İşlem ve Opsiyon Piyasası'nda işlem gören 20 hisse senedine ait veriler kullanılmıştır. Örneklem dönemi, Mart 2017'den Temmuz 2018'e kadardır. Tarihi hisse senedi verileri kullanılarak Ortalama varyans ve minimum varyans portföyleri oluşturulmuş ve opsiyon fiyatları kullanılarak portföy optimizasyon modelleri geliştirilmiştir. Örneklemdeki hisse senetlerinin opsiyonlardan elde edilen volatiliteleri, Black-Scholes opsiyon fiyatlama modeli ile hesaplanmıştır. Hisse senetleri arasındaki opsiyonlardan elde edilen korelasyonların hesaplanmasında, Buss ve Vilkov'un modeli kullanılmıştır. Opsiyonlardan elde edilen ortalama varyans ve minimum varyans portföyleri, kovaryans matrisinde tarihi bilgilerin yerine, opsiyonlardan elde edilen bilgilerin konulması ile oluşturulmuştur. Portföylerin başarılarının değerlendirilmesinde, şu üç kriter kullanılmıştır: portföyün yıllık getirisi, portföyün yıllık volatilitesi ve portföyün Sharpe Rasyosu. Daha sonra, opsiyonlardan elde edilen bilgilerle portföy oluşturmanın, opsiyonlardan elde edilen bilgileri dikkate almayan portföylerden daha iyi başarı ölçütleri sağlayıp sağlamadığı sınanmıştır. Bulgular, opsiyonlardan elde edilen bilgilere dayanan optimal portföylerin, tarihi bilgilere dayanan optimal portföylerden daha başarılı olduğunu göstermektedir.

Anahtar Sözcükler: Portföy optimizasyonu, Tarihi volatilite, Opsiyonlardan elde edilen volatilite, Portföy başarısı, Borsa İstanbul

ABSTRACT

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This study explores if optimal portfolios based on option-implied information perform better than optimal portfolios based on historical information. We used option prices of 20 stocks, which have been trading in the Futures and Options Market of Borsa İstanbul. The sample period is from March 2017 to July 2018. We developed portfolio optimization models using option prices as well as mean variance and minimum variance portfolios using historical stock price data. We calculated implied volatility of the sample stocks from option prices using Black-Scholes option pricing model. We employed Buss and Vilkov's model for the calculation of implied correlations between stocks. Option implied mean variance and option implied minimum variance portfolios are based on the covariance metrics developed after historical information is replaced by option-implied information. For the evaluation of portfolio performance, we used the following three criteria: annualized portfolio return, annualized portfolio volatility and portfolio Sharpe ratio. Then, we test if creating portfolios with option-implied information can yield better performance measures than portfolios that ignore option-implied information. The findings show that optimal portfolios based on option-implied information perform better than optimal portfolios based on historical information.

Keywords: Portfolio optimization, Historical volatility, Option-implied volatility, Portfolio performance, Borsa İstanbul

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22/08/2019

STATEMENT OF COMPLIANCE WITH ETHICAL PRINCIPLES AND RULES

I hereby truthfully declare that this thesis is an original work prepared by me; that I have behaved in accordance with the scientific ethical principles and rules throughout the stages of preparation, data collection, analysis and presentation of my work; that I have cited the sources of all the data and information that could be obtained within the scope of this study, and included these sources in the references section; and that this study has been scanned for plagiarism with "scientific plagiarism detection program" used by Anadolu University, and that "it does not have any plagiarism" whatsoever. I also declare that, if a case contrary to my declaration is detected in my work at any time, I hereby express my consent to all the ethical and legal consequences that are involved.

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1 INTRODUCTION

Portfolio optimization is one of the most important topics in the field of investment management. Investment management is a science about making good investment decisions. Investments should match investors' objectives and assets should be allocated properly by balancing risk against return to attain superior performance. Researchers use different empirical methodologies during the asset allocation and portfolio optimization process. When we talk about portfolio optimization, the first theory that comes to mind is Harry Markowitz's Modern Portfolio Theory (MPT), which is also known as "Portfolio Management Theory". In fact, the mean–variance model developed by Markowitz laid the basis of modern portfolio theory. The mathematical problem of portfolio optimization was investigated by Professor Harry Markowitz in 1952 and he was awarded the Nobel Prize in economics in 1990. The Modern Portfolio Theory presumes that investors focus on minimizing risk while obtaining the highest possible return. According to this theory, it is possible for different portfolios to have different levels of return and risk. The fundamental goal of Modern Portfolio Theory (MPT) is to maximize portfolio's expected return for a given amount of risk. This means that an investor can determine how much risk they are willing to take on, and after that they can diversify their portfolios accordingly.

Markowitz' model has had an important impact on the investment community and is still widely used for solving hedging, asset allocation, portfolio construction and other portfolio management problems. Mean variance optimization (MVO) is an easily solvable model, therefore it can be used as an optimizer even, when there are thousands of assets in consideration. In Markowitz's approach, the optimization problem is formulated with respect to two criteria: the reward of a portfolio that should be maximized and the portfolio risk that should be minimized. Investors have always been exceedingly aware of returns. Risk is related to the volatility of future outcomes. The higher is the risk, the higher will be the expected return, as there is a trade-off between return and risk. Total risk of investment is measured by variance or standard deviation.

It is not necessary for an investor to accept the total risk, as he/she can make diversification and some individual investment risk can be avoided by diversification. If the investor decides to put all his funds in a single security, this indicates acceptance of total risk.

Thus, it is not recommended for the investor to put all his/her funds in a single investment, because it exposes the investor to more risk than necessary. Simply, the essence of diversification is: “do not put all your eggs in one basket”. However, not all risk can be avoided by diversification. The risk related to the movements in an economy is defined as market risk and sometimes is considered as non-diversifiable risk (Dobbins et al., 1994).

The main perception of MPT is that risk and return should not be considered alone, they should be estimated by how an investment influences the portfolio’s overall return and risk. Generally, the lower the correlation between securities, the lower is the risk of portfolio. Hence, risk-averse investors tend to select securities, which have low correlations. Markowitz (1959) MPT makes a presumption that investors are risk-averse, which means that investors preferred less risky portfolio rather than a riskier one for a given level of return. This suggests that an investor is willing to take more risk if she or he expects more reward. In the presence of two criteria, there is not a single optimal solution (portfolio), but a set of optimal portfolios, the so-called efficient portfolios, which tradeoff between risk and return (Anagnostopoulos & Mamanis, 2011). Modern Portfolio Theory suggests that it is possible to construct an “efficient frontier” of optimal portfolios that offers maximum return for a given level of risk.

The Turkish stock market like other emerging markets attracts a large amount of funds from developed markets and depend on capital inflows from foreign investors.

The primary aim of this empirical study is to examine the performance of portfolios constructed with historical (back-forwarding) and implied (looking-forward) information and by comparing their performances confirming that portfolios based on looking-forward information overperforms the portfolios based on historical information.

First, we calculated historical and implied volatilities. Historical volatility was computed based on 3 years rolling window method and implied volatility was calculated by Black-Scholes model.

1.1 Modern Portfolio Theory

Markowitz is considered as the father of modern portfolio theory on the basis of his work on portfolio selection in 1952. However, Roy (1952) is the forgotten father of modern portfolio theory, who claims the equal portion of this honor. In this section we summarize their contributions.

Markowitz's (1952) article is about portfolio selection which presented variance of return V and the expected, mean return E of the portfolio. This article assumes that predictions and beliefs about the securities follow the same probability rules that random variables do. According to this assumption, the expected or future return of a portfolio is the average of expected returns of individual securities in the portfolio and also the variance of the portfolio return is a function of the variances of the individual securities as well as the covariances between individual securities and the weights of these securities in the portfolio. Markowitz’s paper Markowitz (1952) distinguishes between inefficient and efficient portfolios. Markowitz describes efficient frontier as a set of efficient mean-variance combinations and suggests investors select the set for the desired risk-return combination. He uses geometrical analyses of three and four security examples in order to illustrate properties of efficient sets by assuming non-negative investments. In the 1952 article, Markowitz showed that the set of efficient portfolios is linear and the set of efficient mean-variance combination is parabolic.

Mean variance model is a mathematical structure, which helps investors to maximize expected return with a given level of risk. Modern Portfolio Theory (MPT) is an extension and formalization of diversification in investment, the idea that claims that owning different kind of assets is less risky rather than owning just one type of asset. MPT uses variance of asset prices as a proxy of risk. According to MPT, there is trade-off between return and risk. For years, investment managers and advisers were focused on returns, as risk was not understood. MPT pays attention to risk at least as much as return. Investors can make decisions about the risk they are ready to take, though they cannot make any decisions about the return they will achieve, as it depends on factors beyond their control. Yet, they can predict that the greater the risk, the higher will the future return (Markowitz, 1952)

Roy (1952) also suggested to make portfolio choices on the basis of variance and mean of the portfolio entirely. Concretely, he recommended to choose the portfolio which maximizes portfolio's (E-d)/σ. Where, d is a devastating return and σ is a standard deviation of the return. Roy's formula is similar to Markowitz's formula and includes the covariances between the returns of securities. The principal difference is that Markowitz's formula requires non-negative investments, while Roy's allows the amount invested in any security to be negative or positive. The second difference is that Markowitz suggests the investors to choose desired portfolios from the efficient frontier, while Roy proposes to choose a specific portfolio.

The main focus of Markowitz (1959) is to explain portfolio theory to the readers who lack advanced mathematics. He explains the concept of mean, mean-variance analysis, variance and covariance and obtains formulas for mean and variance of the portfolio, defines efficiency of mean-variance and represents geometric analysis of efficient sets. He explains analysis of portfolios with large number of securities and focuses on portfolio selection rather than securities. He claims that analysis of the large portfolio which consists of many different kinds of assets has many covariances. The portfolio problem is defined as the choice of the averages and variances of the portfolios composed of different securities (Markowitz, 1959).

Another important issue that Markowitz (1991) deals with is the relationship between securities. The assets should be chosen not only based on their own characteristics, but also on their relations with other entities. However, if securities' returns are not correlated, the risk may be eliminated by diversifying the portfolio. The correlation among returns is not the same for all kind of securities. Generally, it is expected that the return of the securities from the same industries are more correlated than those from different industries. In order to reduce risk, we should avoid a portfolio with securities which all are highly correlated to each other. For investors the most desired is portfolio on the efficient frontier, because it contains portfolios with the highest return for different levels of risk. In MPT, portfolio optimization is based on the mean-variance model, where risk is defined as variance from the efficient frontier. Markowitz's optimization model has disadvantages as well. It has difficulties with computing large quadratic problems.

Markowitz's diversification is the actual type of diversification actively used by portfolio analysis. This kind of diversification is different from naive diversification, which is commonly used by asset salespeople and in some investment publications. These sources describe diversification as “not putting all eggs in one basket”. Naive diversification ignores the covariance between results and securities in unessential diversification. Markowitz’s diversification includes combination of assets with less rather than perfect positive correlation to reduce risk in the portfolio, without sacrificing any return of the portfolio. Generally, the lower the correlation between securities of the portfolio, the less risky is the portfolio.

> “Not only does the E-V hypothesis imply diversification, it implies the "right kind" of diversification for the "right reason." The adequacy of diversification is not thought by investors to depend solely on the number of different securities held. A portfolio with sixty different rail-way securities, for example, would not be as well diversified as the same size portfolio with some railroad, some public utility, mining, various sort of manufacturing, etc. The reason is that it is generally more likely for firms within the same industry to do poorly at the same time than for firms in dissimilar industries. Similarly, in trying to make variance small it is not enough to invest in many securities. It is necessary to avoid investing in securities with high covariances among each other. We should diversify across industries because firms in different industries, especially industries with different economic characteristics, have lower covariances than firms within an industry” (Markowitz, 1952, “portfolio selection”, pg.89).

1.1.1 Mean-variance model

To achieve an optimal trade-off between return and risk is a challenge for every portfolio manager. The Markowitz model considers the first two moments of the asset return, mean and variance, in order to measure the risk and return of the portfolio. In the financial world, this model is known as Markowitz’s MVO (mean-variance optimization).

The weight of the asset in the portfolio is the proportion of total funds invested in that asset. The risk of a portfolio is estimated as a quadratic function of the weights, which is the variance of the portfolio and the return of portfolio is estimated as linear function in the weights, representing the expected return of portfolio. The trade-off between risk and return is solved by simple quadradic programming (Ceria. S & Sivaramakrishnan. K.K, 2013).

$$\max_{w} \alpha^T \mathrm{w} - \lambda \mathrm{w}^T \mathrm{Qw} \tag{1.1}$$

Where, α is the expected return, Q is the covariance matrix of returns and $\lambda > 0$ is the risk aversion parameter which presents the investor's preference about how to tradeoff risk and return.

The solution to the QP determines the asset weights in an efficient portfolio – the portfolio with the minimum risk level for a given level of the expected return or (equivalently) the one with the largest expected return for a given level of allowed risk (Francis J.C & Kim.D, 2013).

When λ is smaller, the portfolio risk contribution is small, guiding to higher portfolio risk with higher return of portfolio. On the contrary, large λ is generates less risky portfolios with low return. Solving MVO model is possible using different λ values starting from zero in order to construct portfolio with different return and risk. The set of all those portfolios determine efficient frontier that gives a chance to investors to choose the portfolio from the efficient frontier depending on their risk and return wishes and mandates. MVO method does not return single optimal portfolio, but a family of them lying on the efficient frontier. For a given target of return the portfolio that lies on the efficient frontier gives the least risky portfolio. The same way for a given target of risk, the portfolio lying on the efficient frontier gives the portfolio with highest return (Ceria. S & Sivaramakrishnan. K.K, 2013).

The efficient frontier is the set of optimal portfolios, which offer the greatest expected return for a given level of risk, or the lowest risk for a defined level of future or expected return (Markowitz, 1952). Portfolios below the efficient frontier are sub-optimal as they do not provide enough return for a given level of risk. Portfolios on the right of the efficient frontier are sub-optimal, since they have higher level of risk for a given level of (Frank et al., 2011).

The efficient frontier for MVO model is shown below in figure 1.1. The main goal of portfolio theory is to define the optimal allocation among different assets. Though there are many models to determine the optimal allocation, mean-variance optimization (MVO), which was developed by Harry Markowitz in 1952, is one of the most widely used models in the investment industry. The two key reasons which makes it popular are its simplicity and aesthetic attractiveness. Traditional MVO optimizer creates only one efficient frontier, which allows the users to make comparative efficiency of two or more portfolios easily (Frank et al., 2011).

The aim of MVO is straightforward: helping the user to determine "efficient" portfolios. According to Markowitz, a portfolio is efficient if there is no other portfolio with a higher expected return for a given level of risk or lower risk for a given level of expected return. For MVO model, three parameters are needed: standard deviation, returns and correlations. These estimates create the efficient frontier (Markowitz, 1952). Figure 1.1 shows these combinations of investments with the highest return per unit of risk.

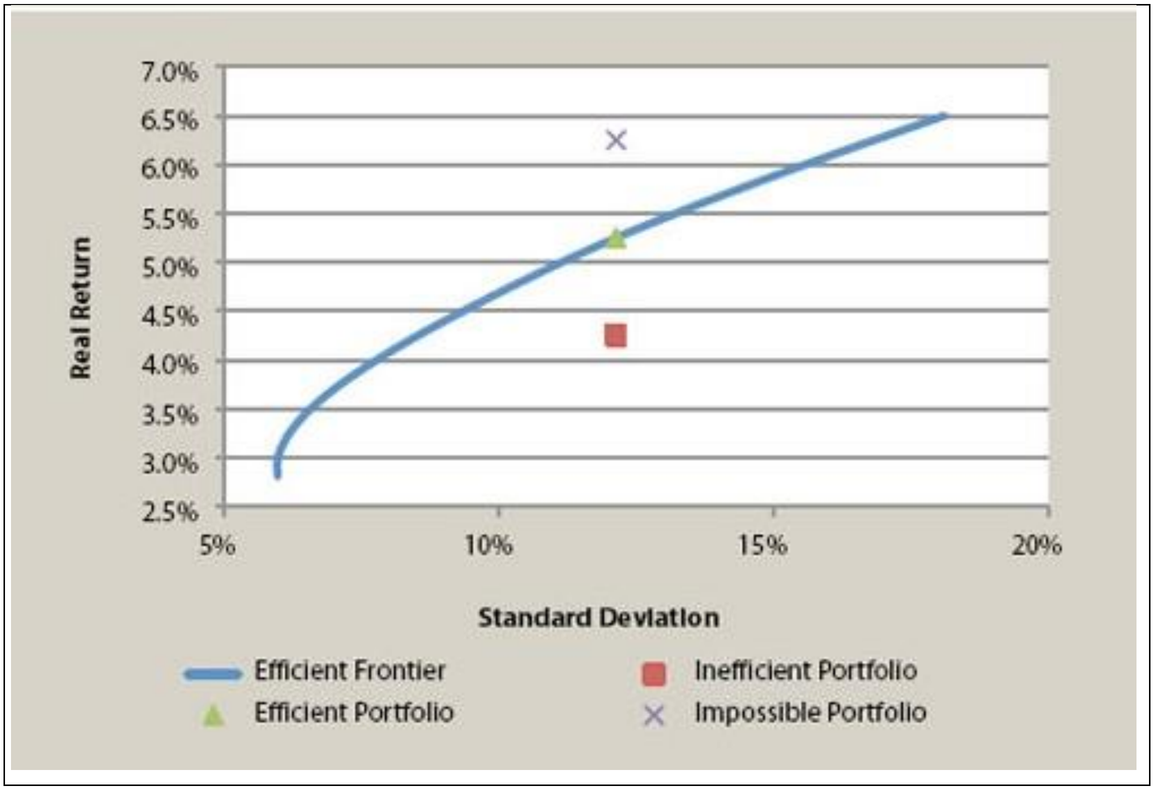

Figure 1. 1 *Efficient Frontier for MVO*

(source:https://www.onefpa.org/journal/Pages/Incorporating%20Time%20into%20the%20Efficient%20Frontier.aspx)

1.1.2 Semi-variance model

Markowitz's mean-variance approach, which leads to optimal the decision of investments, has two important limitations. First, if underlying return data is not normally distributed, the estimation of variance can generate misleading results. Several studies have shown that investment returns are not normally distributed ((Dennis W. Jansen and Casper G. de Vries, 1991) and (Fama & Roll, 1968). Asset returns are supposed to be asymmetrically distributed. They usually follow a lognormal distribution. If the returns are not normally distributed and investors use standard deviation or variance for measuring risk, they are going to lead to wrong asset allocation decisions. Second, the mean-variance approach ignores the investors' risk aversion. As the variance can only measure the dispersion of the returns around the mean, it cannot be tailored to account for individual investors' risk aversion (Boasson et al. 2011).

In Markowitz's mean-variance approach, risk is measured in terms of the variance of portfolio's expected return. The main assumption of using variance as an appropriate measure of risk is that investors can estimate the probability of negative returns equally against positive returns. According to several researchers, variance measures both downside and upside movements of the asset's return. Hence, it is an improper risk measurement. To construct the efficient frontier with an improper risk measurement may lead to irrational results in portfolio optimization.

Semi-variance is the measurement, which can be used for estimating the investment portfolio's potential downside risk. Downside risk first was modeled by Roy in 1952, which is almost the same time when Markowitz was working on developing the mean-variance theory. Markowitz also realized the weakness of variance as a risk measure. He concluded that the variance measurement and downside risk measurement can give the same results when return distribution is normal. However, when the return distribution is not normal, the downside risk measurement is thought to provide a better solution (Markowitz, 1970). Due to the shortcomings of the mean-variance model, Markowitz, in his article in 1991, developed the semi-variance model, which can make portfolio optimization more effective since it is only concerned with adverse deviations. Markowitz (1991) claimed "semi-variance is a more plausible measure of risk" rather than his mean-variance model, because for investors the risk of loss is a more significant concern than the probability of gain. Hence, semi-variance is a more appropriate risk measure for investors, rather than variance (Markowitz, 1991).

Semi-variance is almost identical to variance, but it considers only observations which fall below the target value or mean of the set of data. Semi-variance is useful in asset analysis or portfolio construction as it provides measurement of downside risk. Semi-variance considers merely negative fluctuations of asset returns. We can use semi-variance to compute the average loss of a portfolio, as it neutralizes all values that are above investor's target return or above the mean. Semi-variance is calculated as:

$$\sum_{for\ all\ X_i \leq B} (X_i - B)^2/(n-1) \qquad (1.2)$$

Where: B is the target and n is the number of observations (CFA Institute, 2017).

In short, semi-variance is calculated by measuring the dispersions of all observations which fall below the target value or mean of the dataset. Semi-variance is the average of the squared deviations of values less than the mean. Since investors are more concerned about the downside risk rather than general volatility, measuring risk by semi-variance instead of variance can enable constructing better portfolios (Markowitz, 1959).

1.2 Capital Asset Pricing Model (CAPM)

After Markowitz's two-parameter portfolio analysis model, researchers started investigating what would happen if every investor used Markowitz's model during their investment decisions. As a result of this investigation, the capital asset pricing model (CAPM) was developed. The CAPM is also known as a security market line (SML).

The CAPM describes relationship between an asset's, especially a stock's, future return and systematic risk. Below is the formula for calculating an asset's expected return for a given level of risk (Dobbins et al., 1994).

$$ER_i = R_f + \beta_i(ER_m - R_f) \qquad (1.3)$$

Where:

ER_i = Expected return of investment

R_f = Risk-free rate

β_i = Beta of the investment

ER_m = Expected return of market

$(ER_m - R_f)$ = Market risk premium (Dobbins et al., 1994).

The aim of CAPM formula is to assess if the stock is fairly valued, when its time value of money and risk are compared to its future return. Constructing a portfolio using CAPM helps investors to manage their risk. If investors are able to use CAPM for optimizing portfolio's return relative to its risk, then they will choose portfolios on the efficient frontier as shown on the graph below:

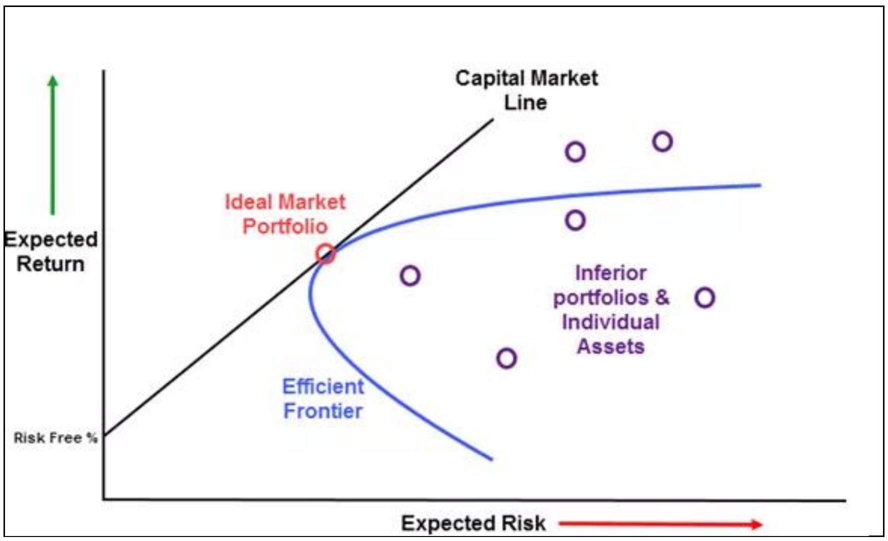

Figure 1. 2 *The CAPM and Efficient Frontier* (Source: https://www.investopedia.com/terms/c/capm.asp)

On the graph, we can see that greater expected return requires greater risk. MPT recommends that starting with risk free rate, the future return of a portfolio increases relatively as risk increases. Any kind of portfolio on the capital market line (CML) is better rather than any portfolio on the right of the line. Efficient frontier and CML illustrates the trade-off between increased risk and increased return. As in real life it is not possible to construct a portfolio on the CML, it is more common for investors to take more risk in order to attain additional return (Markowitz, 1952).

1.3 Options

Many different kinds of options, swaps, forward contracts and other derivatives are regularly traded by fund managers and financial institutions. We can define a derivative as a financial instrument, whose value depends on the underlying asset. For example, a stock option is a derivative with the value dependent on the stock price.

Though the history of options covers over some centuries, it was not till 1973 that exchange-listed, government regulated and standardized options became obtainable (Friedentag, 2009). Call and put options were first introduced in London in 1694. More than two and half centuries later, call and put options became favorite speculative tools of the Wall Street's old time Wolves. When the Put and Call Dealers Association and Brokers established standards and rules which caused a degree of the reputability for call and put options, stock options became commonly used and better understood by investors as a tool of hedging against price changes and gaining possible tax savings (Friedentag, 2009).

Since the importance of derivatives as both a risk-management tools and investment vehicle become broadly familiar, option markets opened all over the world. Options are not traded just only on traditional products, such as, commodities, stocks, interest rate and foreign currencies, but also on new products as well, such as, insurance, inflation, pollution and real estate. Not just number of option markets have increased dramatically, but also the knowledge of investors have become more and more advanced. Currently, many retail customers have the same level of knowledge as a professional trader (Natenberg, 2014).

Options are traded in the over-the-counter market as well as on organized exchanges. However, most trading of stock options takes place on the exchanges. In the USA the major exchanges are the Philadelphia Stock Exchange, the Chicago Board Option Exchange, the American Stock Exchange, the Boston Options Exchange and the International Securities Exchange. Trading of stock options include more than 1000 stocks. One contract gives the holder the right to sell or buy 100 shares at a particular strike price (Hull, 2003).

The largest exchange for trading the stock options in the world is the Chicago Board Options Exchange. The Chicago Board Options Exchange (CBOE) started trading call option contacts on 16 stocks in 1973. Options had been trading before to 1973, though CBOE succeeded in creating an orderly market with elaborated contracts. In 1977 it started trading of put option contracts. Currently, CBOE trades options on over 1000 stocks and many various stock indices. Similar to futures, options have shown to be very popular contracts. Nowadays, many different exchanges all over the world trade options (Hull, 2003).

Despite the complexity, options can be defined by a single word: choice. Except options, all financial contracts are based upon locked-in assurances for the seller and the buyer. Buying the option gives the investor a choice. We should underline that option gives the right the holder to do something and the holders do not have to exercise this right. This characteristic makes options different from other derivatives such as futures and forwards, where the holders have to sell or buy the underlying asset. (Hull, 2003).

While to enter into futures or forward contracts costs nothing, there is cost of obtaining an option. The price the investors pay for options is called “option premium”, When the investor buys an option, she or he buys a price insurance. The investor’s risk is limited. He/she can be protected against certain price movements and can receive monetary compensation. On the other hand, if the investors sell the option, they grant insurance to someone else. If an unfavorable case happens, the option seller compensates the other party and it can be highly expensive (Ward, 2004).

There are two kind of options: a call option and a put option. The call option gives right the holder to buy an underlying asset by a specific date for a specific price. The put option gives right the holder to sell an underlying asset by a specific date for a specific price (Hull, 2003). Options are known by the underlying asset involved. If the underlying asset is one of the particular indexes, such as Standard & Poor (S&P 100), then these options are called the index options. If the underlying product includes common stock, then such kind of options are called equity options. Except equity and index options, options on treasury securities, interest rates, futures and commodities are available (Eisen, 2000).

The price noticed in the contract is known as the strike price or exercise price. The date determined in the contract is known as, maturity or expiration date. American options can be exercised included at any time up to the expiration time, but European options can only be exercised on the expiration date. On exchanges, mostly American options are traded. In the exchange-traded equity options market, within one contract, there is an agreement to sell or buy 100 shares. Analysis of European options are usually easier rather than American ones and some of the characteristics of the American options are often deduced from its corresponding European ones (Hull, 2003).

1.4 Volatility

Investors and options traders are interested in the direction of the market and they are sensitive to speed of the market reactions. In a way, volatility is measured according to market speed. If the market moves slowly, it means that market is low-volatility market. On the other hand, a market, which moves fast, is a high-volatility market (Natenberg, 2014).

The second name of volatility is the standard deviation and the Greek letter sigma (σ) is a traditional symbol of the standard deviation. Similar to interest rates, volatility is also expressed as an annualized number. Annual volatility shows us the possibility of price changes through shorter periods of time. While interest rate is proportional to time, volatility is proportional to the square root of time. In order to calculate the standard deviation, for over more than one-year period we have to multiply the annual volatility by the squared root of time. Where *t* expresses years (Natenberg, 2014).

$$Volatility_t = Volatility_{annual}\ X\ \sqrt{t} \tag{1.4}$$

Generally, investors and traders calculate volatility by observing price changes with regular intervals. For calculation of volatility many traders assume that there are 256 trading days per year, since the square root of 256 is a whole number, 16. Traders can make the following presumption: (Natenberg, 2014). In order to approximate daily volatility, traders can divide the annual volatility by 16.

$$Volatility_{daily} = volatility_{annual} \text{x} \sqrt{1/256} = volatility_{annual} \text{x} 1/16 = \frac{volatility_{annual}}{16} \quad (1.5)$$

Among all inputs, which are required for option evaluation, volatility is the most difficult one for investors to understand. At the same time, volatility plays the main role in actual trade decisions. Changes in the investors' presumptions about the volatility may have a dramatic impact on the option's price (Natenberg, 2014).

Volatility is a measure of fluctuations in the stock price. The underlying share's price volatility influences the option premium. The greater the volatility, the higher is the premium (Friedentag, 2009).

Stock price volatility is a measure of how uncertain the investors are about the future movements of the stock price. For the investors who holds the stock, these, two kind, of consequences tend to balance each other. However, this can impact on the owner of call and put differently. The call owner benefits when the price increases, with restricted downside risk in case of price decreases. Likewise, the put option's owner benefits from price decreases, with restricted risk in case of price increases. Therefore, both call and put option's values increase as volatility increases. Generally, typical stock volatility is between 15% and 60% (Hull, 2003).

1.5 Variance as a Measure of Risk

The word risk can be defined as the dispersion of outcomes around the future value. Simply, the word riskier means there is more dispersion around the future outcome. Mathematically, the term standard deviation and variance measure the dispersion around the future return (Fabozzi & Markowitz, 2011).

The variance of the variable is the measure of a variability of the possible outcomes around the expected return. The variance can be calculated with the following formula:

$$var(R_i) = \sum_{i=1}^{N} p_n \, [r_n - E(R_i)]^2 \tag{1.6}$$

Standard deviation is the square root of variance.

1.6 Covariance of Returns

Occasionally, one random variable is connected with another random variable. Statistically the measurement of the connection between two variables is called covariance. It represents the direction of the connection. The covariance is positive if variables tend to move in the same direction, and it is negative when they move in the opposite direction.

This statistical perception can be used to analyze the situation when a price movement of one asset is connected with other assets. In this situation, the covariance between the returns of two assets, *i* and j, symbolized as σ_{ij} and can be calculated as follow (Francis J.C & Kim.D, 2013):

$$\sigma_{ij} = E\{[r_i - E(r_i)][r_i - E(r_i)]\} = \sum_{s=1}^{s} P_s \, \{[r_{is} - E(r_i)]\left[r_{js} - E\left(r_j\right)\right]\} \tag{1.7}$$

Where, r_{is} is a rate of *i* asset when state s happens. Some variable's covariance with itself equals that variable's variance and when i=j, then the above equation becomes:

$$\sigma_{ii} = E[r_i - E(r_i)]^2 \; = \; \sigma_i^2 \tag{1.8}$$

1.7 Correlation of Returns

Correlation coefficient is another statistical measure between two random variables and is obtained from the covariance. The difference between these two kinds of measures is that the correlation coefficient is standardized by dividing the covariance by the product of standard deviation of two variables. The correlation coefficient between two variables X and Y is computed as follows:

$$P_{XY} = \frac{\sigma_{XY}}{\sigma_X \sigma_Y} \tag{1.9}$$

The correlation coefficient is always less rather than or equal to 1 and bigger than or equal to -1. $-1 \leq P_{XY} \leq +1$

For constructing a diversified portfolio, the analyst must know the correlation coefficients between all assets under consideration. If p_{ij} is +1, then the returns of assets i and j are perfectly positively correlated, at the same time they move to the same direction. If p_{ij} is 0, then the return of assets i and j are not correlated. If p_{ij} is -1, assets i and j fluctuate conversely and are perfectly negatively correlated. (Francis J.C & Kim.D, 2013)

Covariance can be determined in terms of the standard deviation and correlation.

$$\sigma_{ij} = p_{ij}\sigma_i\sigma_j \tag{1.10}$$

1.8 Emerging Markets and Borsa Istanbul

Investing in emerging markets (EM) has become attractive especially since 2010. According to estimation of International Monetary Fund[1], emerging economies are expected to grow two to three times more than developed economies. An important benefit that emerging markets offer to investors is diversification, as they perform differently from developed markets and have inverse relationship with mature west economies. Morgan

[1] https://www.forbes.com/pictures/eglg45gdjd/why-invest-in-emerging-markets-2/#46485a2572e0

Stanley's Emerging Markets Index was launched in 1988[2] as an index for EM, which consisted of just 10 countries, that was 1% of the world market capitalization. Nowadays, EM consists of 24 countries which is 10% of the world market capitalization. This index is available for a number of market segments, regions and covers about 85% of the free float-adjusted market capitalization for each of the 24 countries: Brazil, Chile, Columbia, Mexico, Peru, Czech Republic, Egypt, Greece, Hungary, Poland, Qatar, Russia, South Africa, Turkey, United Arab Emirates, China, India, Indonesia, Korea, Malaysia, Pakistan, Philippines, Taiwan, Thailand. Among 10 big emerging markets (China, India, Argentina, Poland, Turkey, Brazil, Indonesia, South Africa, Mexico and South Korea). Turkish stock market is one of the biggest emerging markets, with domestic and international investor interest. As Table 1.1 shows, foreign investors' trade share is increasing, which means that Turkish market is becoming more attractive internationally. On Borsa Istanbul 402 companies are traded with TL 795 billion market capitalization which makes Borsa Istanbul the second most liquid trading platform in the world, with 242% of share turnover ratio. Comparing to 2017 the derivatives trading value has increased by 46% and 6% of total consolidated revenues is contributed to the derivatives. In the derivative markets the highest daily trading value was on 10th of August in 2018, with TL 13.3 billion, which broke the record.[3]

Table 1. 1 *Breakdown of Domestic and Foreign Investor's Shares Trades in Borsa Istanbul Stock Market* **(source:** https://www.borsaistanbul.com/en/data/data/viop-derivatives-market**)**

PERIOD	DOMESTIC INVESTORS %	FOREIGN INVESTORS %
2013	75.31	24.69
2014	76.29	23.71
2015	71.57	28.43
2016	72.68	27.32
2017	71.96	28.04
2018	65.21	34.79
2019	66.12	33.88
JANUARY	68.93	31.07
FEBRUARY	63.11	36.89

[2] MSCI Emerging markets Index https://www.msci.com/emerging-markets
[3] Borsa Istanbul annual report, https://www.borsaistanbul.com/docs/default-source/kurumsal-yonetim/borsa-2018-annual-report.pdf?sfvrsn=6

Istanbul Stock Exchange is the first Turkish organization which is providing trading of bonds, equities, bills, private sector bonds and international securities. ISE was established as autonomous organization in early 1986. Borsa Istanbul (BIST) is the stock exchange organization of Turkey which combines the former Istanbul Stock Exchange (ISE), Derivatives Exchange Istanbul (VOB) and Istanbul Gold Exchange (IAB) under one umbrella. It was established on April 3, 2013 as a consolidated company with capital 423,234,00 TL. BIST started operating on April 5, 2013 with the slogan “worth investing”. The biggest shareholders of BIST is the Turkish Government with 49%. The rest of the shares are divided as follows: IMK with 41%, VOB with 5%, IMKB members with 4%, IMKB brokers with 1% and IAB members 0.3%.

There are four main markets in Borsa Istanbul: debt securities market, equity market and precious metals, diamond market and derivatives market. Turkish derivatives market (hereafter VIOP) is designed for trading options and futures contracts based on capital market instruments and financial or economic indicators and other derivative products electronically. Borsa Istanbul Derivative Market (VIOP) and Turkish Derivatives Exchange (TURKDEX) started trading on 5th of August in 2013. All option and future contracts in Turkey can be traded at the single platform under the VIOP.

BIST Stock indices have been created in order to measure the return and price performances of group of stocks which are traded on Borsa Istanbul. BIST 30, BIST 100, BIST Industrials and BIST Banks Price Indices are once calculated during the session and spread in real-time. Foreign currency and return indices which are calculated and spread once at the end of the session. BIST30 Index includes 30 stocks which are traded on the BIST Main markets and BIST Stars and the real estate investment venture and trust capital investments trusts are traded on the Structured and Collective Products Market.

1.8.1 MSCI indices

MSCI denotes Morgan Stanley Capital International. The MSCI Index is a measure of performance of stock markets in a specific area. Morgan Stanley published the Capital International Indexes in 1968. These indexes were first indexes for markets outside the United States. It took almost 20 years, for the Emerging Markets Index to be published in 1987 and All Country Indexes for emerging markets and developing markets. The exchange-traded funds follow the MSCI indexes. Managed mutual funds try to outperform them by picking better stocks.[4] Each Index sums up the total value of market capitalization of all stocks. The market caps are computed both in local currency and U.S dollar, which gives the idea how index is doing without exchange rates impact. MSCI has the indexes for different geographic areas and global indexes for stock categories from small to large-cap. MSCI Emerging Market Index and MSCI All Country Index are one of the most popular tracks.

The MSCI Emerging Markets Index represents the performance of large and mid-cap securities in the following 26 developing countries: China, Korea, Brazil, Columbia, Mexico, Peru, Greece, Malaysia, Pakistan, Hungary, Egypt, Argentina, Turkey, Poland, Russia, Philippines, Qatar, South Africa, Saudi Arabia, Thailand, Taiwan, United Arab Emirates, Czech Republic, Chile, Indonesia, India. According to, data of December 2018, it had more than 1100 constituents covering about 85% of free float-adjusted capitalization in each country.[5]

On the chart below we can see the percentage allocation of county in MSCI Emerging Markets Index. The chart is according to March 29, 2019.

[4] https://www.thebalance.com/msci-index-what-is-it-and-what-does-it-measure-3305948

[5] https://www.msci.com/documents/1296102/15035999/USLetter-MIS-EM-May2019-cbren.pdf/fb580e1e-d54c-4c68-1314-977bbff69bd7?t=1559125400402

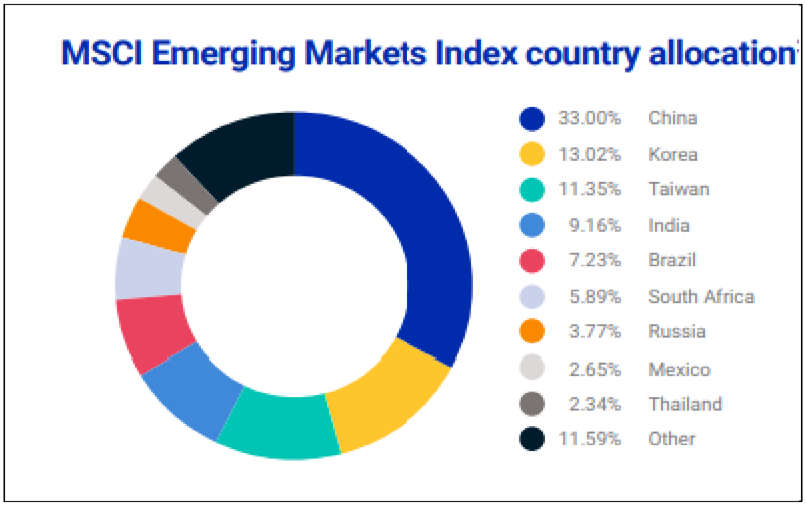

Figure 1. 3 *MSCI EM Index country allocation*

(***source:*** *https://www.msci.com/documents/1296102/15035999/USLetter-MIS-EM-May2019-cbr-en.pdf/fb580e1e-d54c-4c68-1314-977bbff69bd7?t=1559125400402)*

The MSCI ACWI Index represents the performance of the large and mid-cap stocks of 23 developed countries (Australia, Ireland, Germany, Italy, Norway, Austria, Switzerland, Portugal, Belgium, Denmark, Japan, Spain, the U.K, New Zealand, Finland, France, Hong Kong, Israel, Canada, Netherlands, Singapore, Sweden, and the U.S.) and 26 emerging markets. With 2844 constituents, the index covers about 85% of the global investable equity opportunity set. On the chart below we can see the MSCI ACWI country allocation.

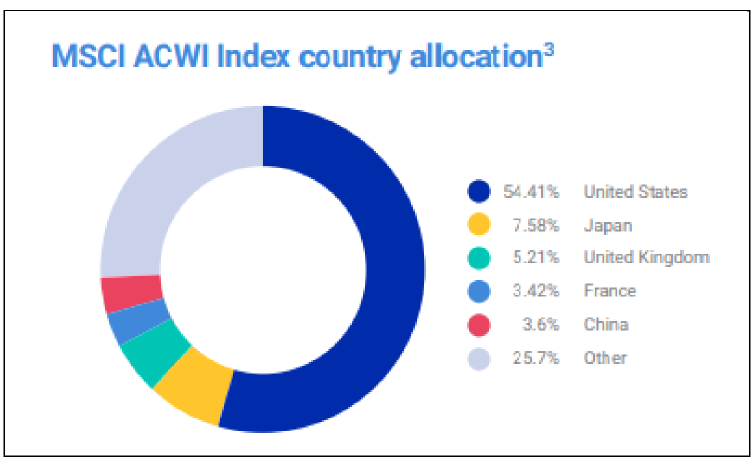

Figure 1. 4 *MSCI AC Index country allocation*

(source: *https://www.msci.com/documents/1296102/15035999/USLetter-MIS-ACWI-Apr2019-cbr-en.pdf/9de006ef-9cf0-8bd7-62ec-e67430e9155f?t=1559105406131)*

1.8.2 MSCI Turkey index

MSCI Turkey Index is developed to measure the performance of the large and mid-cap segments of the Turkish market. With 16 constituents, the index covers approximately 85% of the equity universe in Turkey, below we can see the performance of MSCI Turkey Index comparable to MSCI EM and MSCI AC. According to annual performance, the lowest performance for MSCI Turkey was in 2008 and 2018 (-60.34 per cent and -38.45 per cent) The highest performances were in 2005 and 2009 (79.84 per cent and 91.35 per cent). These ups and downs are caused by political and economic factors inside and outside of Turkey, which impacts Turkish stock market performance.

Table 1. 2 *Annual performance of MSCI Turkey, MSCI EM and MSCI AC*

ANNUAL PERFORMANCE (%)

Year	MSCI Turkey	MSCI Emerging Markets	MSCI ACWI IMI
2018	-38.45	-10.26	-5.54
2017	21.53	20.59	8.87
2016	-5.72	14.51	11.60
2015	-24.10	-5.23	8.96
2014	35.19	11.38	18.24
2013	-29.92	-6.81	18.21
2012	61.69	16.41	14.60
2011	-33.25	-15.70	-4.81
2010	29.20	27.14	22.29
2009	91.35	72.94	32.16
2008	-60.34	-50.92	-39.36
2007	57.02	25.74	0.26
2006	-16.99	18.20	8.19
2005	79.84	54.41	28.54

Source: (*https://www.msci.com/documents/10199/ae0d3e1e-ef7f-47ed-a2a3-970532651d23)*

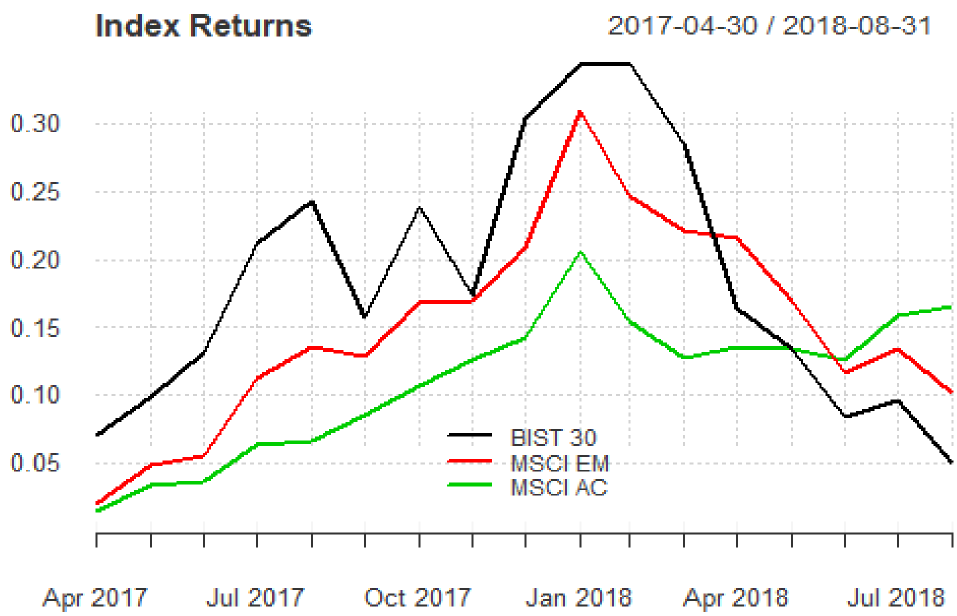

Figure 1. 5 *The return indexes of BIST30 and MSCI Indexes (MSCI All Country World and MSCI Emerging Markets).*

The chart on the Figure 1.5 above shows performance comparison of the return indexes of BIST30 and MSCI Indexes (MSCI All Country World and MSCI Emerging Markets).

Table 1. 3 *The net returns of MSCI Turkey, MSCI EM and MSCI AC*

INDEX PERFORMANCE — NET RETURNS (%) (JUL 31, 2019)

					ANNUALIZED			
	1 Mo	3 Mo	1 Yr	YTD	3 Yr	5 Yr	10 Yr	Since Dec 29, 2000
MSCI Turkey	13.88	19.67	4.53	13.83	-7.83	-8.59	-0.49	1.66
MSCI Emerging Markets	1.03	-2.03	2.80	12.15	8.58	5.66	7.12	7.89
MSCI ACWI IMI	2.59	1.02	7.24	19.54	10.08	10.41	12.09	4.73

(source: https://www.msci.com/documents/10199/ae0d3e1e-ef7f-47ed-a2a3-970532651d23)

When MSCI EM, MSCI ACWI are compared, we find that MSCI Turkey seems to be the most volatile index with the highest turnover ratio. Moreover, MSCI Turkey index has the lowest Sharpe Ratio.

Table 1. 4 *Index risk and return characteristics of MSCI Turkey, MSCI EM and MSCI AC*

(source: https://www.msci.com/documents/10199/ae0d3e1e-ef7f-47ed-a2a3-970532651d23).

INDEX RISK AND RETURN CHARACTERISTICS (JUL 31, 2019)

		ANNUALIZED STD DEV (%) [2]			SHARPE RATIO [2, 3]				MAXIMUM DRAWDOWN	
	Turnover (%) [1]	3 Yr	5 Yr	10 Yr	3 Yr	5 Yr	10 Yr	Since Dec 29, 2000	(%)	Period YYYY-MM-DD
MSCI Turkey	13.43	33.12	29.70	29.90	-0.07	-0.14	0.13	0.22	68.07	2013-05-15—2018-08-13
MSCI Emerging Markets	7.11	11.06	13.47	13.63	0.84	0.50	0.57	0.42	59.79	2007-10-29—2008-10-27
MSCI ACWI IMI	2.61	10.73	11.90	10.80	0.99	0.92	1.11	0.29	53.48	2007-06-15—2009-03-09

[1] Last 12 months [2] Based on monthly net returns data [3] Based on ICE LIBOR 1M

Table 1. 5 *MSCI Turkey's top 10 constituents*

(source: https://www.msci.com/documents/10199/ae0d3e1e-ef7f-47ed-a2a3-970532651d23)

TOP 10 CONSTITUENTS

	Float Adj Mkt Cap (EUR Billions)	Index Wt. (%)	Sector
TURKIYE GARANTI BANKASI	3.67	13.14	Financials
AKBANK	3.49	12.47	Financials
BIM BIRLESIK MAGAZALAR	3.22	11.50	Cons Staples
TUPRAS TURKIYE PETROL	2.83	10.12	Energy
TURKCELL ILETISIM HIZMET	2.32	8.29	Comm Srvcs
KOC HOLDING	2.30	8.22	Industrials
EREGLI DEMIR CELIK FABRI	1.68	6.01	Materials
TURKIYE IS BANKASI C	1.59	5.67	Financials
SABANCI HLDG (HACI OMER)	1.46	5.23	Financials
TURK HAVA YOLLARI	1.11	3.97	Industrials
Total	23.67	84.62	

On the table above, we see top 10 constituents among 20 MSCI Turkey constituents (those stocks are in our sample except for BIM Birleşik Mağazalar).

1.8.3 Single stock options contract specification on Borsa Istanbul

With the important merger of Borsa Istanbul and Turkish Derivatives Exchange, all derivative contracts are traded on VOB and VIOP and are integrated on a single platform and all contracts are traded under the one roof of Borsa Istanbul Futures and Options Market. There are following contracts trading:

- Single stock options contracts
- Single stock futures contracts
- BIST 30 options contracts

- BIST 30 futures contracts
- TRYUSD futures contracts
- TRYEUR futures contracts
- EUR/USD cross currency futures contracts
- Gold futures contracts
- USD/Ounce gold futures contracts
- Aegean cotton futures contracts
- Anatolian red wheat futures contracts
- Base-Load electricity futures contracts

There are two kind of option classes, call and put options.

Exercise style of options listed on VIOP is European, which means that the options can only be exercised on the expiry date.

Contract size, there are 100 shares of underlying stock in per contract.

Tick size, prices are offered for the premium value of one underlying asset. TL 0.01 per underlying asset = TL 1.00 per contract (contracts size 100 shares) (https://www.borsaistanbul.com/en/products-and markets/products/options/single-stock-options).

1.8.4 BIST 30 index options contract specification

The name of the underlying asset is BIST 30 Price Index, with options class: call and put. With European exercise style, which can be exercised on the expiration date.

Contract size, as the underlying security is the 1/1000 of the index value. Index options' contract size is 100 underlying securities (https://www.borsaistanbul.com/en/products-and markets/products/options/equity-index-options/bist30-index-options-contract-specification).

1.9 Research Problem

Recent studies on portfolio optimization have suggested that portfolios based on the historical data yield poor performance and that measures derived from option prices such as option-implied volatility risk premium are helpful in predicting stock returns.). (Plyakha, Uppal, & Vilkov, 2012). Historical volatility shows how stock prices fluctuate on a day to day basis over one-year period. Implied volatility does not depend on historical prices of stock, but it forecasts how the marketplace anticipates stock's volatility in the future, based on the changes in the stock options prices (Canina & Figlewski, 1993).

We investigate if implied information in stock options prices can be used in order to improve portfolio's performance. Using option-implied volatility as forward-looking information to create a portfolio strategy, we develop a multi-stage portfolio optimization model using the option prices to simplify market risk. We explore if using option-implied volatility and option-implied correlations can help improve portfolio performance.

In our research, we calculated implied volatility using Black-Scholes option pricing model. BSOPM calculates the theoretical value of a European option using the option's strike price, expected interest rate, time of expiration, expected volatility, expected dividends and current stock price. For comparison, we used the performance of the benchmark portfolio, which is the BIST30 Index. We measured portfolio performance with the following measures: Annualized return, annualized portfolio volatility (standard deviation) and Sharpe ratio.

1.9.1 Significance of the study

The findings may enable local and foreign investors, future investors, researchers and policy makers to utilize forward–looking information implied based on option prices in order to improve the out-of-sample performance of their portfolios.

1.9.2 Goals of the study

The main purpose of this study is to examine whether investors can use forward-looking information to improve the selection of portfolios and improve their out-of-sample performance. We aim to investigate if optimizing portfolios using information in option-prices can yield better performance measures rather than optimizing portfolios ignoring option-implied information. For the evaluation of portfolio performance, we used the following three criteria: annualized portfolio return, annualized portfolio volatility (standard deviation), portfolio Sharpe ratio. Based on our empirical analysis we can demonstrate that prices of stock options contain information that can be useful to improve the out-of-sample performance of portfolios.

1.9.3 Limitations of the study

In this study we use option price data of stock options traded at Borsa Istanbul. Our sample consists of 20 stock options and the sample period is from March 2017 to July 2018.

1.9.4 Research questions

The research aims to answer the following questions:

- Is forward-looking information helpful in portfolio selection process?
- Does option-implied volatility improve portfolio performance?
- Is the performance of portfolio based on option-implied volatility superior to mean-variance or minimum variance portfolio?

1.9.5 Research hypothesis

H_0 : Optimal portfolios based on option implied information do not perform better than optimal portfolios based on historical information.

H_1: Optimal portfolios based on option implied information perform better than optimal portfolios based on historical information.

This thesis is organized as follows: Having introduced the topic in chapter 1, chapter 2 presents review of related literature. Chapter 3 presents the data used and the methodology of the study. Chapter 4 presents the empirical findings and chapter 5 presents conclusion and recommendations.

2 LITERATURE REVIEW

Since portfolio selection is an important issue among investors and policy makers, researchers have given a special attention to the subject of portfolio optimization. There are numerous papers about portfolio optimization using different methodologies and techniques. According to many studies, option-implied volatility is a strong forecaster of future volatility in the equity market. Thus, option implied information is used in several studies for portfolio optimization purposes. This chapter presents the summary of relevant literature about portfolio optimization and option implied volatility. Previous studies are sorted out according to similar methodologies and techniques.

H. Markowitz, who is considered as father of Modern Portfolio Theory (MPT) wrote his first paper titled “Portfolio Selection” in 1952. This seminal paper is about portfolio optimization and helps investors to allocate investments between different securities. He introduced the efficient set of portfolios with maximum return with a given level of risk. Mean-variance optimization (MVO) is a quantitative tool, which allows investors to make allocation by considering the trade-off between return and risk. Markowitz (1959) in his book titled “Portfolio Selection: Efficient Diversification of Investments” importantly extended findings from his paper portfolio selection (1952).

James Tobin (1958) in his article titled “Liquidity Preferences as Behavior towards Risk” based on Markowitz work created the “efficient frontier” and “capital market line”. According to Tobin’s model, investors, no matter what their risk tolerance level is, will maintain stock portfolios in the same proportion since they have similar expectations about the future. As a result, Tobin concludes that their investment portfolios will diverge just in their proportion of bonds and stocks.

Later, Harry Markowitz (1959) suggested another measurement of risk, semi-variance of returns. Markowitz (1959) discussed semi-variance and claimed that portfolios based on semi-variance have better performance compared to those based on variance. Moreover, Markowitz (1991) suggested that semi-variance is a more reasonable measure of risk and claimed that as investors worry about underperformance than over-performance, semi-variance is a more suitable measure of risk rather than variance.

Boasson et al. (2011) used mean-semi-variance approach in order to measure the downside risk in optimal portfolio selection. They measured return dispersions below the expected return of the investment return. They used the sample of 7 exchange traded index funds which includes several kinds of securities to test and compare the differences between asset allocations and optimal portfolios. They constructed portfolios using mean-semi-variance approach and traditional mean-variance approach. They showed that semi-variance approach provides desirable benefits. Portfolio optimization under the semi-variance model improved portfolio’s expected return, while minimizing its downside risk exposure.

Konno, Waki, & Yuuki (2002) explained the attractiveness and importance of using lower partial risk also called downside risk in portfolio management. The aim of this work is to review the important characteristics of this measurement and to obtain alternative measurements, such as lower semi-absolute deviation, lower semi-variance, conditional value-at risk and below the target risk. They proposed that these risk measurements are useful for controlling the downside risk, when the asset distribution in non-symmetric. Their sample consists of 104 assets and 105 scenarios. According to results MCVaR, M-LSAD and mean-lower partial risk models can control portfolio’s downside risk, when the distribution of returns are not symmetric nor normal.

Grootveld & Hallerbach (1999) explained that the popularity of downside risk among the investors has been growing. The paper focuses on the similarities and differences between downside risk and variance measures, from the theoretical and empirical point of view in USA. Empirical results showed the differences between portfolios which are based on variance and semi-variance. According to the findings, downside risk approach tends to favor stocks with the minimum risk point. The differences in portfolio composition with downside risk optimization are significantly large compared to the portfolio optimization with mean-variance approach.

Galsband (2012) investigated the downside risk of international stock returns in 14 major industrialized economies worldwide, including Canada, the US and twelve major EAFE countries between 1975-2010. His findings show that the world’s largest equity markets can be rationalized by differences in sensitivities of international asset returns to the downside shocks. Generally, international value stocks are sensitive to market’s permanents downside shocks.

Yan, Miao, & Li (2007) utilized Markowitz's semi-variance portfolio selection model. They formulated multi-period semi-variance model. For solving this model, they use genetic algorithm (GA) and particle swarm optimizer (PSO). According to the authors, for the need of investors and reality of financial markets, measuring the risk by variance should be replaced by semi-variance as the findings showed that the method is evidently superior.

Chang, Christoffersen, Jacobs, & Vainberg (2012) found option-implied skewness and volatility as a strong predictor of future beta, and suggest that company beta can be estimated from skewness and option-implied volatility measures from index options and equity. They compare option-implied beta with historical beta and find out that option-implied betas contain information which is not contained in historical beta and option-implied betas have important predictive power for future betas which increase beyond options' maturities. According to the empirical findings of this paper, if the underlying risk-neutral distribution is more negatively skewed, then option-implied betas are higher. They also prove that the stock's option-implied beta is relatively determined by the difference between the skew of the index's risk-neutral distribution and stock's risk-neutral distribution.

Kempf & Korn (2012) developed new estimators of the covariance matrix, which totally relied on forward-looking information. This estimator required implied volatility and implied correlation. Second, the core contribution is that they tested this new method on GMVP, which is based on the covariance matrix of implied estimators and found that it performs much better in the out-of-sample for a sample of 30 stocks from Dow Jones Industrial Average (DJIA). The findings of the empirical study give 2 recommendations to investors. First, strategies which are based on fully-implied estimators outperform the benchmark strategies, and mostly are not beaten by other strategies. Second, if investors want to use historical estimators, they should use the most recent date and shrinkage estimators.

Plyakha et al. (2012) used option-implied information in order to improve the selection of the mean-variance portfolio, with large number of stocks and to identify which option-implied information is the most useful to improve the out-of-sample performance. They measure the performance of portfolios using Sharpe ratio, turnover and volatility. As the benchmark, they use 1/N portfolio. Their findings prove that using option-implied volatility reduces volatility of the portfolio. Also using implied volatility and option implied model-free skewness can achieve higher Sharpe ratio rather than by ignoring option-implied information. They conclude that stock options prices include information which can be used to improve the performance of the out-of-sample performance.

Kostakis, Panigirtzoglou, & Skiadopoulos (2010) used market option prices (also known as option-implied distribution) for asset allocation problem. They used risky and risk-free assets, implied spread, which was taken from S&P500 futures options and then converted to risk-adjusted ones. They used the stock index implied distribution as an input for calculating the optimal portfolio. According to empirical findings using option-implied information increases obtained risk-adjusted return of investor's which makes meaningfully better off compared with the historical distribution.

Driessen, Maenhout, & Vilkov (2013) developed a model of correlation risk pricing for stock returns. They used two samples, which are, the narrow stock market index DJ30, from 1997 to 2012 and the broad stock market index S&P500 from 1996 to 2012. According to findings, there is a big negative risk premium in 1996-2012, with average option-implied correlation 39.5% for S&P500 and 46% for DJ30, with average realized correlation 32.6% for S&P500 and 35.5% for DJ30. Index options seem expensive in the sense that risk-neutral expected correlation in their prices is considerably higher than average realized correlation, but this high price reflects insurance cost against the unexpected correlation increases in risk and resulting loss in diversification benefits. Another core result is that implied correlation has significant predictive power for future stock market excess returns, particularly, at the 6-month and 1-year horizons.

Bahaludin, Abdullah, & Tolos (2017) used option-implied distribution as an input in asset allocation. The data is divided into two, option prices data and historical prices data. The data consists of stocks, which are listed in Dow Jones Industrial Average (DJIA) index. The sample period is 1/1/2009-12/31/2015. As an alternative of using historical prices, they used option prices to build a portfolio. The performance is measured by Sharpe ratio and standard deviation of portfolio. Findings showed that the portfolio based on risk-world and risk-neutral densities exhibit statistically significant differences. Moreover, the portfolio constructed from the risk-world density performs better rather than from the risk-neutral density.

Rehman & Vilkov (2011) used U.S. exchange-traded individual stock option data and showed that ex ante skewness is positively correlated with future stock returns. They measured ex ante by using the model free implied skewness (MFIS) of the distribution of risk-neutral return and explained that high MFIS stocks perform better, rather than low MFIS. Using MFIS, it is possible to identify the variation of a value of a firm from its basic value. The most overrated stocks have the most negative ex ante skewness. In contrast to historical skewness estimates, ex ante skewness is related positively to the future returns of the stocks.

Vilkov & Xiao (2013) proposed a forward-looking variable, which directly evaluates the expected stock or market crash size, and condition of realization. With this variable, they made predictions of crash in the cross-section stocks and market index. The main aim of their paper is to estimate a stock specific and market wide tail loss measure (TLM) from the put options, traded at out of money and to show that an ex ante magnitude is positively correlated to the future realized crashes. When there is no crash, higher loss expectations through the immediate decline in the value of asset are related with the higher premium to be gained for taking risk. TLM contains information about the future stock returns over implied correlation (IC) and variance risk premium (VRP) to predict the future returns of market. For getting final results they proceed some steps: first, they use (EVT) extreme value theory, in order to estimate the TLM from perceived out-of-the-money option prices. Then, they conduct time-series test, using the robust regression and show that when standard deviation increases in the TLM, it causes ultimate positive change in the weekly return of market. Next, they exercise optimization of portfolios using option-implied variables and compare it to a non-informative portfolio, and find out that the only variable which gives significantly useful information is implied correlation. On the final stage they study stock-specific TLM and stock returns cross-section and they conclude that TLM is positively correlated with the expected return.

Buss, Schoenleber, & Vilkov (2017) used variance risk premium and implied correlation for predicting market returns, with variance risk premium predicting market return only one quarter ahead. They address two core issues in the paper: first, they build an ex ante covariance (correlation) matrix from the option prices without using any historical information to estimate linear factor model obtained by current option prices, and second they identify which link of implied correlation forecasts the market returns. They found that correlation and variance risk premiums differ according to sectors of economy. That means that stocks in various sectors have heterogeneous exposure to underlying factor. Implied correlation forecasts systematic diversification risk and option-implied covariance matrix which is based on implied correlation between and within economic sectors and it might be used for extracting statistical factors which better explain dynamics of stock returns.

Davari-ardakani, Aminnayeri, & Seifi (2016) considered multistage portfolio optimization model with NYSE options and stocks, they developed optimization model, which employs options for mitigating market risk. In their methodology, they used dependence structure of different security returns and they considered serial correlations of each security return. They used Black-Scholes model for determining European option's call and put prices. They also used back-testing simulations for comparing multistage models to single-stage ones. According to their results, multistage models show better performance and options can be considered as a core instrument for controlling investment portfolio's market risk.

Vial (2013) analyzed option-implied information in the context of portfolio optimization. Option-implied information is obtained from DJIA and S&P100 index, with sample period from January 1996 to January 2012. The paper focused on the derivation of the option-implied covariance and the stability of forward-looking covariance was increased by Frobenius norm, also known as an, Euclidean norm. Nevertheless, option-implied portfolios performed better than portfolios based on historical information. However, the difference often is insignificant. The results for different estimation periods and strategies are robust assuming that forward-looking information is inherent in exchange-traded options.

Vilkov & Xiao (2013) computed extreme returns using option-implied information. From the observed option prices, they estimated forward-looking tail loss measure (TLM). TLM predicts the probability of market crashes and market returns. In addition to TLM, they also consider the option-implied predictors of asset returns such as model free implied variance, kurtosis and skewness, variance risk premium and implied correlation. They found that TLM is positively correlated to the future market returns, especially in the short run. The other variables, which importantly explain future market returns are implied correlation and variance risk premium. They also find that constructing portfolios using implied correlation gives better results than using historical information.

3 DATA AND METHOGOLOGY

This chapter presents the methods employed in the study in order to construct different kind of portfolios using stock options date which is obtained from Borsa Istanbul and includes 20 companies. We compare portfolios performances constructed using historical and implied information.

In R, we used PortfolioAnalytics to calculate the portfolio weights. The method that is used was to generate thousands of portfolios and estimate what their risk-return characteristics are based on the covariance matrix used. Then the weights of the portfolio that minimizes variance and the portfolio that maximizes the mean-variance are used to calculate subsequent performance. The portfolios were constrained to be long-only, such that asset weights were >=0%, fully invested, such that the sum of asset weights = 100%, and individual asset weights were capped at 20%. The DerivMkts package was used to estimate the implied volatility of the options. Options were chosen as the call price with the strike closest to the call and the expiration date was selected as the closest month where there was both an option for the underlying asset and the BIST30. The reason for this was that the BIST30 implied volatility was required to estimate the implied correlation and it was consistent to use options that had the same expiration.

Our research utilizes option-implied volatility in order to improve the out-of-sample performance of portfolios. We employ Black-Scholes option pricing model (BSOPM), which is the most widely used model in option pricing. For constructing mean-variance and minimum-variance portfolios we use Markowitz's models. In this part, we define our data and explain different portfolio construction methods and the metrics used in order to compare the benchmark portfolio with the portfolios based on historical and option implied information.

3.1 Data

We utilize stock price data and option price data of 20 stocks, traded in the Futures and Options Market of Borsa Istanbul since March of 2017. The data set includes daily data between March 2017 until July 2018.

From 2013 to 2015 only 10 stock options were traded on Borsa Istanbul. From 2015 onwards number increased gradually and became 20. We decided to employ data starting from 2017, as it includes 20 stocks and until 2017 the trading volumes of stock options were minimal. The risk free (RF) rate was computed from the benchmark (with a maturity of 2 years) government bond data.

We built different portfolios from these 20 stocks with 20% maximum weight restriction of assets in the portfolio. For the benchmark portfolio, we used BIST30 index data, as our sample consists of 20 stock options.

Stock closing prices and stock total returns were obtained from Thomson Reuters Database. The discrete return of an individual investment or portfolio can be calculated as follows (Dobbins et al., 1994):

$$R_t = \frac{P_t - P_t - 1 + D_t}{P_t - 1} \tag{3.1}$$

Where,

R is a periodical return

P_{t-1} is an initial price of the period

P_t-P_{t-1} is a capital loss or gain

D_t is a dividend in the end of the period

Pt is the last price of the period.

Historical volatilities of returns were computed using 3 years “rolling window method” from the stocks total return. Stock option data was obtained from Borsa Istanbul. Total number of the months of the data are 17 months from March 2017 to July 2018. We decided to work on monthly basis, thus, for calculations we chose the last trading dates of each month.

Data was filtered as follows: we choose the last trade date of each month, then we choose the expiry date, which is at least one month ahead (1 month or 2 months ahead) and we considered the option contract (when both the BIST30 Index Option and the stock option were traded on the same date), which had the closest strike price to the observed stock close price on the expiry date of the option contract.

The data includes 20 stock options and one index option. In total, data consists of 340 stock options and 17 index options. The list of the companies included in the study for portfolio construction purposes is as follows:

Table 3. 1 *The list of sample companies*

AKBNK.E	Akbank T.A.Ş.
ARCLK.E	Arçelik A.Ş.
EKGYO.E	Emlak Konut Gayrimenkul Yatırım Ortaklığı A.Ş.
EREGL.E	Ereğli Demir ve Çelik Fabrikaları T.A.Ş.
GARAN.E	T. Garanti Bankası A.Ş.
HALKB.E	Türkiye Halk Bankası A.Ş.
ISCTR.E	T. İş Bankası A.Ş.
KCHOL.E	Koç Holding A. Ş.
KRDMD.E	Kardemir Karabuk Demir Celik Sanayi ve Ticaret A. Ş.
PETKM.E	Petkim Petrokimya Holding A.Ş.
PGSUS.E	Pegasus Hava Tasimaciligi A. Ş.
SAHOL.E	Hacı Ömer Sabancı Holding A.Ş
SISE.E	Türkiye Şişe ve Cam Fabrikaları A.Ş
TCELL.E	Turkcell Iletişim Hizmetleri A.Ş
THYAO.E	Türk Hava Yolları A.O.,
TOASO.E	Tofas Turk Otomobil Fabrikasi A.Ş.
TTKOM.E	Türk Telekomünikasyon A.Ş.
TUPRS.E	Türkiye Petrol Rafinerileri A.Ş.
VAKBN.E	Türkiye Vakıflar Bankası T.A.O.
YKBNK.E	Yapi ve Kredi Bankasi A.Ş.

3.2 Portfolio Optimization

Optimal portfolios were constructed with R code (the R codes are presented in the appendix). We applied monthly rebalancing during portfolio optimization. Rebalancing is a process of buying and selling of securities to bring the portfolio for setting the weights of each asset back to its original state according to an investor’s tolerance of risk and investment strategy has changed. The investor can use rebalancing to readjust the weighting of each asset or security class in the portfolio in order to fulfill a newly devised asset allocation.

3.2.1 Minimum variance optimization

Minimum variance portfolio also known as minimum risk portfolio is a risk-based approach in portfolio construction. It is different from Markowitz portfolio selection in that instead of using both return and risk, minimum variance portfolio is constructed using only measure of risk. If an investor desires investing in the portfolio with the lease risk, he/she does not think about the expected return, however the only thing that he/she wants is the lowest possible risk. The minimum variance portfolio is related with modern portfolio theory and efficient frontier. It is the only portfolio lying on the efficient frontier, with a minimum level of standard deviation. In this situation, the variance is also minimal. That is the reason why this portfolio is also called the minimum variance portfolio.

The minimum variance portfolio can be calculated by minimizing the variance to the necessary constraint. This is called as budget constraint, which is the amount of capital the investor has to invest. The reason why investors want to optimize the minimum variance portfolio is that it is very hard to estimate the future or expected return, but it is easy to measure the risk.

The optimization problem of mean-variance portfolio can be written as:

$$\min_{w} \quad w^T \hat{\Sigma} w - w^T \hat{\mu} \tag{3.2}$$

$$\text{s.t } \; w^T e = 1 \tag{3.3}$$

The purpose of the first equation is to minimize the difference of portfolio return variance $w^T \hat{\Sigma} w$ and its mean $w^T \hat{\mu}$. The second equation assures that the portfolio weights resume to one.

The minimum-variance portfolio problem solution is as follows:

$$w_{min} = \frac{\hat{\Sigma}^{-1} e}{e^T \hat{\Sigma}^{-1} e} \tag{3.4}$$

The covariance matrix $\hat{\Sigma}$ can be decompounded into correlation and volatility matrices.

$\widehat{\Sigma} = \text{diag}(\hat{\sigma})\hat{\Omega}diag(\hat{\sigma})$

Where, $\text{diag}(\hat{\sigma})$ is the diagonal matrix with stock volatilities on the diagonal, and diagonal and $\hat{\Omega}$ is the matrix of correlation. For obtaining the optimal weights of portfolio on the covariance matrix sample, we need to estimate two quantities: correlations $\hat{\Omega}$ and volatilities $(\hat{\sigma})$.

3.2.2 Mean-variance optimization

Mean-variance model of portfolio selection was published in 1952 by Harry Markowitz. This model is one part of Modern Portfolio Theory (MPT) and still inspires empirical and theoretical research today. Markowitz suggests that portfolio selection process can be divided into two stages: the first stage starts with observations and beliefs about future performance of assets and in the second stage above mentioned beliefs are used for constructing the portfolio (Markowitz, 1952). The most important aim of the portfolio theory is optimally allocating investment budget among different assets. Mean-variance optimization (MVO) is the quantitative tool, which allows the investor to make allocation according to trade-off among risk and return. As we have already mentioned, Markowitz divides portfolio selection process into 2 stages, mean variance approach is about second stage, portfolio selection.

Every investor's aim is to maximize the capitalized or discounted return of future or expected return. Although the future is not known exactly, it is "anticipated" or "expected" return which Markowitz discounts. The hypothesis indicates that the investor puts all her or his funds in the asset with the highest discount value. If two or more assets have the same value, then any of these combinations of these is as good as any other. Let us assume there are N securities, and consider that r_{it} is an anticipated return, at time t per dollar invested in asset i, d_{it} be a rate at which the return on i^{th} asset at time t will be discounted back to the present. If we assume that X_i is the relative amount invested in asset i. Markowitz excludes short sales, so $X_i \geq 0$ for all i. Then the discounted expected return of a portfolio is calculated with the following formula: (Markowitz, 1952).

$$R = \sum_{t=1}^{\infty} \sum_{i=1}^{N} d_{it} r_{it} X = \sum_{i=1}^{N} X_i \left(\sum_{t=1}^{\infty} d_{it} r_{it}\right) \tag{3.5}$$

$$R_i = \sum_{t=1}^{\infty} d_{it} r_{it} \tag{3.6}$$

this is the discounted return of the i^{th} asset, thus

$$R = \sum X_i R_i \tag{3.7}$$

Where,

R_i is independent of X_i since $X_i \geq 0$ for all i and $\sum X_i = 1$

R is a weighted average of R_i, with X_i as non-negative weights

In order to maximize R, Markowitz consider $X_i = 1$ for i with maximum R_i

$R = \sum X_i R_i$ defines the flow of returns from the portfolio as an entire.

If the investor's aim is to maximize the expected return, then he or she should place all the funds in the assets with the maximum expected return. Markowitz's model does not derive n-asset case, but his model represents for 3 or 4 asset cases. The portfolios return (R) is a sum of the random variables, investor can choose the weights of assets. The expected return of the portfolio is calculated as follows:

$$E = \sum_{i=1}^{N} X_i \mu_i \tag{3.8}$$

The variance of portfolio is calculated as follows:

$$V = \sum_{i=1}^{N} \sum_{j=1}^{N} \sigma_{ij} X_i X \tag{3.9}$$

Investors have to choose different combinations of E and V according to their choice of portfolio $X_1, \ldots X_N$. Assume that the set of all accessible (E, V) combination is as on figure below.

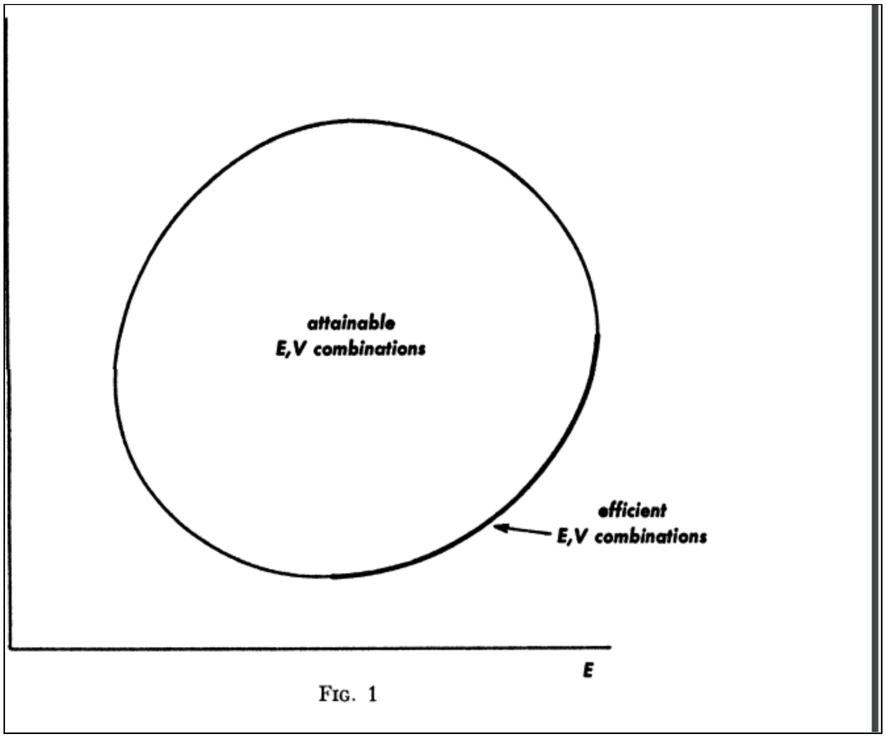

Figure 3. 1 *E-V combination (Source: Adopted from H. Markowitz (1952)*

According to E-V rule the investors should choose one of those portfolios that give increase to the E-V combinations shown as efficient in the figure above. With minimum V for a given E and maximum E for a given V. In order to calculate the efficient surface, two conditions are needed. First, the investors must act according to E-V maxim and second, we had to be able to arrive at reasonable μ_i and σ_{ij}.

In the three securities case, Markowitz's model reduces to the following:

$$E=\sum_{i=1}^{3} X_i \mu_i \tag{3.10}$$

$$V=\sum_{i=1}^{3}\sum_{j=1}^{3} X_i X_j \sigma_{ij} \tag{3.11}$$

$$\sum_{i=1}^{3} X_i = 1 \tag{3.12}$$

$X_i \geq 0$ for i = 1, 2, 3.

3.2.3 Optimization with option implied information

We built a new estimator of covariance matrix using forward-looking information from a cross-section of option prices. The minimum variance portfolio is based on the covariance metrics where historical information is replaced by option implied information. For this, we employed:

- Option-implied volatility
- Option-implied correlation

3.2.3.1 Implied volatility ("Black-Scholes option pricing model")

Expectations regarding the volatility or any other moments of returns are usually estimated according to historical data. However, historical data may not be accurate for providing the best volatility forecast of expected risks, as it contains past information. In contrast, option trading data contains forward-looking information. Thus, implied volatility derived from option trading data may have significant information for expected risk, which historical data lacks (Francis J.C & Kim.D, 2013). Implied volatility ignores historical information, alternatively, determines the option σ based on the actual prices of options. While historical volatility is a backward-looking estimator, implied volatility is a forward-looking estimator.

Implied volatility is not directly measurable therefore it is usually derived from the "Black-Scholes Option Pricing Model", which was developed by three economists: Myron Scholes, Fischer Black and Robert Merton and is possibly the most well-known option pricing model in the world. The above, mentioned economists introduced this model in their paper in 1973, called "The Pricing of Options and Corporate Liabilities". The economists were awarded Nobel Prizes in economics in 1997 for their work of a new method in determining the value of derivatives. The model makes certain assumptions (Black & Scholes, 1973).

- The option is European and will be expired only on expiration date
- There is no dividend during the lifetime of option;
- Markets are efficient (for example, market movements cannot be anticipated);

- There are no any transaction costs during buying the options
- The volatility and risk-free rate are known and constant
- The returns on the underlying are normally distributed

Black-Scholes formula is also appropriate for corporate bonds, corporate liabilities and warrants. Generally, the formula can also be used to obtain the discount which should be applied to the corporate bond for default possibility.

For call option,

$$C = SN(d_1) - N(d_2)X_e^{-rT} \tag{3.13}$$

and for put option,

$$P = X_e^{-rT}N(-d_2) - SN(-d_1) \tag{3.14}$$

Where,

C is price of the call option

P is price of the put option

S is current stock price

t is time until option exercise

K is option strike price

r is risk free interest rate

N is cumulative normal standard distribution

e is exponential term.

Calculation of implied volatility requires using the following above mentioned five other model inputs:

1. The underlying price of the stock
2. The market price of the option
3. The expiration time
4. The strike price of an option
5. The risk-free interest rate

When we know option prices, the first step is using this information to back out implied volatilities for predicting volatility (Benninga, 2008). By replacing the model price C in the above, mentioned equation with the market price of the call option, we can figure out this equation regarding the volatility parameter σ by repeating the method, as the rest of variables, S, X, r, and T can be detected uniquely. The value which is obtained by backed out of σ is called implied volatility. It is implied by the Black-Scholes option pricing formula and values of other parameters (Benninga, 2008).

Implied volatility has several advantages and disadvantages. Regarding advantages: first, it might shift forward-looking information of underlying asset. Second, since the underlying asset's trading price is available, implied volatility can be calculated. Hence, implied volatility can be calculated any time the market is operates.

Regarding disadvantages: first, the estimation of implied volatility depends on the choice of option pricing model, so if the option price is calculated incorrectly, then implied volatility will provide deceiving assessment of expected risk. If market is efficient and option pricing model is specified correctly, implied volatility obtained by options with the same expiration date and with the same underlying asset should be the same, even if strike price is different. However, according to empirical studies, implied volatility calculated by Black-Scholes option pricing model displays a suspicious pattern across strike price. Second, implied volatility computed from deep-out-of-the-money and deep-in-the-money options has, a tendency to be higher rather than calculated from at-the-money options on the same underlying asset. Third, if there is no closed-form solution for the option, computing of implied volatility would be difficult, though several model free methods can be suggested. Fourth, there are measurement problems in implied volatility. These problems generally come from nonsynchronous trading. To specify more, timing of option closing prices and underlying asset may be different. For example, for rarely traded options the last trade of the option might appear before to the last trade of the day. Transaction cost is another reason of measurement error and fifth one, if there is no actively traded option on the underlying asset, estimation of implied volatility is impossible (Francis J.C & Kim.D, 2013).

According to preliminary studies, historical volatility provides better forecasting for future volatility rather than implied volatility. However, recent studies demonstrate that implied volatility is much more informationally efficient for forecasting future volatility rather than historical volatility. Yet, the comparison results can be sensitive to the options, option pricing model, sample period, market data.

In empirical finance, usually, historical information was used to build the future expectations. Option prices contain valuable information about the future moments of asset prices (Black & Scholes, 1973). In order to improve the quality of moment estimations, a number of researchers use forward-looking information instead of historical data. Strategies based on fully-implied estimators are a good choice as they significantly outperform the benchmark strategies in most cases and are never beaten by any other strategies (Kempf & Korn, 2012).

For our research, we need to calculate historical and implied volatilities. For this purpose, researches use different kind of models. For calculating historical volatility, we decided to use the rolling-window methodology with 3 years window length. For calculating implied volatility, some researchers follow Black-Scholes model and some of them follow Bakshi, Kapadia and Madan’s model.

3.2.3.2 Implied correlations

After predicting the volatility using options, we consider another option-implied information, implied correlation, as a portfolio optimization under the unbiased measure.

Implied correlations are not calculated directly from the options’ price. (Buss & Vilkov, 2012) create their model for computing implied correlations. They list the technical conditions and empirical formal observations, which must be satisfied for the implied correlation matrix. Then, they defined a simple parametric equation for calculating implied correlations, which is constant with the following empirical and technical observations.

Especially, for determination of the implied correlations, $\rho_{i,j,t}^{Q}$, they only determine constraints which equates the observed implied variance of a market index $(\sigma_{M,t}^{Q})2$ with the calculated implied variance of the portfolio for all market index elements i=1,….., N: (Buss & Vilkov, 2012)

$$(\sigma_{M,t}^{Q})^2 = \sum_{i=1}^{N}\sum_{j=i}^{N} w_i\, w_j \sigma_{i,t}^{Q} \sigma_{j,t}^{Q} \rho_{ij,t}^{Q} \qquad (3.15)$$

Where,

$\sigma_{i,t}^{Q}$ defines the implied volatility of i stock in the index, and

w_i is the index weight.

Implied correlation should satisfy the following technical conditions: first, correlation matrix should be positive and second, all correlations $\rho_{i,j,t}^{Q}$ should not exceed one. Furthermore, the implied correlation should be constant with two following empirical observations: First, the implied correlation $\rho_{i,j,t}^{Q}$ should not be higher than the correlation under the true measure $\rho_{i,j,t}^{P}$, and second the correlation risk premium should be higher in magnitude for stocks pairs. This provides larger diversification profits (for example, negatively or low correlated stocks), and hereafter are showing to a larger risk of losing diversification in bad times categorized by strong correlations. According to Mueller, Stathopoulos, & Vedolin (2012) the second observation is supported by a negative correlation between the correlation risk premium and correlation under the objective measure.

Taking into consideration the above mentioned empirical and technical conditions (Buss & Vilkov, 2012) developed the following parametric form for calculation of implied correlation $\rho_{i,j,t}^{Q}$:

$$\rho_{i,j,t}^{Q} = \rho_{i,j,t}^{P} - \alpha_t(1 - \rho_{i,j,t}^{P}) \qquad (3.16)$$

Where,

$\rho_{i,j,t}^{P}$ is expected correlation under the objective measure

α_t represents the parameter to be identified, it relates to the implied and realized correlations of the index and is constant in the interval (-1, 1). If α is supposed to be negative the implied correlation is going to be higher rather than the realized correlation, and if α is positive the opposite result will be obtained. (Eklund & Estaifo, 2018).

Replacing the implied correlations (3.15) into constraints (3.16), α can be calculated as following.

$$\alpha_t = -\frac{\left(\sigma_{M,t}^{Q}\right)^2 - \sum_{i=1}^{N}\sum_{j=i}^{N} w_i w_j \sigma_{i,t}^{Q}\sigma_{j,t}^{Q}\rho_{ij,t}^{Q}}{\sum_{i=1}^{N}\sum_{j=i}^{N} w_i w_j \sigma_{i,t}^{Q}\sigma_{j,t}^{Q}(1-\rho_{ij,t}^{\rho})} \tag{3.17}$$

After that, we should use equation (2) in order to distinguish the full implied correlation matrix Γ_t^Q, with $\rho_{i,j,t}^{Q}$ element.

If -1<α_t≤0, then the model is satisfied for technical conditions mentioned above, and also constant with empirical observations (Buss & Vilkov, 2012).

3.3 Performance Measurement

Most investors assess their portfolios' performance based only on returns, just few of them consider risk. There are several kind of performance measurement tools, which are able to evaluate portfolio performance. Sharpe and Treynor ratios combine risk and return performance in a single value, though each of them is slightly different. These performance measures are based on security market line and capital market line. (Francis J.C & Kim.D, 2013).

For the evaluation of portfolio performance, we used the following three criteria: annualized return annualized portfolio return, annualized portfolio volatility (standard deviation), portfolio Sharpe ratio.

3.3.1 Portfolio return

The actual return on a portfolio of securities over some specific period of time is the weighted average of the individual stocks in the portfolio, it can be computed using the following formula (Fabozzi & Markowitz, 2011).

$$R_p = w_1R_1 + w_2R_2 + \cdots w_GR_G \tag{3.18}$$

Where,

R_p is rate of return of the portfolio over the period of time

R_g is the rate of return of asset g over the period of time

w_g is the weight of security g in the portfolio

G is the number securities in the portfolio

Shortly the above formula can be written as following:

$$R_p = \sum_{g=1}^{G} w_g R_g \tag{3.19}$$

3.3.2 Portfolio volatility (Standard deviation)

Volatility or standard deviation is the statistical measurement of annual rate of return of investment. Higher standard deviation indicates greater risk and more volatility. Correlation is the measure of degree to which assets move in relation to each other, in our case between stocks.

In order to calculate the portfolio standard deviation, we need to use correlation and co-variance Using correlation we can calculate portfolio standard deviation, with the following formula:

$$\sigma_\rho = \sqrt{w_1^2\sigma_1^2 + w_2^2}\sigma_2^2 + 2w_1w_2\rho_{12}\sigma_1\sigma_2 \tag{3.20}$$

Where,

σ_ρ is portfolio standard deviation

w is asset weight

σ is asset volatility

$\rho_{1,2}$ is correlation between assets 1 and 2

Co-variance measures the joint variability of two random variables and in finance co-variance matrix is often used for portfolio optimization and risk management processes. We can calculate portfolio standard deviation using covariance. We can collapse the above, mentioned portfolio standard deviation formula into a much simpler from using a covariance matrix, with the following formula:

$$\sigma_{portfolio} = \sqrt{w_T \, . \, \Sigma \, . \, w} \tag{3.21}$$

Where,

$\sigma_{portfolio}$ is portfolio volatility
$\sum$ is a co-variance matrix of returns
W is portfolio weights (w_T is transposed portfolio weights)
. is the dot-multiplication operator

3.3.3 Sharpe ratio

Sharpe ratio was first developed by (Tobin, 1965) as a linear risk-return modeling technique and later extended by (Sharpe, 1966). The Sharpe ratio is the average return earned in excess of the risk-free investment rate (such as U.S. government bond) per unit of total risk or volatility.

According to Modern Portfolio Theory, adding assets to the portfolio, which are weakly correlated with each other (correlation coefficient of less than one) may decrease the risk of the portfolio without losing the return. Such kind of diversifications are made for increasing of portfolio’s Sharpe ratio. The greater the Sharpe ratio the more attractive the return with respect to risk. Sharpe ratio can be computed using the formula below:

$$S_p = \frac{\text{Excess return (or risk premium)}}{\text{Total risk}} = \frac{\overline{r_P} - r_f}{\sigma_P} \tag{3.22}$$

Where, $\overline{R_P}$ is average return on portfolio, r_f is average return of risk-free interest rate and σ_P is standard deviation of portfolio.

The average holding period return (HPR) above the risk-free interest rate is known as the risk premium or excess return. It measures the additional return, that is earned by investing in risky assets. Portfolio's risk premium is divided by the standard deviation of returns, σ, which is the measure of portfolio's total risk (Francis J.C & Kim.D, 2013).

Using Sharpe's ratio, the managers can check if a portfolio's excess mean return is sufficient in order to compensate for higher risk taken by investing in risky asset portfolio rather than the market portfolio. Using Sharpe ratio, we can rank and compare performance of investment portfolios with different risk classes, which have different average return and risk.

3.3.4 T-test

Eventually, to test our above, mentioned hypothesis, in other words, to compare the performance of portfolios constructed with historical information and forward-looking information, we conduct t-test. T-test is a statistically significant test used testing hypothesis to compare two group means and the probability differences of the samples (Fraser, 2016).

4 FINDINGS

This chapter covers the results and interpretations of results of the thesis. The first section shows the relationship between implied and historical volatilities and stock total returns. The second section presents analysis based on implied and historical information (implied and historical correlation matrixes). The third section presents evaluation of stock performance, while the fourth and fifth sections show the summary of portfolio weights and portfolio performance.

4.1 Historical and Implied Volatilities

As indicated in the charts below, for every stock, the historical volatility is flatter than implied volatility, which means that implied volatility fluctuates more and shows a similar pattern adjusted returns of the stocks.

The stock volatility impacts the value of call and put options positively. It means that the higher the volatility, the higher is the chance to make a profit for the buyer, thus the higher the value of option. This is the reason why options are more valuable during volatile times.

The full purpose of understanding the concept of option implied volatility is to get some important insights about how to trade options. Implied volatility measures the implied risk, which traders assign to the price of option. Generally, implied volatility is an indicator of how much risk traders are attaching to the stock. Higher implied volatility means that the market is becoming riskier and vice versa. Traders usually look at implied volatility to measure the market direction. Implied volatility is purely about expected volatility. Increasing implied volatility means that the volatility expectations are going up.

Historical volatility is calculated using the 3 years rolling -window method. As reported in the charts, historical volatility line seems flat for most of the stocks. For equity markets volatility is a risk measure. However, both call and put options benefit from greater volatility.

According to implied volatilities, we can say that the riskiest stocks are AKBNK, YKBNK and PETKM. On the other hand, KRDMD, PGSUS and THYAO have almost the same historical and implied volatilities. Both lines are close to each other. These stocks are fairly stable (data with respect to implied and historical volatilities as well as stock returns are presented in appendix).

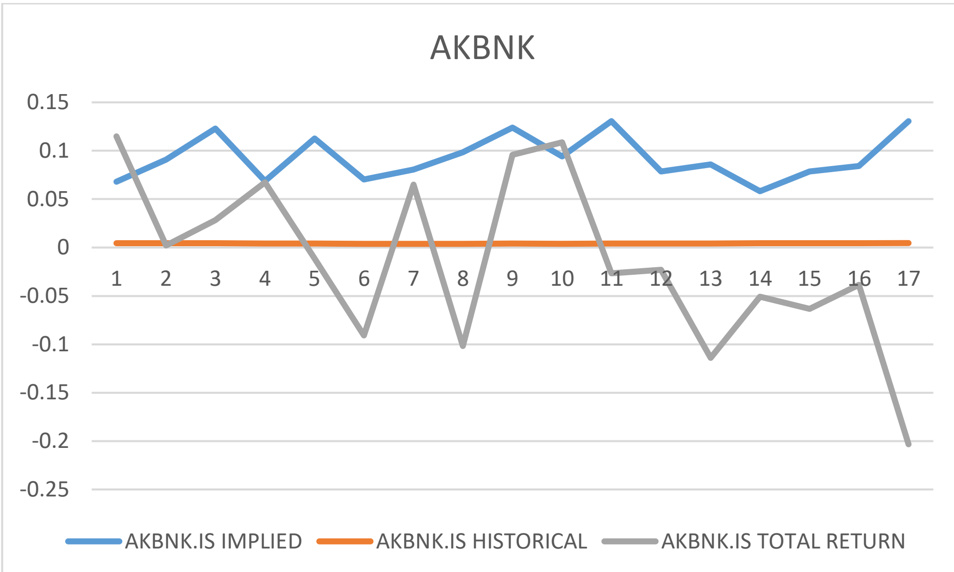

Figure 4. 1 *The relationship between implied and historical volatilities and stock total return (AKBNK).*

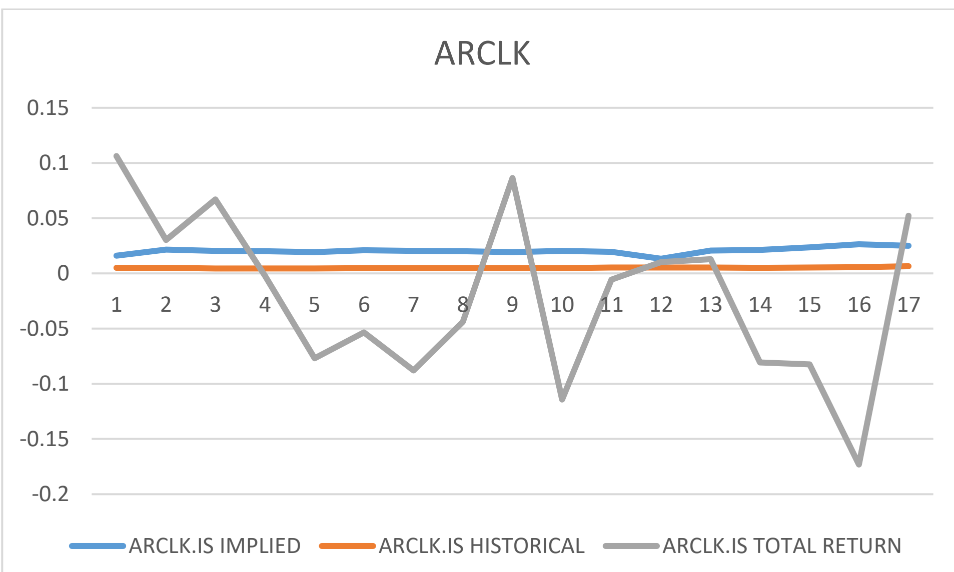

Figure 4. 2 *The relationship between implied and historical volatilities and stock total return (ARCLK).*

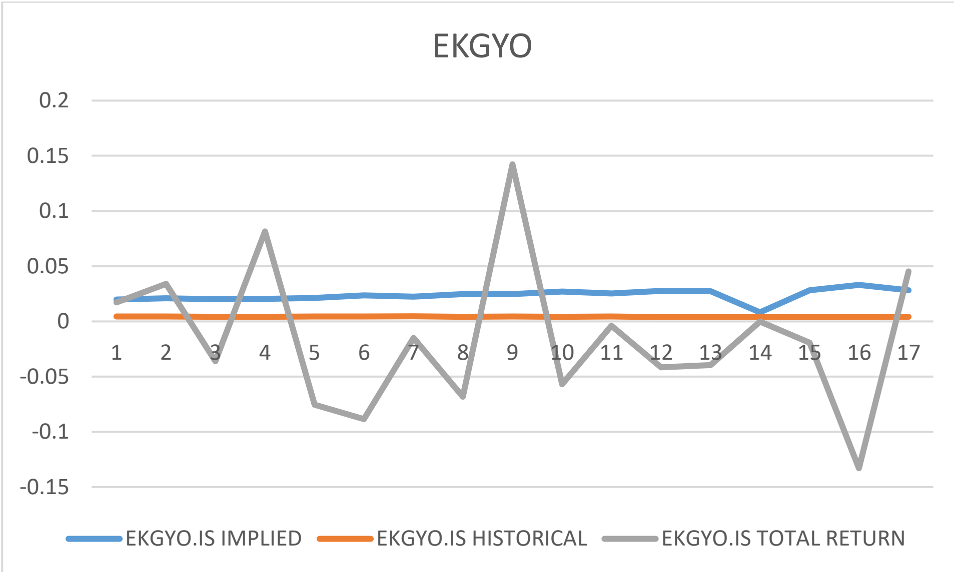

Figure 4. 3 *The relationship between implied and historical volatilities and stock total return (EKGYO).*

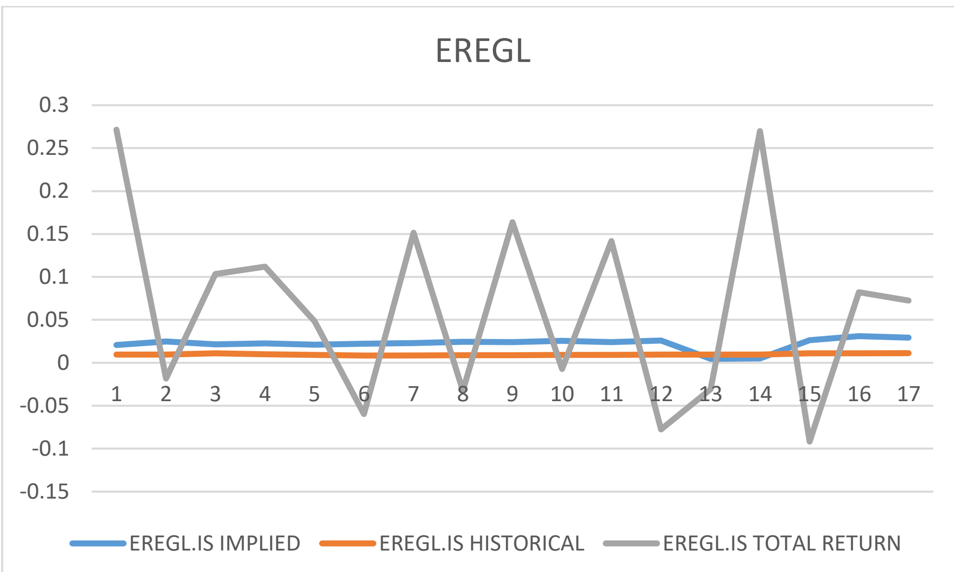

Figure 4. 4 *The relationship between implied and historical volatilities and stock total return (EREGL).*

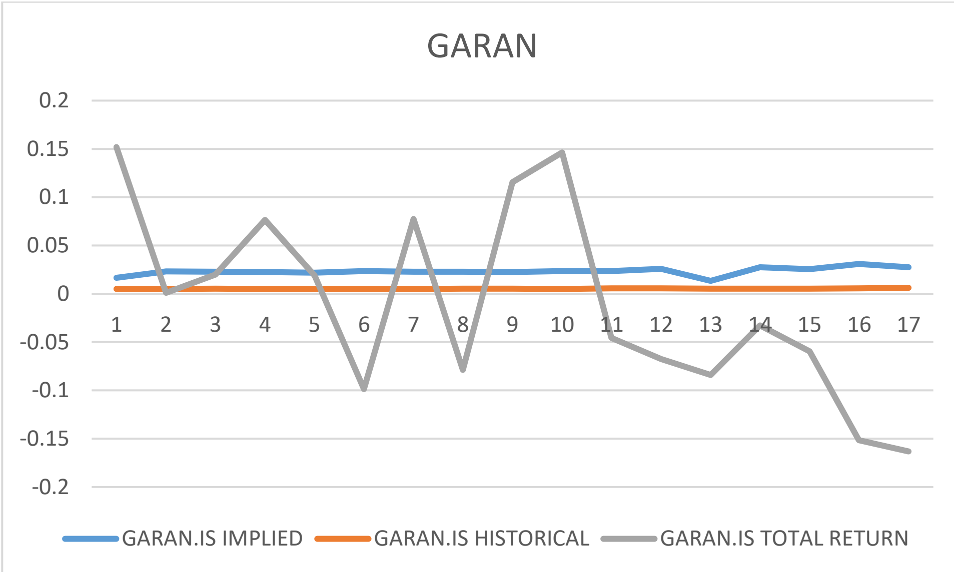

Figure 4. 5 *The relationship between implied and historical volatilities and stock total return (GARAN).*

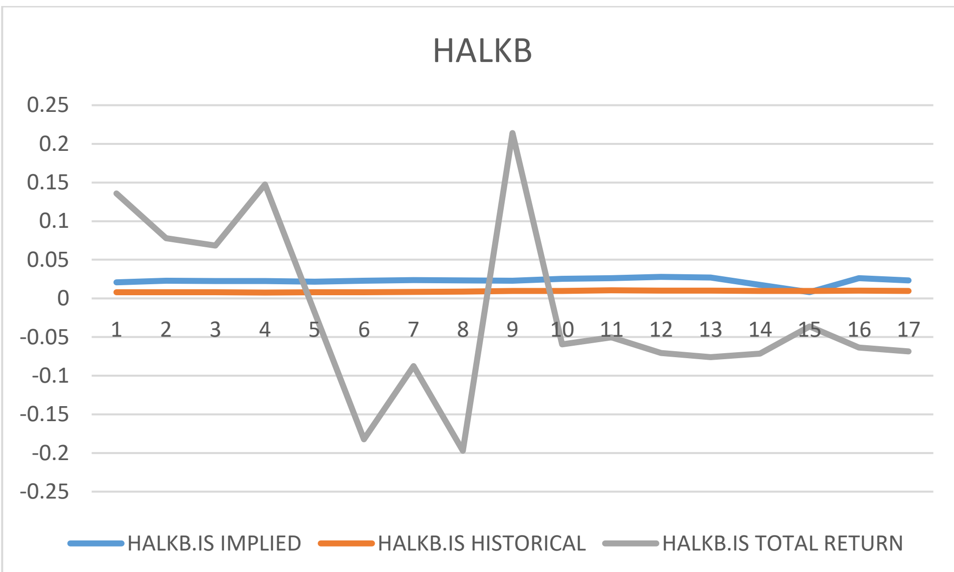

Figure 4. 6 *The relationship between implied and historical volatilities and stock total return (HALKB).*

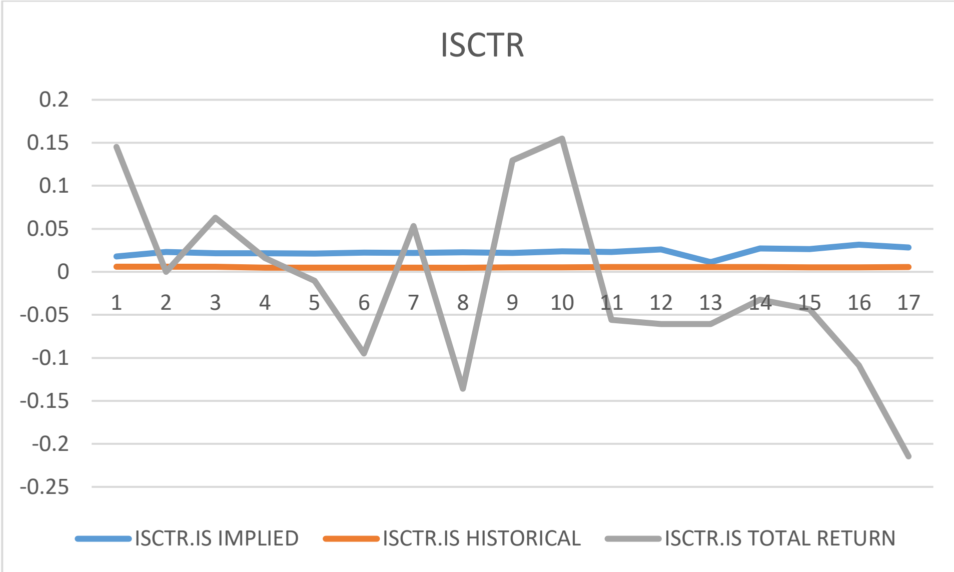

Figure 4. 7 *The relationship between implied and historical volatilities and stock total return (ISCTR).*

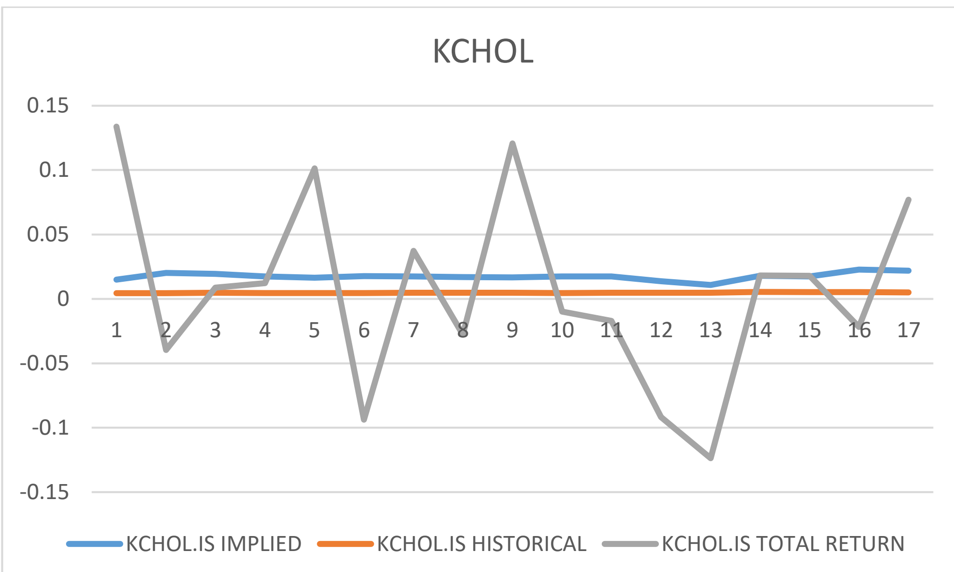

Figure 4. 8 *The relationship between implied and historical volatilities and stock total return (KCHOL).*

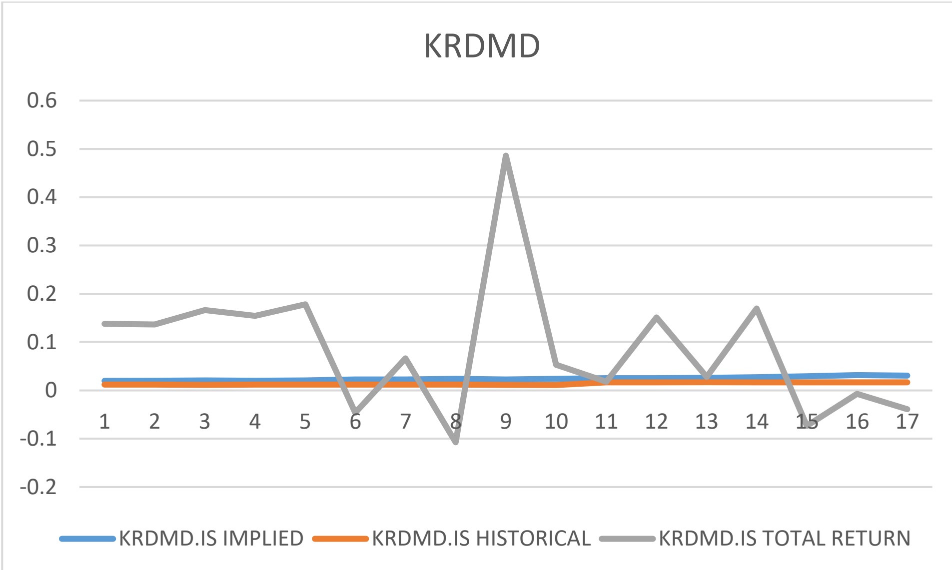

Figure 4. 9 *The relationship between implied and historical volatilities and stock total return (KRDMD).*

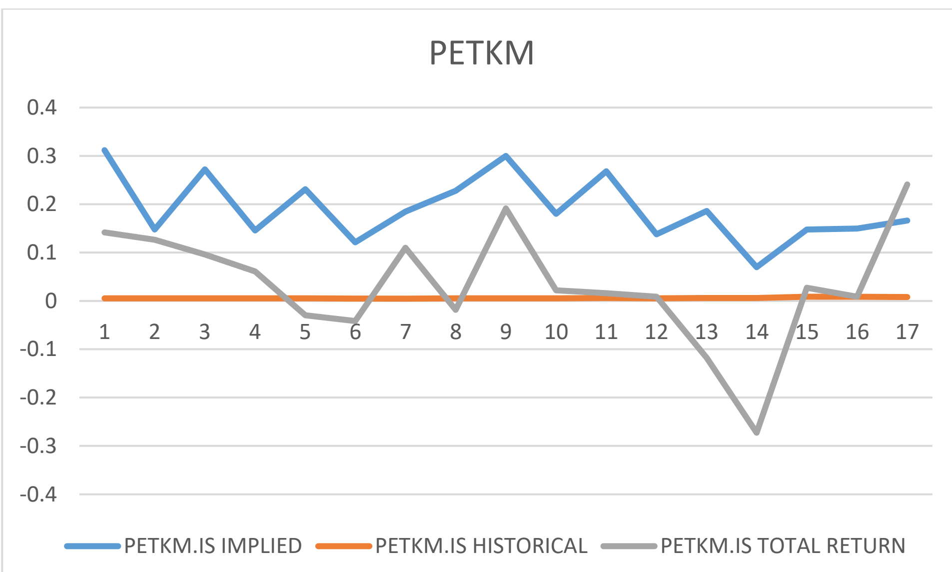

Figure 4. 10 *The relationship between implied and historical volatilities and stock total return (PETKM).*

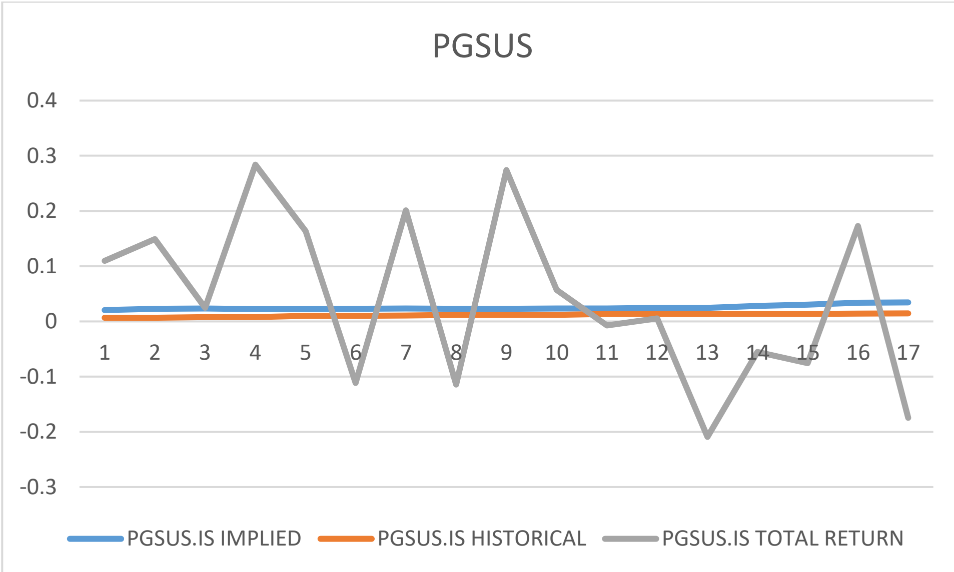

Figure 4. 11 *The relationship between implied and historical volatilities and stock total return (PGSUS).*

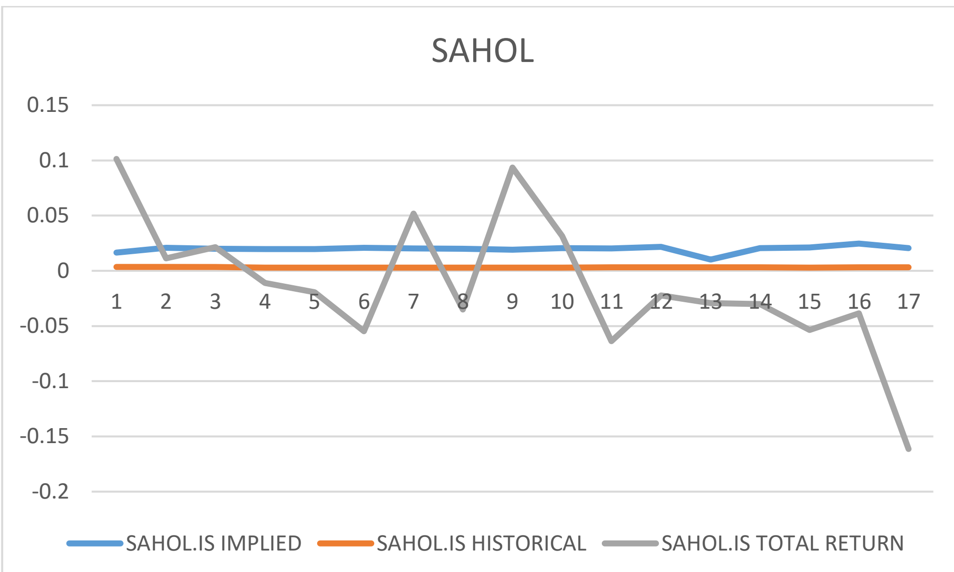

Figure 4. 12 *The relationship between implied and historical volatilities and stock total return (SAHOL).*

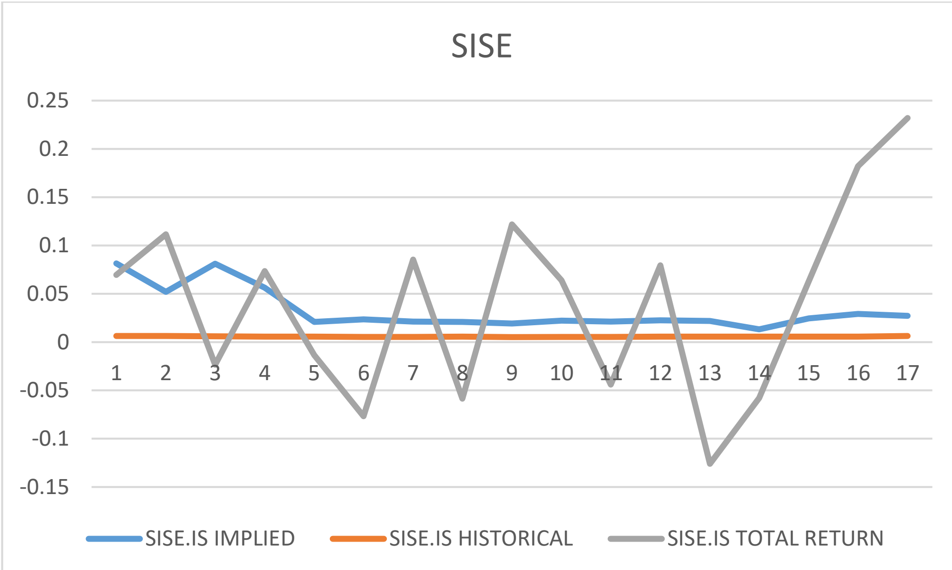

Figure 4. 13 *The relationship between implied and historical volatilities and stock total return (SISE).*

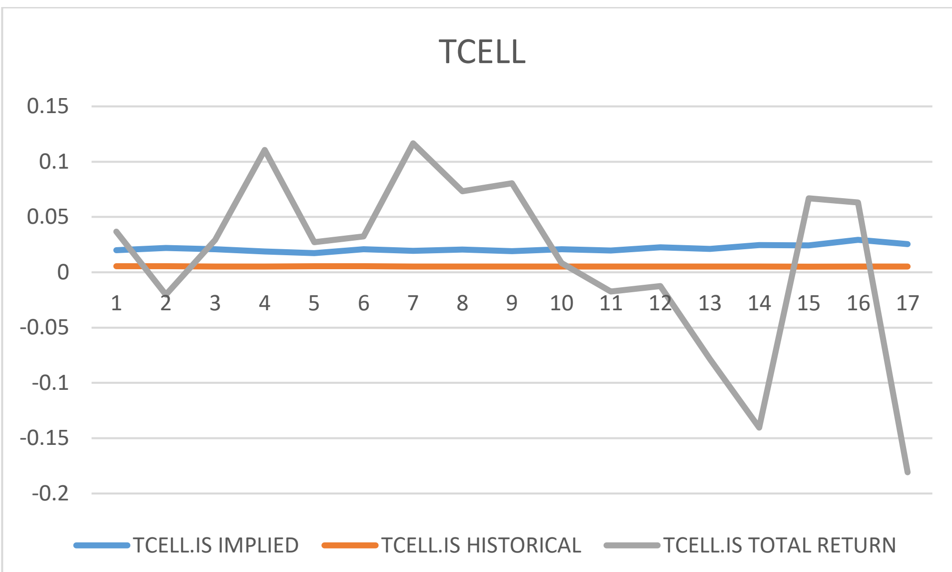

Figure 4. 14 *The relationship between implied and historical volatilities and stock total return (TCELL).*

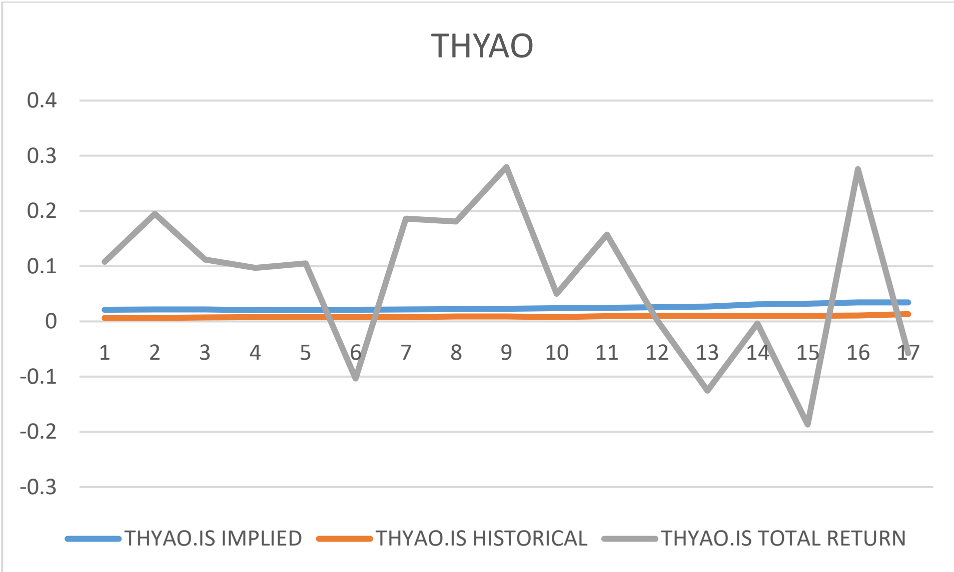

Figure 4. 15 *The relationship between implied and historical volatilities stock total return (THYAO).*

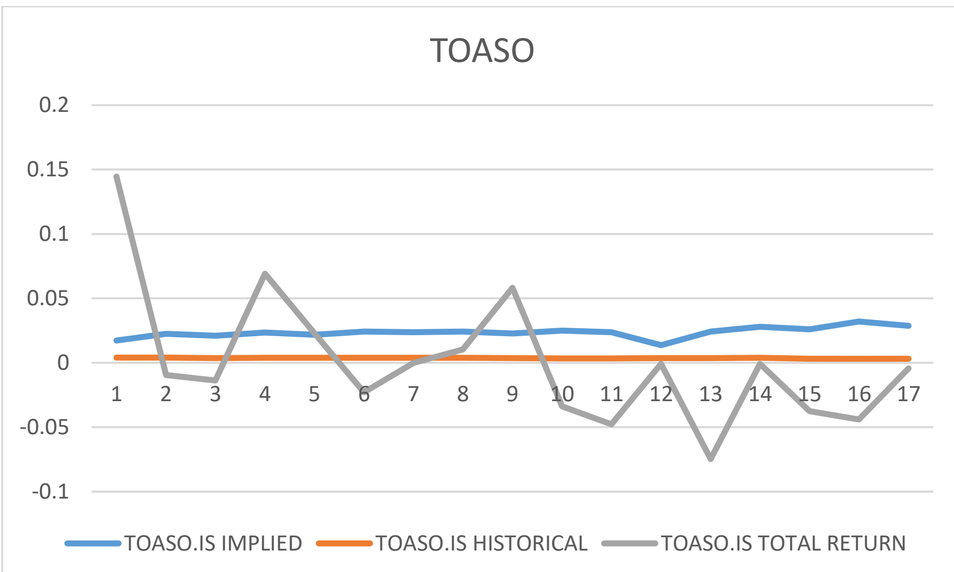

Figure 4. 16 *The relationship between implied and historical volatilities and stock total return (TOASO).*

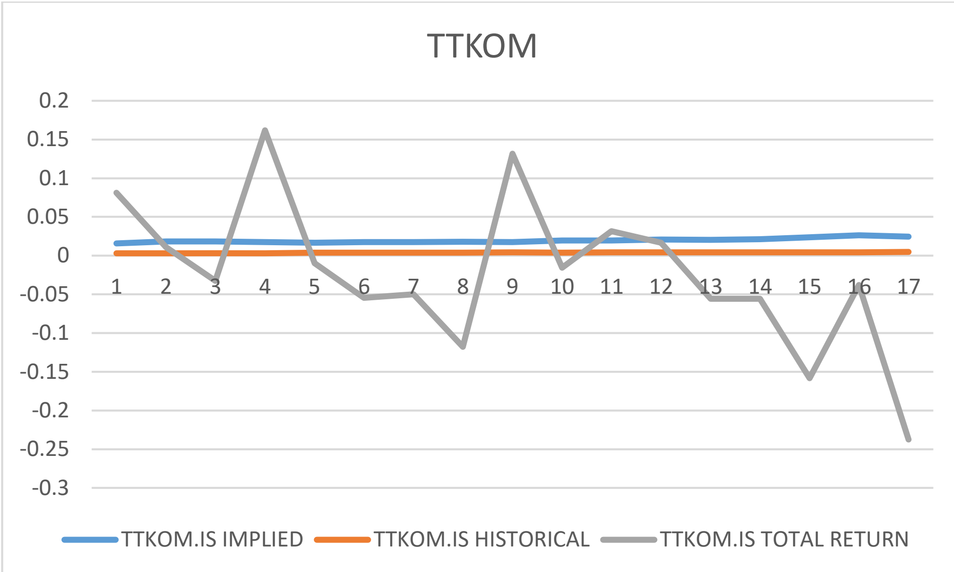

Figure 4. 17 *The relationship between implied and historical volatilities and stock total return (TTKOM).*

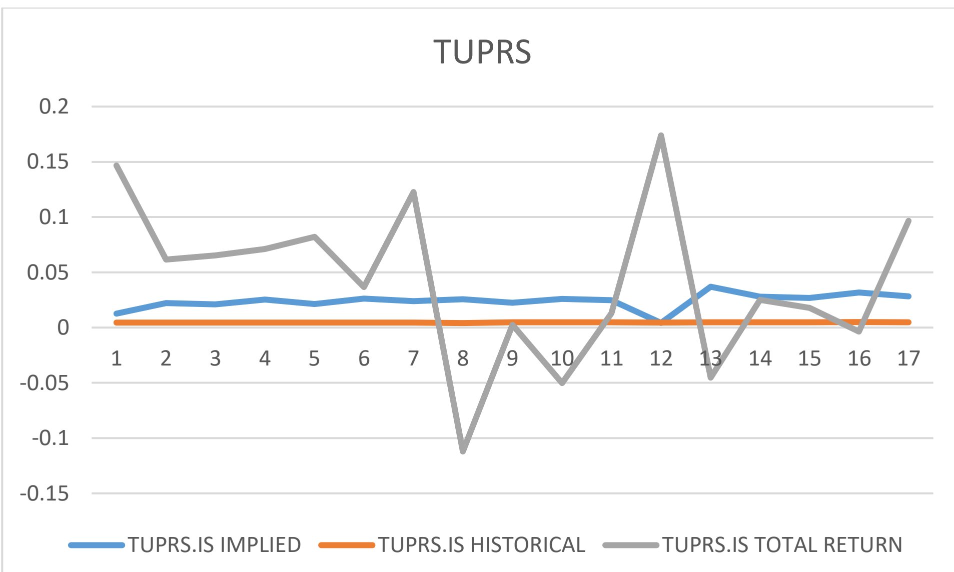

Figure 4. 18 *The relationship between implied and historical volatilities and stock total return (TUPRS).*

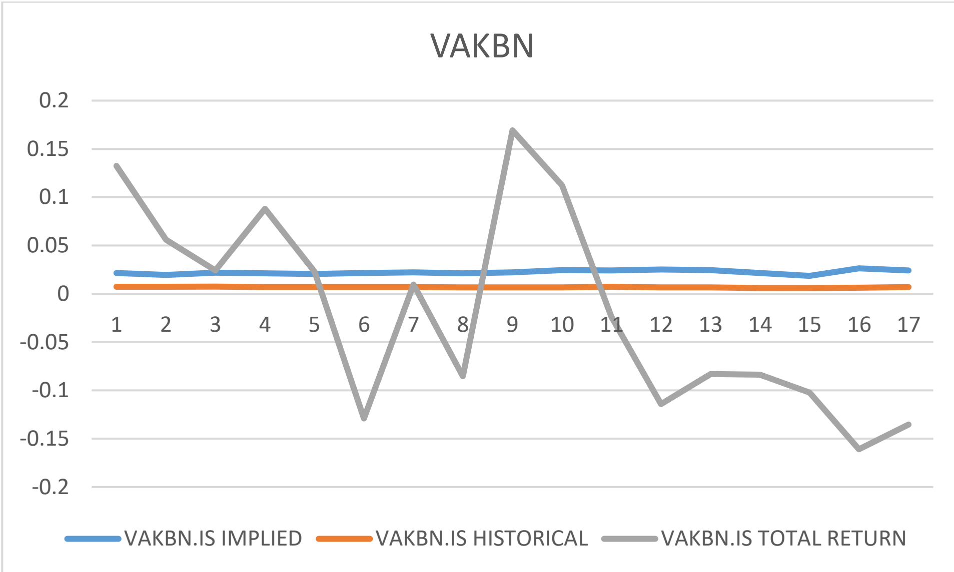

Figure 4. 19 *The relationship between implied and historical volatilities and stock total return (VAKBN).*

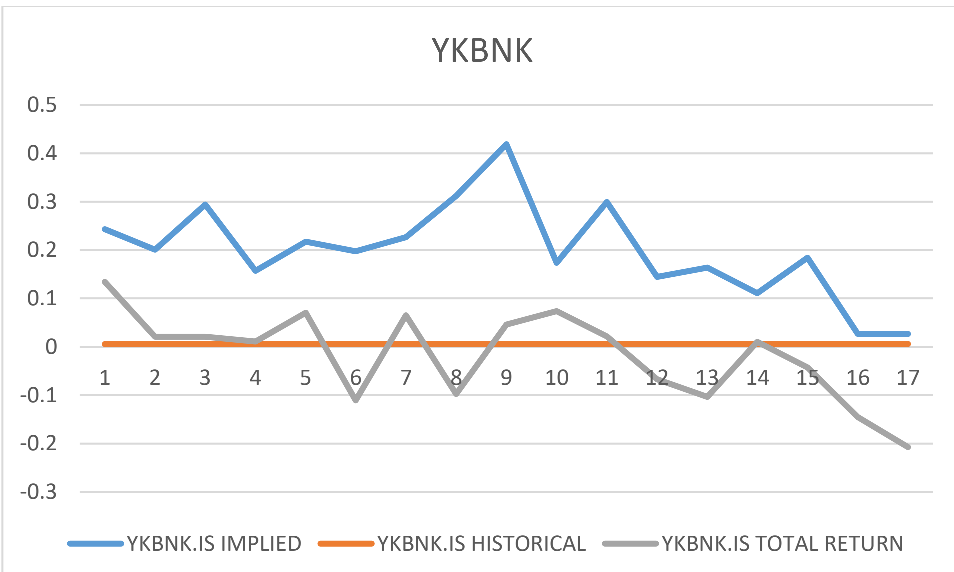

Figure 4. 20 *The relationship between implied and historical volatilities and stock total return (YKBNK).*

4.2 Historical Correlations

Correlation is a statistical measure of relationship of two variables. In our research, we have 20 stocks and we consider each stock's correlations with other stocks. The relationship of per stock with all other stocks are as the following:

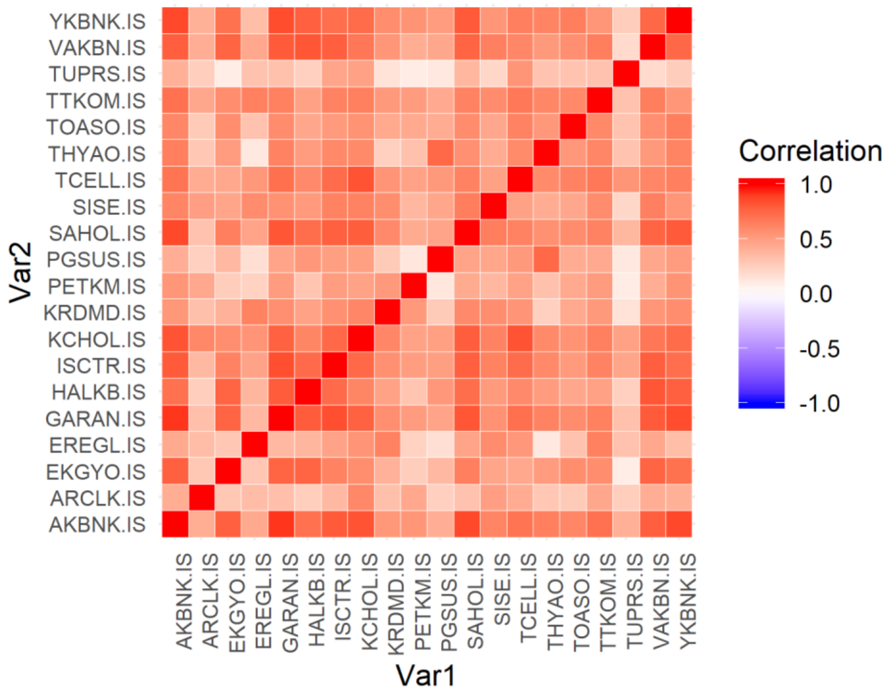

Figure 4. 21 *The implied correlation matrix, between stock adjusted returns (Heat map).*

- AKBNK.IS has the lowest positive correlation with TUPRS.IS and PGSUS.IS, (0.409294 and 0.418239), and has the highest positive correlation with YKBNK.IS and GARAN.IS (0.865758 and 0.920365).
- ARCLK.IS has the lowest positive correlation with PGSUS.IS and HALKB.IS, (0.249732 and 0.0.2572), and has the highest positive correlation with SISE.IS and KCHOL.IS (0.499434 and 0.605018).
- EKGYO.IS has the lowest positive correlation with TUPRS.IS and PETKM.IS (0.094682 and 0.261217), and has the highest correlation with VAKBN.IS and AKBNK.IS (0.765747 and 0.778438).
- EREGL.IS has the lowest positive correlation THYAO.IS and PGSUS.IS (0.122076 and 0.166356) and the highest positive correlation with KRDMD.IS and TTKOM.IS (0.636876 and 0.638668).

- GARAN.IS has the lowest correlation with TUPRS.IS and ARCLK.IS (0.326065 and 0.332568) and the highest positive correlation with YKBNK.IS and AKBNK.IS (0.850539 and 0.920365).
- HALKB.IS has the lowest correlation with TUPRS.IS and ARCLK.IS (0.24569 and 0.2572), and has the highest correlation with GARAN.IS and VAKBN.IS (0.797995 and 0.81843).
- ISCTR.IS has the lowest positive correlation with ARCLK.IS and TUPRS.IS (0.363373 and 0.46631), and has the highest correlation with AKBNK.IS and GARAN.IS (0.80061 and 0.844211).
- KCHOL.IS has lowest positive correlation with PETKM.IS and TUPRS.IS (0.482676 and 0.485753) and has the highest correlation with AKBNK.IS and TCELL.IS (0.828906 and 0.830927).
- KRDMD.IS has the lowest correlation with TUPRS.IS and THYAO.IS (0.152279 and 0.245183), and has the highest correlation with KCHOL.IS and EREGL.IS (0.612896 and 0.705496).
- PETKM.IS has the lowest correlation with TUPRS.IS and PGSUS.IS (0.101229 and 0.1254), and the highest correlation with AKBNK.IS and TUPRS.IS (0.54018 and 0.550839).
- PGSUS.IS has the lowest correlation with TUPRS.IS and PETKM.IS (0.12205 and 0.1254) and the highest correlation with HALKB.IS and THYAO.IS (0.531596 and 0.746089).
- SAHOL.IS has the lowest correlation with ARCLK.IS and TUPRS.IS (0.317981 and 0.372048), and has the highest correlation with GARAN.IS and AKBNK.IS (0.817031 and 0.865264).
- SISE.IS has the lowest correlation with TUPRS.IS and PETKM.IS (0.209242 and 0.377284), and has the highest correlation with VAKBN.IS and SAHOL.IS (0.641646 and 0.654696).
- TCELL.IS has the lowest correlation with ARCLK.IS and EKGYO.IS (0.431604 and 0.450934) and has the highest correlation with ISCTR.IS and KCHOL.IS (0.731675 and 0.830927).

- THYAO.IS has the lowest correlation with EREGL.IS and KRDMD.IS (0.122076 and 0.245183), and the highest correlation with AKBNK.IS and PGSUS.IS (0.647053 and 0.746089).
- TOASO.IS has the lowest correlation with ARCLK.IS and TUPRS.IS (0.27407 and 0.314875) and has the highest correlation with TCELL.IS and YKBNK.IS (0.628133 and 0.65352).
- TTKOM.IS has the lowest correlation with TUPRS.IS and PGSUS.IS (0.316638 and 0.439006) and has the highest correlation with TCELL.IS and AKBNK.IS (0.675538 and 0.710175).
- TUPRS.IS has the lowest positive correlation with EKGYO.IS and PETKM.IS (0.094682 and 0.101229) and has the highest correlation with KCHOL.IS and TCELL.IS (0.485753 and 0.545106).
- VAKBN.IS has the lowest correlation with TUPRS.IS and ARCLK.IS (0.193777 and 0.419738) and the highest correlation with GARAN.IS and HALKB.IS (0.800962 and 0.81843).
- YKBNK.IS has the lowest correlation with TUPRS.IS and EREGL.IS (0.260332 and 0.340138) and the highest correlation with GARAN.IS and AKBNK.IS (0.850539 and 0.865758).

4.3 Implied Correlation

As we have already mentioned implied correlation was calculated by Buss&Vilkov method, (the data of results is in the appendix). The implied correlation of per stock with all other stocks are as the following:

- AKBNK.IS has the lowest negative correlation with TUPRS.IS and PGSUS.IS (-0.18141 and -0.16352), the lowest positive correlation is with KRDMD.IS and PETKM.IS (0.069473 and 0.08036), and the highest correlation is with YKBNK.IS and GARAN.IS (0.731516 and 0.84073).

- ARCLK.IS has the lowest negative correlation with PGSUS.IS and HALKB.IS (-0.500536 and -0.485600), and the highest positive correlation with KCHOL.IS 0.210036.
- EKGYO.IS has the lowest negative correlation with TUPRS.IS and PETKM.IS (-0.810635 and -0.477567) and the lowest positive correlation with THYAO.IS and KCHOL.IS (0.027165 and 0.152399), and has the highest correlation with VAKBN.IS and AKBNK.IS (0.531495 and 0.556876).
- EREGL.IS has the lowest negative correlation with THYAO.IS and PGSUS.IS (-0.755885 and -0.667288), while TCELL.IS and KCHOL.IS have the lowest positive correlation of (0.062933 and 0.091282), and KRDMD.IS and TTKOM.IS have the highest positive correlation of (0.273752 and 0.277336).
- GARAN.IS has the lowest negative correlation with TUPRS.IS and ARCLK.IS (-0.347871 and -0.334863), the lowest positive correlation with PETKM.IS and SISE.IS (0.028579 and 0.118852) and the highest positive correlation with YKBNK.IS and AKBNK.IS (0.806464 and 0.819711).
- HALKB.IS has the lowest negative correlation with TUPRS.IS and ARCLK.IS (-0.508620 and -0.485601), while the lowest positive correlation is with THYAO.IS and SISE.IS (0.022718 and 0.038520), and the highest positive correlation is with GARAN.IS and VAKBN.IS (0.595989 and 0.636860).
- ISCTR.IS has the lowest negative correlation with ARCLK.IS and TUPRS.IS (-0.273254 and -0.067380), has the lowest positive correlation with PETKM.IS and TOASO.IS (0.007787 and 0.041978), and has the highest positive correlation with AKBNK.IS and GARAN.IS (0.601220 and 0.688421).
- KCHOL.IS has lowest negative correlation with PETKM.IS and TUPRS.IS (-0.034645 and -0.028495), has lowest positive correlation with EREGL.IS and TOASO.IS (0.091282 and 0.109032) and has the highest positive correlation with AKBNK.IS and TCELL.IS (0.0.657812 and 0.661854).
- KRDMD.IS has the lowest negative correlation with TUPRS.IS and THYAO.IS (-0.695443 and -0.509633), has the lowest positive correlation with TTKOM.IS and

PETKM.IS (0.047593 and 0.052338), and has the highest positive correlation with KCHOL.IS and EREGL.IS (0.225792 and 0.273752).

- PETKM.IS has the lowest negative correlation with TUPRS.IS and PGSUS.IS (-0.797542 and -0.749200), has the lowest positive correlation with ISCTR.IS and TTKOM.IS (0.007787 and 0.026722), and has the highest positive correlation with AKBNK.IS and YKBNK.IS (0.080360 and 0.101677).
- PGSUS.IS has the lowest negative correlation with TUPRS.IS and PETKM.IS (-0.7559 and -0.7492), has the lowest positive correlation with YKBNK.IS and TCELL.IS (0.026829 and 0.048405), and has the highest positive correlation with HALKB.IS and THYAO.IS (0.063193 and 0.492178).
- SAHOL.IS has the lowest negative correlation with ARCLK.IS and TUPRS.IS (-0.36404 and -0.2559), has the lowest positive correlation with THYAO.IS and TOASO.IS (0.133603 and 0.169289), and has the highest positive correlation with GARAN.IS and AKBNK.IS (0.634063 and 0.730528).
- SISE.IS has the lowest negative correlation with TUPRS.IS and PETKM.IS (-0.58152 and -0.24543), has the lowest positive correlation with HALKB.IS and YKBNK.IS (0.03852 and 0.083282), and has the highest positive correlation with VAKBN.IS and SAHOL.IS (0.283293 and 0.309392).
- TCELL.IS has the lowest negative correlation with ARCLK.IS and EKGYO.IS (-0.13679 and -0.09813), has the lowest positive correlation with PGSUS.IS and EREGL.IS (0.048405 and 0.062933), and has the highest positive correlation with ISCTR.IS and KCHOL.IS (0.46335 and 0.661854).
- THYAO.IS has the lowest negative correlation with EREGL.IS and KRDMD.IS (-0.75585 and -0.50963), has the lowest positive correlation with HALKB.IS and EKGYO.IS (0.022718 and 0.027165), and has the highest positive correlation with AKBNK.IS and PGSUS.IS (0.294107 and 0.492178).
- TOASO.IS has the lowest negative correlation with ARCLK.IS and TUPRS.IS (-0.45186 and -0.37025), has the lowest positive correlation with ISCTR.IS and THYAO.IS (0.041978 and 0.062764), and has the highest positive correlation with TCELL.IS and YKBNK.IS (0.256267 and 0.307041).

- TTKOM.IS has the lowest negative correlation with TUPRS.IS and PGSUS.IS (-0.36672 and -0.12199), has the lowest positive correlation with PETKM.IS and KRDMD.IS (0.026722 and 0.047593), and has the highest positive correlation with TCELL.IS and AKBNK.IS (0.351076 and 0.42035).
- TUPRS.IS has the lowest negative correlation with EKGYO.IS and PETKM.IS (-0.81064 and -0.79754) and has the highest positive correlation with TCELL.IS 0.090211.
- VAKBN.IS has the lowest negative correlation with TUPRS.IS and ARCLK.IS (-0.61245 and -0.16052), has the lowest positive correlation with THYAO.IS and KRDMD.IS (0.053247 and 0.074821), and has the highest positive correlation with GARAN.IS and HALKB.IS (0.601924 and 0.63686).
- YKBNK.IS has the lowest negative correlation with TUPRS.IS and EREGL.IS (-0.47934 and -0.31972), has the lowest positive correlation with PGSUS.IS and SISE.IS (0.026838 and 0.0832282), and the highest correlation with GARAN.IS and AKBNK.IS (0.701078 and 0.731516).

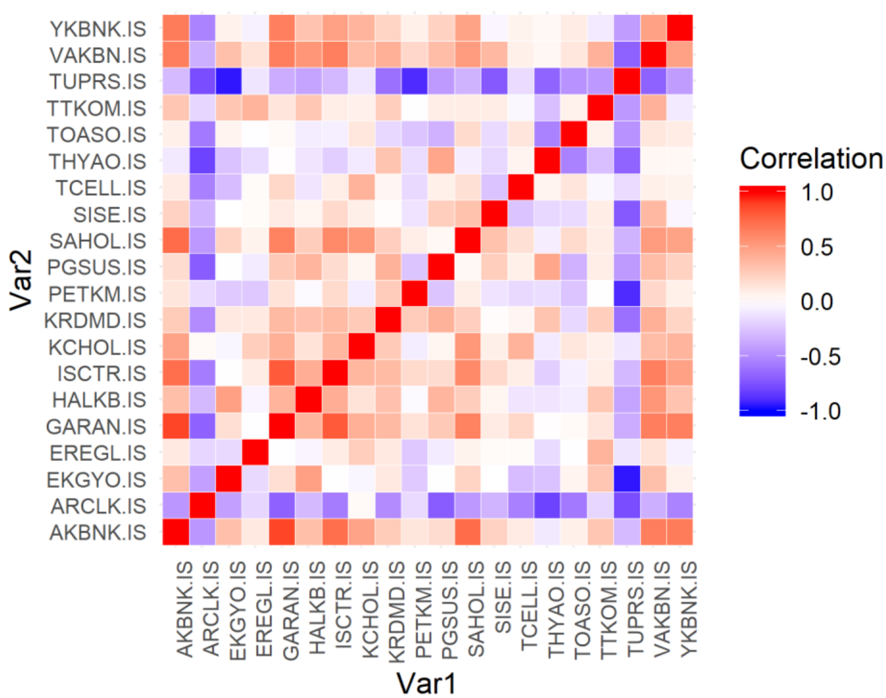

Figure 4-1 *The implied correlation matrix, between stock adjusted returns (Heat map).*

4.4 Stock Performance

We present information about annualized return, annualized standard deviation and annualized Sharpe ratio of each individual stocks. The three companies with the lowest annualized return are TTKOM with -0.276, ARCLK with -0.251, and HALKB with -0.238, and the three companies with the highest annualized return are KRDMD with 1.039, THYAO with 0.898 and EREGL with 0.776.

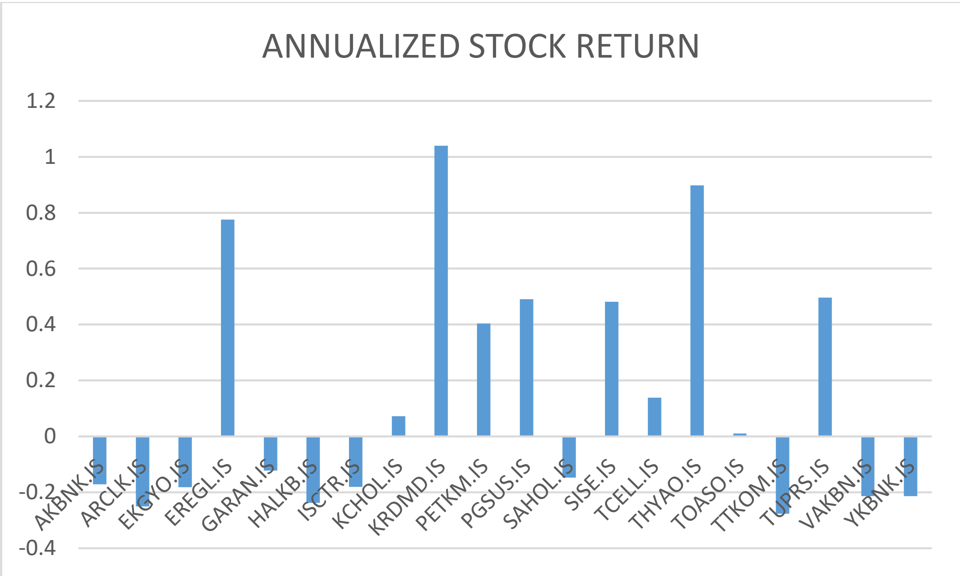

Figure 4. 22 *Annualized stock return*

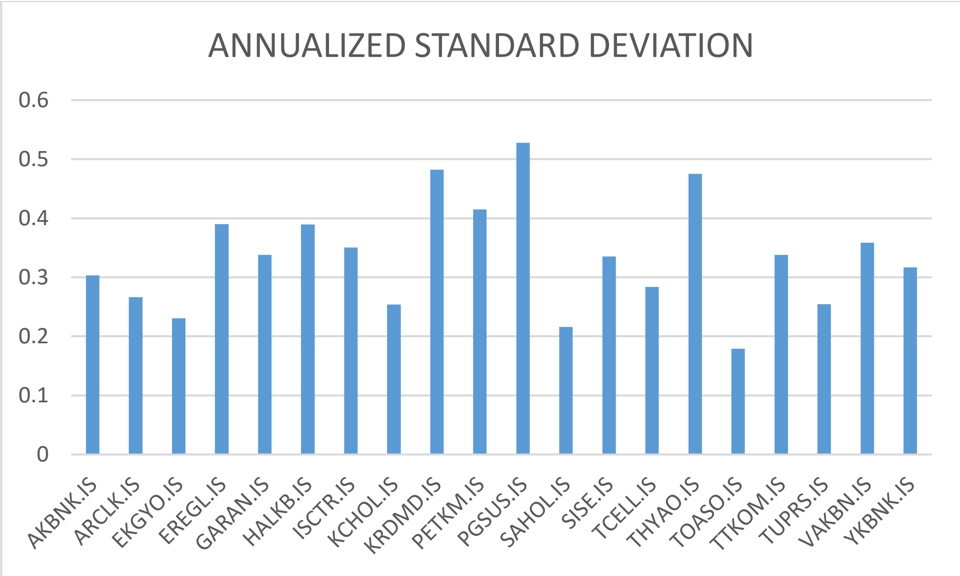

Figure 4. 23 *Annualized standard deviation*

The next performance measure is annualized standard deviation. Standard deviation is used for measuring stock volatility. The more an individual stock's returns fluctuates from its mean return, the more volatile the stock. The higher the volatility, the riskier the stock, and vice versa.

In our sample, stocks with the highest annualized standard deviation are the following three companies: PGSUS (Pegasus), KRDMD (Kardemir) and THYAO (Turkish Airlines), which are respectively 0.528, 0.482 and 0.475.

The stocks with the lowest annualized standard deviation are TOASO (Tofas), SAHOL (Sabanci Holding) and EKGYO (Emlak Konut) which are respectively 0.179, 0.216 and 0.230.

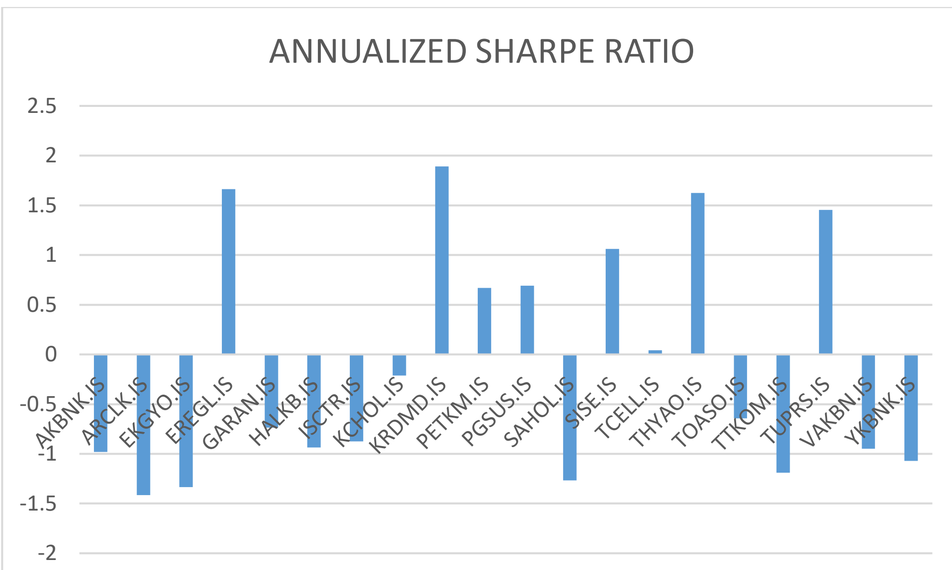

Figure 4. 24 *Annualized Sharpe ratio*

The last measure of stock performance is the Sharpe ratio. Simply, the Sharpe ratio is a return per unit of risk, which is characterized by variance. The higher the Sharpe ratio, the better is the combination of return and risk.

In our sample, the following 3 stocks have the highest Sharpe ratios: KRDMD (Kardemir), EREGL ((Eregli) and THYAO (Turkish Airlines) which are respectively 1.892, 1.664 and 1.623. The stocks with the lowest Sharpe ratio are: ARCLK (Arcelik), EKGYO (Emlak Konut) and SAHOL (Sabanci Holding), which are respectively -1.416, -1.336 and -1.266.

4.5 Optimized Portfolio Weights

The technique of portfolio analysis does not indicate the amount of money to be invested in each security. Preferably, the proportion of each security the optimum portfolio should presume. The proportions or weights is symbolized as $w_i s$. Thus, w_i is the of portfolio's total value which should be invested in *i* security.

Assuming the all funds in the portfolio are accounted for, the following constraint is placed for all portfolios:

$$\sum_{i=1}^{n} w_i = 1$$

In other words, the total sum of investment in portfolio is 100%.

The graphs below show weights of portfolios developed by different methods. We can see that weights obtained by using implied volatility show much greater variation of weights throughout the time. Especially, the minimum variance implied weights changes throughout the time, which indicates that it is dynamically adjusting the weights based on how the market perceives future volatility. The flat graphs of portfolios based on historical information reflect the slight changes in the mean returns and the sample covariance matrix from month to month, thus the sample weights are the nearly the same. When option-implied information is used in portfolio optimization, implied volatility changes more from one month to another and as a result the weights of optimal portfolios fluctuate in time.

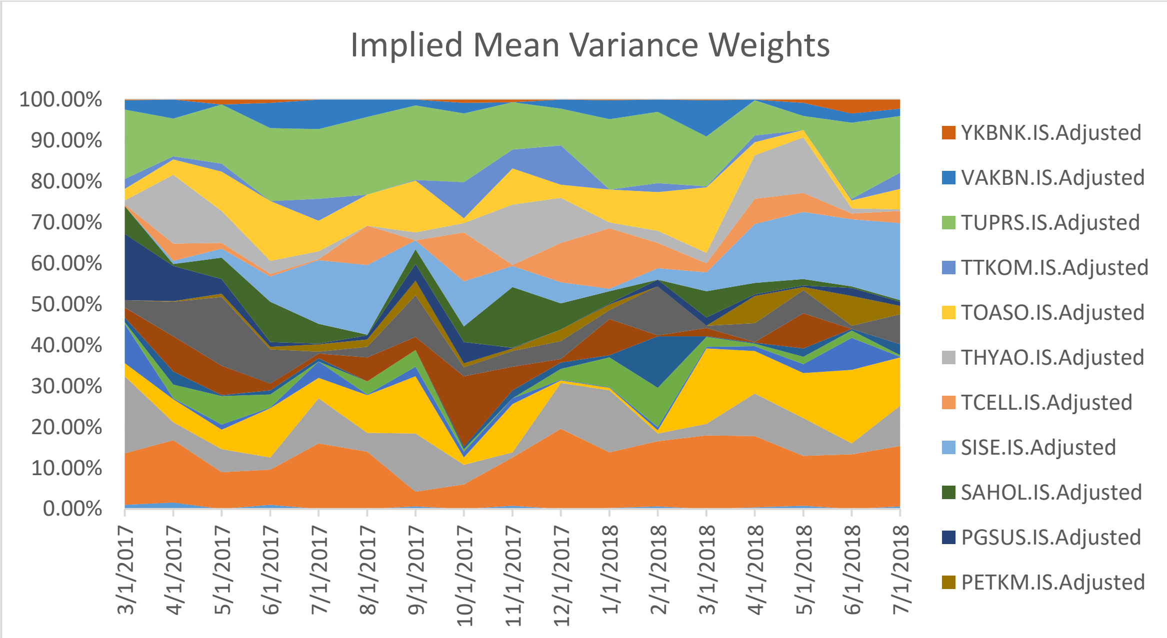

Figure 4. 25 *Implied mean variance weights allocation*

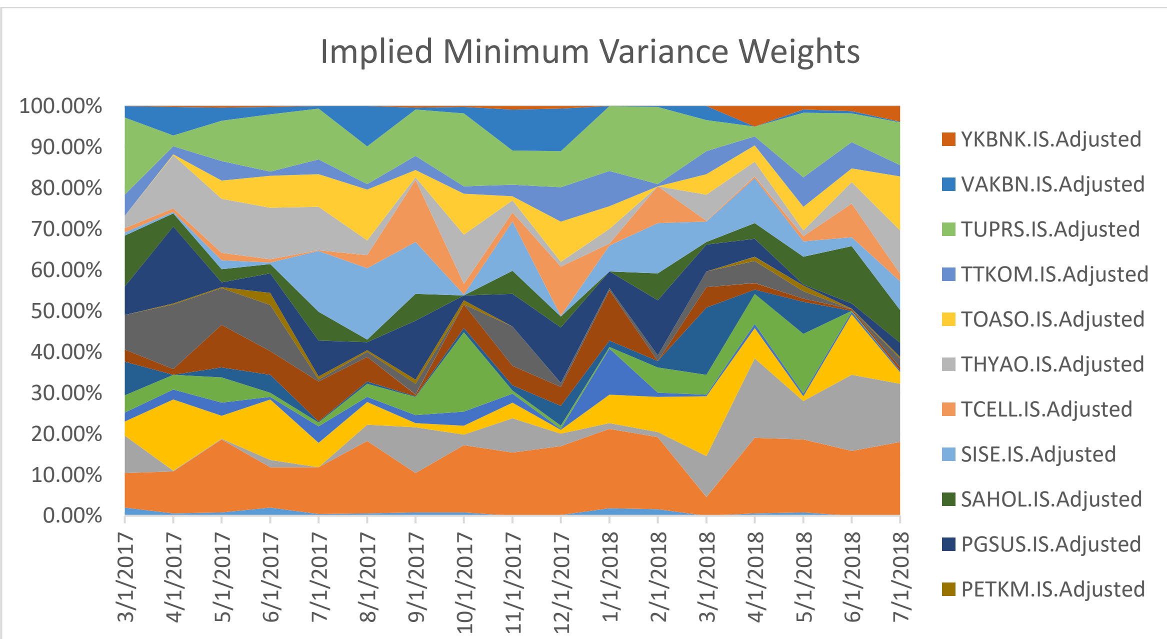

Figure 4. 26 *Implied minimum variance weights allocation*

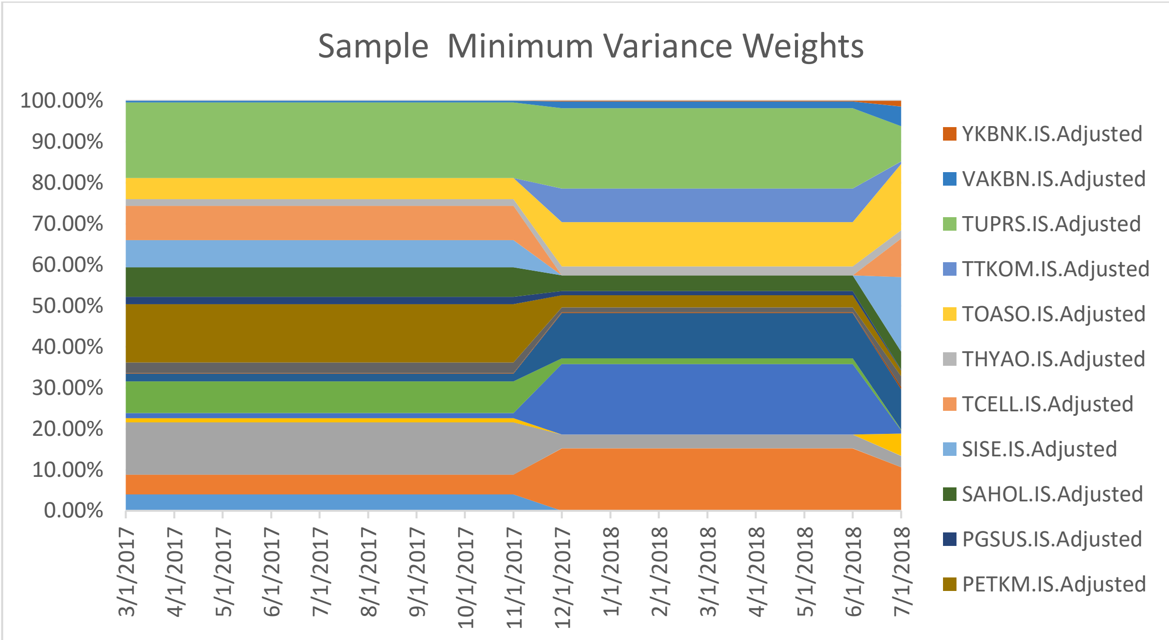

Figure 4. 27 *Sample (historical) minimum variance weights allocation*

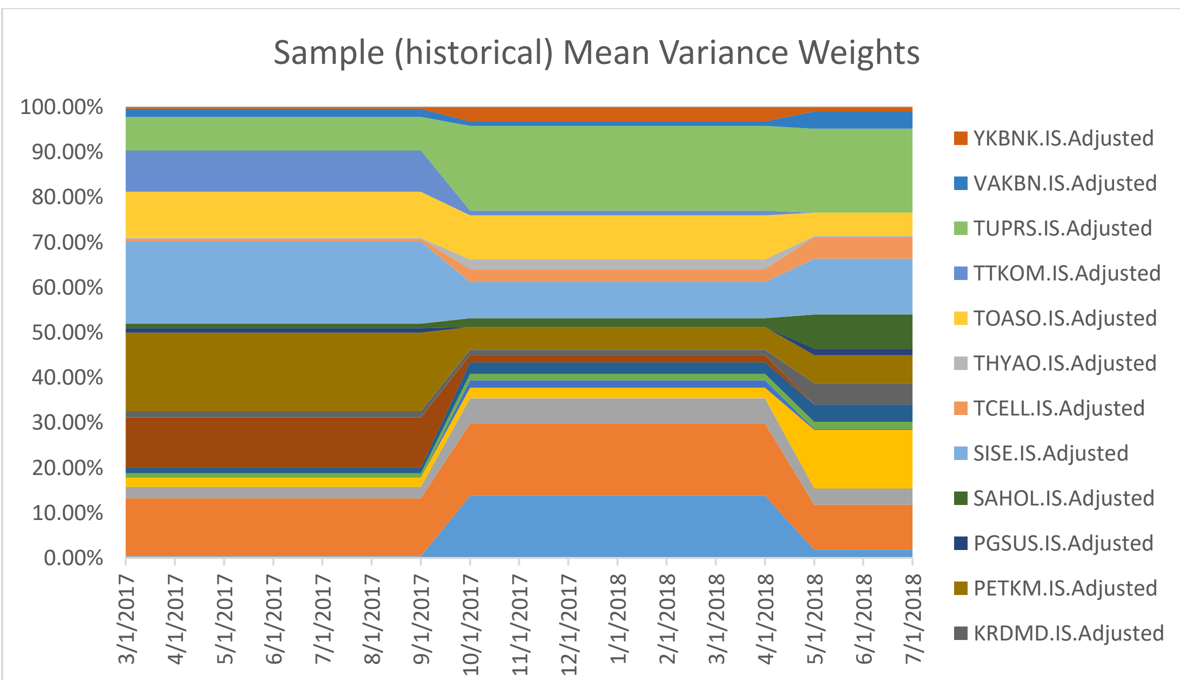

Figure 4. 28 *Sample (historical) mean variance weights allocation*

4.6 Portfolio Performances

This chart shows the cumulative returns of 5 different portfolios:

- Benchmark portfolio,
- Minimum Variance Sample (historical) portfolio,
- Minimum Variance Implied portfolio,
- Mean Variance Sample (historical) portfolio,
- Mean Variance Implied portfolio.

We can see that Minimum Variance Implied and Mean Variance Implied portfolios provide the highest level of cumulative return, especially Minimum Variance Implied, which has the highest return among the five portfolios during the whole sample period. The increase in the cumulative returns is noticeable beginning from November 2017, from about 0.42% to 0.60%. From April 2017 to November 2017, the benchmark portfolio had the least variance. From November 2017 to the end of sample period, the minimum variance sample (historical) portfolio had the least variance.

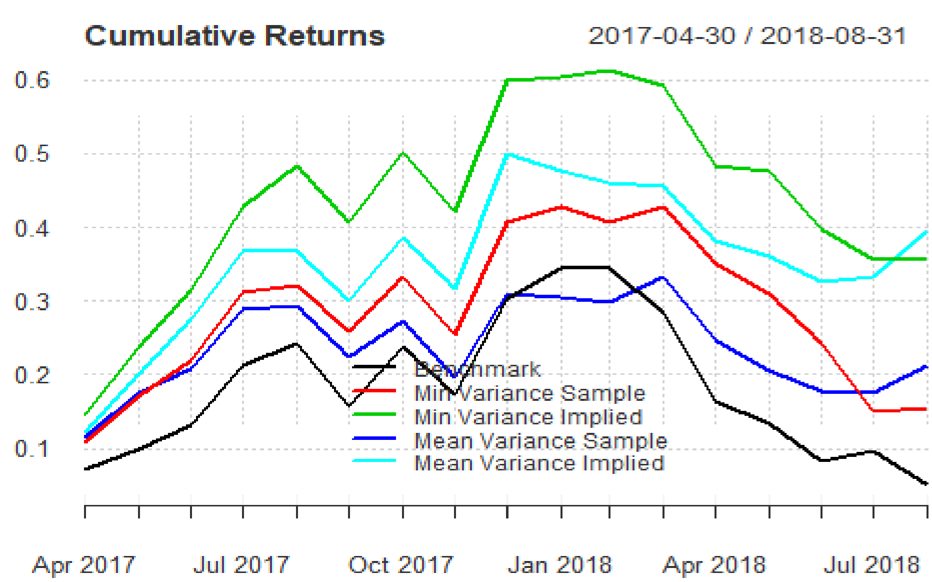

Figure 4. 29 *Cumulative returns of 5 different portfolios (benchmark, minimum variance sample (historic), minimum variance implied, mean variance sample (historic) and mean variance implied)*

In the Table 4.1 below, we make the comparison of performance of different portfolios. We can see that the Minimum Variance Implied portfolio has the highest annualized standard deviation (0.2272) and Mean Variance Implied portfolio has the highest annualized return and annualized Sharpe ratio (0.2657 and 0.5717).

Table 4. 1 *Performance of portfolios (benchmark, minimum variance sample (historical), minimum variance implied, mean variance sample (historical) and mean variance implied)*

	Bench mark	Minimum Variance Sample	Minimum Variance Implied	Mean Variance Sample	Mean Variance Implied
Annualized Return	0.0347	0.1068	0.2411	0.1452	0.2657
Annualized Std Dev	0.1994	0.2082	0.2272	0.1807	0.2051
Annualized Sharpe (Risk-free ratef=12.64%)	-0.4444	-0.1175	0.4192	0.0545	0.5717

Portfolios constructed using option implied information have higher annualized returns and annualized Sharpe ratios compared to portfolios constructed using historical information. In fact, Mean Variance Implied portfolio has the highest annualized return and annualized Sharpe ratio. Mean Variance Implied outperforms the Mean Variance Sample (historical) and benchmark portfolio. Implied Minimum Variance portfolio outperforms Minimum Variance Sample (historical) portfolio and benchmark portfolio.

The Table 4.2 below shows comparison of two portfolios: Implied mean variance and sample (historical) mean variance. This comparison illustrates our thesis hypothesis, compares two type of portfolios, portfolio based on the historical information and portfolio based on the implied information, namely mean variance sample (historical) and mean variance implied. The t-test results show that the mean returns of these two portfolios are different (statistically significant at 5% level) and our H_0 hypothesis (optimal portfolios based on option implied information do not perform better than optimal portfolios based on historical information) can be rejected, which means that the portfolio based on implied information performs better rather than the portfolio based on historical information.

Table 4. 2 *T-test of portfolio performance (Mean Variance Implied versus Mean Variance Sample)*

	Mean Variance Implied	*Mean Variance Sample*
Mean	0.021419367	0.012621574
Variance	0.003505145	0.002722029
Observations	17	17
Pearson Correlation	0.959318809	
Hypothesized Mean Difference	0	
df	16	
t Stat	2.09166207	
P(T<=t) one-tail	0.026386877	
t Critical one-tail	1.745883676	
P(T<=t) two-tail	0.052773753	
t Critical two-tail	2.119905299	

In the Table 4.3 below, we compare two portfolios: Minimum Variance Implied and Minimum Variance Sample (historical) portfolio. The t-test results show that the mean returns of these two portfolios are different (statistically significant at 5% level) and our H_0 hypothesis (optimal portfolios based on option implied information do not perform better than optimal portfolios based on historical information), can be rejected, which means that the portfolio based on implied information performs better than the portfolio based on historical information.

Table 4. 3 *T-test of portfolio performance (Minimum Variance Implied versus Minimum Variance Sample)*

	Min Variance Implied	*Min Variance Sample*
Mean	0.020118846	0.010153347
Variance	0.004300208	0.003613047
Observations	17	17
Pearson Correlation	0.953678957	
Hypothesized Mean Difference	0	
df	16	
t Stat	2.067251785	
P(T<=t) one-tail	0.02764188	
t Critical one-tail	1.745883676	
P(T<=t) two-tail	0.055283761	
t Critical two-tail	2.119905299	

In the Table 4.4, we compare the portfolios based on Mean Variance Implied and Benchmark portfolios. The t-test results show that the mean returns of these two portfolios are different (statistically significant at 5% level) and our H_0 hypothesis (optimal portfolios based on option implied information do not perform better than optimal portfolios based on historical information) can be rejected, which means that the Mean Variance Implied portfolio based on implied information performs better than the Benchmark portfolio.

Table 4.4 *T-test of portfolio performance (Mean Variance Implied versus Benchmark)*

	Mean Variance Implied	*Benchmark*
Mean	0.016820691	0.004398389
Variance	0.003302536	0.003314774
Observations	17	17
Pearson Correlation	0.927401208	
Hypothesized Mean Difference	0	
df	16	
t Stat	2.336772122	
P(T<=t) one-tail	0.016390914	
t Critical one-tail	1.745883676	
P(T<=t) two-tail	0.032781829	
t Critical two-tail	2.119905299	

In the Table 4.5, we compare two portfolios: Minimum Variance Implied and Benchmark portfolios. The t-test results show that the mean returns of these two portfolios are not different and our H_0 hypothesis (Optimal portfolios based on option implied information do not perform better than optimal portfolios based on historical information) cannot be rejected, which means that the Minimum Variance Implied portfolio based on implied information does not performs better than the Benchmark portfolio.

Table 4.5 *T-test of portfolio performance (Minimum Variance Implied versus Benchmark)*

	Min Variance Implied	*Benchmark*
Mean	0.007487883	0.004398389
Variance	0.002928062	0.003314774
Observations	17	17
Pearson Correlation	0.901092936	
Hypothesized Mean Difference	0	
df	16	
t Stat	0.508207171	
P(T<=t) one-tail	0.309121411	
t Critical one-tail	1.745883676	
P(T<=t) two-tail	0.618242822	
t Critical two-tail	2.119905299	

4.7 Comparison of the Results with Other Similar Research Results

As we have already mentioned in literature review, there is a limited number of similar research papers. Some frameworks used in our study are similar to the studies of other researchers. The following studies and results are similar to our studies:

One study by Kempt, Korn and Sassning (2012), developed the first estimator of the covariance matrix which is totally forward-looking, using information just from options. They tested the quality of the new estimator by examining the out-of-sample performance of a global minimum variance (GMVP) strategy and compared its performance with benchmark strategies based on historical information. According to their results, the implied estimator outperforms the historical estimator.

The aim of DeMiguel, Plyakha, Uppal and Vilkov (2013) in their study is to examine if option-implied information can be used in order to improve the selection of mean-variance portfolios with a large number of assets. They measure the performance of portfolio using

Sharpe ratio, volatility and turnover. Their empirical evidence shows that option-implied volatility helps to decrease volatility of the portfolio. Using the option-implied volatility, skewness and risk premium in order to adjust returns leads to a significant improvement in Sharpe ratio.

Vial (2013) in his master thesis analyzes the option-implied information in the portfolio optimization framework. They analyze different portfolio allocation strategies and study if option implied portfolios outperform historical portfolios. Their results show that options add forecasting power to the portfolio optimization problem. Although option-implied portfolios outperform the historical ones, differences are mostly insignificant. In specific situations, option-implied information is a rational alternative to the historical moment estimators.

Our findings are in line with the findings of Kempt, Korn (2012) and Sassning DeMiguel, Plyakha, Uppal and Vilkov (2013) and Vial (2013). As the findings imply, forward-looking information derived from option prices are superior indicators of future stock returns and it can be utilized to make better investment decisions.

5 CONCLUSION

Portfolio optimization is one of the most important topics in the field of investment management. Various portfolio optimization methodologies and techniques have been developed by researchers and practitioners to examine and determine optimal portfolios. According to many studies, option-implied volatility is a strong forecaster of future volatility in the equity market. According to other studies, option implied information is a strong forecaster of future volatility, that was also improved by our study and results. Thus, option implied information is used in several studies for portfolio optimization purposes.

The aim of our thesis was to construct different type of portfolios using different methods (based on historical and implied information); compare their performances and most importantly determine whether portfolios based on implied information outperform the portfolios constructed based on the historical information in the Turkish Stock Market. For this aim, we decided to compare 5 portfolios: the benchmark portfolio (BIST30 index), minimum variance sample (historical) portfolio, which was built using historical information, minimum variance implied portfolio, which was built using implied information, mean variance sample (historical) based on historical information, and mean variance implied portfolio based on implied information.

We used option prices of 20 stocks, which have been trading in the Futures and Options Market of Borsa İstanbul. The sample period is from March 2017 to July 2018. We developed portfolio optimization models using option prices as well as mean variance and minimum variance portfolios using historical stock price data.

We calculated implied volatility of the sample stocks from option prices using Black-Scholes option pricing model. We employed Buss and Vilkov's model for the calculation of implied correlations between stocks. Option implied mean variance and option implied minimum variance portfolios are based on the covariance metrics developed after historical information is replaced by option-implied information. After obtaining the optimal weights of portfolios, we made monthly rebalancing and we measured the performances of portfolios using annualized return, annualized standard deviation and annualized Sharpe ratio.

According to our results, portfolios based on implied information outperform benchmark and sample (historical) portfolios. Portfolios constructed using option implied information have higher annualized returns and annualized Sharpe ratios compared to portfolios constructed using historical information. Most importantly, the minimum variance implied portfolio appears to show the best performance among other portfolios with respect to annualized return, standard deviation and Sharpe ratio.

Our study is limited with the sample period and the assumptions of the models employed in the methodology. Our study could be extended with a longer sample size and historical volatilities can be computed with shorter rolling window methods.

Our findings may enable local and foreign investors, future investors, researchers and policy makers to utilize forward –looking information implied in option prices in order to improve the out-of-sample performance of their portfolios. Our findings confirm that portfolios based on implied-information overperform portfolios based on historical information.

Our recommendation to future researchers who will work on the same topic could be to model the relations between implied volatility and stock returns and use implied volatility to forecast future stock returns.

To our knowledge this is one of the first studies in Turkey, which utilizes, stock option data for portfolio optimization purposes.

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APPENDIX

Historical Covariance

	AKBNK.IS	ARCLK.IS	EKGYO.IS	EREGL.IS	GARAN.IS	HALKB.IS	ISCTR.IS	KCHOL.IS	KRDMD.IS	PETKM.IS	PGSUS.IS	SAHOL.IS	SISE.IS	TCELL.IS	THYAO.IS	TOASO.IS	TTKOM.IS	TUPRS.IS	VAKBN.IS	YKBNK.IS
AKBNK.IS	0.004343	0.001989	0.003468	0.002909	0.004304	0.004176	0.004016	0.003657	0.003838	0.002576	0.00225	0.003379	0.003222	0.003352	0.003363	0.002587	0.002611	0.001811	0.004458	0.004344
ARCLK.IS	0.001989	0.004997	0.001386	0.002401	0.001668	0.001618	0.001955	0.002863	0.002547	0.002315	0.001441	0.001332	0.002783	0.00224	0.001632	0.001235	0.001804	0.001224	0.002558	0.002196
EKGYO.IS	0.003468	0.001386	0.004571	0.001955	0.003672	0.004602	0.003234	0.002608	0.002966	0.001278	0.002022	0.002601	0.002481	0.002239	0.002738	0.002492	0.002199	0.00043	0.004463	0.003642
EREGL.IS	0.002909	0.002401	0.001955	0.009774	0.002597	0.003324	0.00357	0.003611	0.006858	0.001684	0.001343	0.002757	0.004568	0.003858	0.000952	0.002006	0.003522	0.002111	0.003859	0.002561
GARAN.IS	0.004304	0.001668	0.003672	0.002597	0.005036	0.005038	0.00456	0.003678	0.004443	0.002641	0.002726	0.003436	0.00313	0.003717	0.003538	0.002651	0.002534	0.001554	0.0049	0.004596
HALKB.IS	0.004176	0.001618	0.004602	0.003324	0.005038	0.007916	0.004985	0.003682	0.004663	0.001963	0.003862	0.003824	0.003642	0.003889	0.003588	0.002581	0.002403	0.001468	0.006277	0.005301
ISCTR.IS	0.004016	0.001955	0.003234	0.00357	0.00456	0.004985	0.005793	0.003792	0.004666	0.002775	0.003087	0.003525	0.00381	0.00409	0.003621	0.002528	0.002682	0.002383	0.005147	0.004194
KCHOL.IS	0.003657	0.002863	0.002608	0.003611	0.003678	0.003682	0.003792	0.004481	0.004469	0.002338	0.002691	0.003137	0.003337	0.004085	0.003164	0.002367	0.002404	0.002184	0.003946	0.003739
KRDMD.IS	0.003838	0.002547	0.002966	0.006858	0.004443	0.004663	0.004666	0.004469	0.011865	0.004147	0.002431	0.003842	0.004954	0.00437	0.002106	0.00311	0.003183	0.001114	0.005046	0.004818
PETKM.IS	0.002576	0.002315	0.001278	0.001684	0.002641	0.001963	0.002775	0.002338	0.004147	0.005236	0.000741	0.001879	0.002152	0.002551	0.001847	0.002042	0.002072	0.000492	0.00267	0.003035
PGSUS.IS	0.00225	0.001441	0.002022	0.001343	0.002726	0.003862	0.003087	0.002691	0.002431	0.000741	0.006666	0.002279	0.002973	0.003143	0.004804	0.002231	0.001999	0.000669	0.003194	0.003192
SAHOL.IS	0.003379	0.001332	0.002601	0.002757	0.003436	0.003824	0.003525	0.003137	0.003842	0.001879	0.002279	0.003511	0.003058	0.002739	0.002649	0.002209	0.002094	0.00148	0.003904	0.003637
SISE.IS	0.003222	0.002783	0.002481	0.004568	0.00313	0.003642	0.00381	0.003337	0.004954	0.002152	0.002973	0.003058	0.006215	0.002823	0.002689	0.002282	0.002593	0.001108	0.00436	0.003251
TCELL.IS	0.003352	0.00224	0.002239	0.003858	0.003717	0.003889	0.00409	0.004085	0.00437	0.002551	0.003143	0.002739	0.002823	0.005392	0.003435	0.002941	0.002767	0.002688	0.0039	0.00362
THYAO.IS	0.003363	0.001632	0.002738	0.000952	0.003538	0.003588	0.003621	0.003164	0.002106	0.001847	0.004804	0.002649	0.002689	0.003435	0.00622	0.002672	0.002705	0.001652	0.00358	0.003758

TOASO.IS	0.002587	0.001235	0.002492	0.002006	0.002651	0.002581	0.002528	0.002367	0.00311	0.002042	0.002231	0.002209	0.002282	0.002941	0.002672	0.004065	0.002137	0.001348	0.003146	0.003173
TTKOM.IS	0.002611	0.001804	0.002199	0.003522	0.002534	0.002403	0.002682	0.002404	0.003183	0.002072	0.001999	0.002094	0.002593	0.002767	0.002705	0.002137	0.003112	0.001186	0.003137	0.002302
TUPRS.IS	0.001811	0.001224	0.00043	0.002111	0.001554	0.001468	0.002383	0.002184	0.001114	0.000492	0.000669	0.00148	0.001108	0.002688	0.001652	0.001348	0.001186	0.004509	0.001122	0.001331
VAKBN.IS	0.004458	0.002558	0.004463	0.003859	0.0049	0.006277	0.005147	0.003946	0.005046	0.00267	0.003194	0.003904	0.00436	0.0039	0.00358	0.003146	0.003137	0.001122	0.00743	0.004925
YKBNK.IS	0.004344	0.002196	0.003642	0.002561	0.004596	0.005301	0.004194	0.003739	0.004818	0.003035	0.003192	0.003637	0.003251	0.00362	0.003758	0.003173	0.002302	0.001331	0.004925	0.005798

Historical Correlation

	AKBNK.IS	ARCLK.IS	EKGYO.IS	EREGL.IS	GARAN.IS	HALKB.IS	ISCTR.IS	KCHOL.IS	KRDMD.IS	PETKM.IS	PGSUS.IS	SAHOL.IS	SISE.IS	TCELL.IS	THYAO.IS	TOASO.IS	TTKOM.IS	TUPRS.IS	VAKBN.IS	YKBNK.IS
AKBNK.IS	1	0.426872	0.778438	0.446487	0.920365	0.71229	0.80061	0.828906	0.534736	0.54018	0.418239	0.865264	0.620178	0.692615	0.647053	0.61573	0.710175	0.409294	0.784705	0.865758
ARCLK.IS	0.426872	1	0.290096	0.343596	0.332568	0.2572	0.363373	0.605018	0.330808	0.452622	0.249732	0.317981	0.499434	0.431604	0.292687	0.27407	0.457451	0.257803	0.419738	0.407887
EKGYO.IS	0.778438	0.290096	1	0.29255	0.765264	0.765089	0.628423	0.5762	0.402725	0.261217	0.366306	0.649262	0.46553	0.450934	0.513582	0.578205	0.583034	0.094682	0.765747	0.707484
EREGL.IS	0.446487	0.343596	0.29255	1	0.370175	0.3779	0.474381	0.545641	0.636876	0.235429	0.166356	0.4707	0.586173	0.531467	0.122076	0.318202	0.638668	0.317915	0.45282	0.340138
GARAN.IS	0.920365	0.332568	0.765264	0.370175	1	0.797995	0.844211	0.774232	0.574817	0.514289	0.470475	0.817031	0.559426	0.713281	0.63208	0.585799	0.639995	0.326065	0.800962	0.850539
HALKB.IS	0.71229	0.2572	0.765089	0.3779	0.797995	1	0.736072	0.618155	0.481219	0.304913	0.531596	0.725403	0.51926	0.595242	0.511359	0.455012	0.484207	0.24569	0.81843	0.782515
ISCTR.IS	0.80061	0.363373	0.628423	0.474381	0.844211	0.736072	1	0.744152	0.562786	0.503894	0.496719	0.781627	0.635031	0.731675	0.603169	0.520989	0.631686	0.46631	0.784527	0.723647
KCHOL.IS	0.828906	0.605018	0.5762	0.545641	0.774232	0.618155	0.744152	1	0.612896	0.482676	0.492313	0.790878	0.632243	0.830927	0.599217	0.554516	0.643885	0.485753	0.683753	0.733558
KRDMD.IS	0.534736	0.330808	0.402725	0.636876	0.574817	0.481219	0.562786	0.612896	1	0.526169	0.27334	0.595323	0.576905	0.546306	0.245183	0.447823	0.523797	0.152279	0.53741	0.580855
PETKM.IS	0.54018	0.452622	0.261217	0.235429	0.514289	0.304913	0.503894	0.482676	0.526169	1	0.1254	0.438226	0.377284	0.480176	0.323714	0.442537	0.513361	0.101229	0.428026	0.550839

PGSUS.IS	0.418239	0.249732	0.366306	0.166356	0.470475	0.531596	0.496719	0.492313	0.27334	0.1254	1	0.471046	0.461897	0.524202	0.746089	0.428558	0.439006	0.12205	0.45387	0.513419
SAHOL.IS	0.865264	0.317981	0.649262	0.4707	0.817031	0.725403	0.781627	0.790878	0.595323	0.438226	0.471046	1	0.654696	0.629365	0.566802	0.584645	0.633566	0.372048	0.764231	0.806089
SISE.IS	0.620178	0.499434	0.46553	0.586173	0.559426	0.51926	0.635031	0.632243	0.576905	0.377284	0.461897	0.654696	1	0.487566	0.432507	0.454058	0.589651	0.209242	0.641646	0.541641
TCELL.IS	0.692615	0.431604	0.450934	0.531467	0.713281	0.595242	0.731675	0.830927	0.546306	0.480176	0.524202	0.629365	0.487566	1	0.593083	0.628133	0.675538	0.545106	0.616119	0.647339
THYAO.IS	0.647053	0.292687	0.513582	0.122076	0.63208	0.511359	0.603169	0.599217	0.245183	0.323714	0.746089	0.566802	0.432507	0.593083	1	0.531382	0.614776	0.311875	0.526623	0.625833
TOASO.IS	0.61573	0.27407	0.578205	0.318202	0.585799	0.455012	0.520989	0.554516	0.447823	0.442537	0.428558	0.584645	0.454058	0.628133	0.531382	1	0.60083	0.314875	0.572434	0.65352
TTKOM.IS	0.710175	0.457451	0.583034	0.638668	0.639995	0.484207	0.631686	0.643885	0.523797	0.513361	0.439006	0.633566	0.589651	0.675538	0.614776	0.60083	1	0.316638	0.652477	0.541946
TUPRS.IS	0.409294	0.257803	0.094682	0.317915	0.326065	0.24569	0.46631	0.485753	0.152279	0.101229	0.12205	0.372048	0.209242	0.545106	0.311875	0.314875	0.316638	1	0.193777	0.260332
VAKBN.IS	0.784705	0.419738	0.765747	0.45282	0.800962	0.81843	0.784527	0.683753	0.53741	0.428026	0.45387	0.764231	0.641646	0.616119	0.526623	0.572434	0.652477	0.193777	1	0.750367
YKBNK.IS	0.865758	0.407887	0.707484	0.340138	0.850539	0.782515	0.723647	0.733558	0.580855	0.550839	0.513419	0.806089	0.541641	0.647339	0.625833	0.65352	0.541946	0.260332	0.750367	1

Implied Covariance

	AKBNK.IS	ARCLK.IS	EKGYO.IS	EREGL.IS	GARAN.IS	HALKB.IS	ISCTR.IS	KCHOL.IS	KRDMD.IS	PETKM.IS	PGSUS.IS	SAHOL.IS	SISE.IS	TCELL.IS	THYAO.IS	TOASO.IS	TTKOM.IS	TUPRS.IS	VAKBN.IS	YKBNK.IS
AKBNK.IS	0.067856	0.014089	0.028546	0.016725	0.030767	0.026887	0.027904	0.026488	0.019609	0.078585	0.015623	0.028829	0.046118	0.025574	0.024346	0.021111	0.023232	0.012016	0.03006	0.111169
ARCLK.IS	0.014089	0.016055	0.005174	0.006261	0.005408	0.004722	0.00616	0.009404	0.005901	0.032029	0.004537	0.005153	0.018065	0.007752	0.005357	0.004571	0.007279	0.003681	0.007821	0.025476
EKGYO.IS	0.028546	0.005174	0.019817	0.005922	0.013825	0.015607	0.011836	0.00995	0.007981	0.020537	0.007394	0.01169	0.018708	0.008998	0.010443	0.010714	0.010307	0.001502	0.015852	0.049094
EREGL.IS	0.016725	0.006261	0.005922	0.020679	0.006831	0.007875	0.009127	0.009625	0.012893	0.018907	0.00343	0.008658	0.024063	0.010833	0.002536	0.006023	0.011533	0.005152	0.009576	0.024111
GARAN.IS	0.030767	0.005408	0.013825	0.006831	0.016469	0.014839	0.014495	0.012189	0.010384	0.036859	0.008658	0.013411	0.020494	0.012975	0.011716	0.009895	0.010314	0.004716	0.015116	0.053805
HALKB.IS	0.026887	0.004722	0.015607	0.007875	0.014839	0.020998	0.014271	0.010988	0.009816	0.024676	0.011046	0.013445	0.02148	0.012226	0.010703	0.008678	0.008811	0.004012	0.01744	0.055895
ISCTR.IS	0.027904	0.00616	0.011836	0.009127	0.014495	0.014271	0.017901	0.012214	0.0106	0.037652	0.00953	0.013376	0.024255	0.013876	0.011657	0.009175	0.010614	0.007031	0.015436	0.047727
KCHOL.IS	0.026488	0.009404	0.00995	0.009625	0.012189	0.010988	0.012214	0.015049	0.010584	0.033068	0.00866	0.012409	0.022141	0.014449	0.010617	0.008954	0.009919	0.006716	0.012335	0.044359
KRDMD.IS	0.019609	0.005901	0.007981	0.012893	0.010384	0.009816	0.0106	0.010584	0.019817	0.041367	0.005518	0.010719	0.023184	0.010901	0.004985	0.008298	0.00926	0.002416	0.011125	0.040307
PETKM.IS	0.078585	0.032029	0.020537	0.018907	0.036859	0.024676	0.037652	0.033068	0.041367	0.311898	0.010043	0.031304	0.06015	0.038012	0.026113	0.03253	0.036004	0.006371	0.035153	0.151644
PGSUS.IS	0.015623	0.004537	0.007394	0.00343	0.008658	0.011046	0.00953	0.00866	0.005518	0.010043	0.020563	0.00864	0.018908	0.010655	0.015453	0.008089	0.007906	0.001972	0.009571	0.036292
SAHOL.IS	0.028829	0.005153	0.01169	0.008658	0.013411	0.013445	0.013376	0.012409	0.010719	0.031304	0.00864	0.01636	0.023905	0.011411	0.010471	0.009843	0.010177	0.005363	0.014375	0.050824
SISE.IS	0.046118	0.018065	0.018708	0.024063	0.020494	0.02148	0.024255	0.022141	0.023184	0.06015	0.018908	0.023905	0.081494	0.019729	0.017834	0.017061	0.021139	0.006732	0.026936	0.07622
TCELL.IS	0.025574	0.007752	0.008998	0.010833	0.012975	0.012226	0.013876	0.014449	0.010901	0.038012	0.010655	0.011411	0.019729	0.020092	0.012143	0.011719	0.012025	0.008708	0.012843	0.045232
THYAO.IS	0.024346	0.005357	0.010443	0.002536	0.011716	0.010703	0.011657	0.010617	0.004985	0.026113	0.015453	0.010471	0.017834	0.012143	0.020863	0.010102	0.011151	0.005077	0.011186	0.044559
TOASO.IS	0.021111	0.004571	0.010714	0.006023	0.009895	0.008678	0.009175	0.008954	0.008298	0.03253	0.008089	0.009843	0.017061	0.011719	0.010102	0.017324	0.009931	0.004671	0.01108	0.042402

TTKOM.IS	0.023232	0.007279	0.010307	0.011533	0.010314	0.008811	0.010614	0.009919	0.00926	0.036004	0.007906	0.010177	0.021139	0.012025	0.011151	0.009931	0.01577	0.004481	0.012049	0.033548
TUPRS.IS	0.012016	0.003681	0.001502	0.005152	0.004716	0.004012	0.007031	0.006716	0.002416	0.006371	0.001972	0.005363	0.006732	0.008708	0.005077	0.004671	0.004481	0.012701	0.003212	0.014463
VAKBN.IS	0.03006	0.007821	0.015852	0.009576	0.015116	0.01744	0.015436	0.012335	0.011125	0.035153	0.009571	0.014375	0.026936	0.012843	0.011186	0.01108	0.012049	0.003212	0.021625	0.054394
YKBNK.IS	0.111169	0.025476	0.049094	0.024111	0.053805	0.055895	0.047727	0.044359	0.040307	0.151644	0.036292	0.050824	0.07622	0.045232	0.044559	0.042402	0.033548	0.014463	0.054394	0.242989

Implied Correlation

	AKBNK.IS	ARCLK.IS	EKGYO.IS	EREGL.IS	GARAN.IS	HALKB.IS	ISCTR.IS	KCHOL.IS	KRDMD.IS	PETKM.IS	PGSUS.IS	SAHOL.IS	SISE.IS	TCELL.IS	THYAO.IS	TOASO.IS	TTKOM.IS	TUPRS.IS	VAKBN.IS	YKBNK.IS
AKBNK.IS	1	-0.14626	0.556876	-0.10703	0.84073	0.424581	0.60122	0.657812	0.069473	0.08036	-0.16352	0.730528	0.240356	0.38523	0.294107	0.23146	0.42035	-0.18141	0.569411	0.731516
ARCLK.IS	-0.14626	1	-0.41981	-0.31281	-0.33486	-0.4856	-0.27325	0.210036	-0.33838	-0.09476	-0.50054	-0.36404	-0.00113	-0.13679	-0.41463	-0.45186	-0.0851	-0.48439	-0.16052	-0.18423
EKGYO.IS	0.556876	-0.41981	1	-0.4149	0.530528	0.530179	0.256846	0.152399	-0.19455	-0.47757	-0.26739	0.298523	-0.06894	-0.09813	0.027165	0.15641	0.166068	-0.81064	0.531495	0.414968
EREGL.IS	-0.10703	-0.31281	-0.4149	1	-0.25965	-0.2442	-0.05124	0.091282	0.273752	-0.52914	-0.66729	-0.0586	0.172346	0.062933	-0.75585	-0.3636	0.277336	-0.36417	-0.09436	-0.31972
GARAN.IS	0.84073	-0.33486	0.530528	-0.25965	1	0.595989	0.688421	0.548463	0.149634	0.028579	-0.05905	0.634063	0.118852	0.426562	0.264161	0.171598	0.27999	-0.34787	0.601924	0.701078
HALKB.IS	0.424581	-0.4856	0.530179	-0.2442	0.595989	1	0.472143	0.236309	-0.03756	-0.39017	0.063193	0.450806	0.03852	0.190485	0.022718	-0.08998	-0.03159	-0.50862	0.63686	0.565029
ISCTR.IS	0.60122	-0.27325	0.256846	-0.05124	0.688421	0.472143	1	0.488304	0.125572	0.007787	-0.00656	0.563254	0.270061	0.46335	0.206338	0.041978	0.263372	-0.06738	0.569054	0.447294

KCHOL.IS	0.657812	0.210036	0.152399	0.091282	0.548463	0.236309	0.488304	1	0.225792	-0.03465	-0.01537	0.581756	0.264485	0.661854	0.198433	0.109032	0.287771	-0.02849	0.367506	0.467116
KRDMD.IS	0.069473	-0.33838	-0.19455	0.273752	0.149634	-0.03756	0.125572	0.225792	1	0.052338	-0.45332	0.190646	0.153811	0.092612	-0.50963	-0.10435	0.047593	-0.69544	0.074821	0.161709
PETKM.IS	0.08036	-0.09476	-0.47757	-0.52914	0.028579	-0.39017	0.007787	-0.03465	0.052338	1	-0.7492	-0.12355	-0.24543	-0.03965	-0.35257	-0.11493	0.026722	-0.79754	-0.14395	0.101677
PGSUS.IS	-0.16352	-0.50054	-0.26739	-0.66729	-0.05905	-0.063193	-0.00656	-0.01537	-0.45332	-0.7492	1	-0.05791	-0.07621	0.048405	0.492178	-0.14288	-0.12199	-0.7559	-0.09226	0.026838
SAHOL.IS	0.730528	-0.36404	0.298523	-0.0586	0.634063	0.450806	0.563254	0.581756	0.190646	-0.12355	-0.05791	1	0.309392	0.258731	0.133603	0.169289	0.267131	-0.2559	0.528463	0.612177
SISE.IS	0.240356	-0.00113	-0.06894	0.172346	0.118852	0.03852	0.270061	0.264485	0.153811	-0.24543	-0.07621	0.309392	1	-0.02487	-0.13499	-0.09188	0.179302	-0.58152	0.283293	0.083282
TCELL.IS	0.38523	-0.13679	-0.09813	0.062933	0.426562	0.190485	0.46335	0.661854	0.092612	-0.03965	0.048405	0.258731	-0.02487	1	0.186166	0.256267	0.351076	0.090211	0.232239	0.294678
THYAO.IS	0.294107	-0.41463	0.027165	-0.75585	0.264161	0.022718	0.206338	0.198433	-0.50963	-0.35257	0.492178	0.133603	-0.13499	0.186166	1	0.062764	0.229552	-0.37625	0.053247	0.251666
TOASO.IS	0.23146	-0.45186	0.15641	-0.3636	0.171598	-0.08998	0.041978	0.109032	-0.10435	-0.11493	-0.14288	0.169289	-0.09188	0.256267	0.062764	1	0.20166	-0.37025	0.144867	0.307041
TTKOM.IS	0.42035	-0.0851	0.166068	0.277336	0.27999	-0.03159	0.263372	0.287771	0.047593	0.026722	-0.12199	0.267131	0.179302	0.351076	0.229552	0.20166	1	-0.36672	0.304953	0.083891
TUPRS.IS	-0.18141	-0.48439	-0.81064	-0.36417	-0.34787	-0.50862	-0.06738	-0.02849	-0.69544	-0.79754	-0.7559	-0.2559	-0.58152	0.090211	-0.37625	-0.37025	-0.36672	1	-0.61245	-0.47934
VAKBN.IS	0.569411	-0.16052	0.531495	-0.09436	0.601924	0.63686	0.569054	0.367506	0.074821	-0.14395	-0.09226	0.528463	0.283293	0.232239	0.053247	0.144867	0.304953	-0.61245	1	0.500734
YKBNK.IS	0.731516	-0.18423	0.414968	-0.31972	0.701078	0.565029	0.447294	0.467116	0.161709	0.101677	0.026838	0.612177	0.083282	0.294678	0.251666	0.307041	0.083891	-0.47934	0.500734	1

Sample Volatility

	AKBNK.IS	ARCLK.IS	EKGYO.IS	EREGL.IS	GARAN.IS	HALKB.IS	ISCTR.IS	KCHOL.IS	KRDMD.IS	PETKM.IS	PGSUS.IS	SAHOL.IS	SISE.IS	TCELL.IS	THYAO.IS	TOASO.IS	TTKOM.IS	TUPRS.IS	VAKBN.IS	YKBNK.IS
1	0.004343	0.004997	0.004571	0.009774	0.005036	0.007916	0.005793	0.004481	0.011865	0.005236	0.006666	0.003511	0.006215	0.005392	0.00622	0.004065	0.003112	0.004509	0.00743	0.005798
2	0.004343	0.004997	0.004571	0.009774	0.005036	0.007916	0.005793	0.004481	0.011865	0.005236	0.006666	0.003511	0.006215	0.005392	0.00622	0.004065	0.003112	0.004509	0.00743	0.005798
3	0.004376	0.004438	0.004264	0.011062	0.005304	0.008102	0.005895	0.004707	0.011719	0.005169	0.007612	0.003431	0.005946	0.005278	0.007383	0.00378	0.003086	0.004538	0.007495	0.005677
4	0.004114	0.004486	0.00425	0.010074	0.005054	0.007563	0.005002	0.004536	0.012139	0.005252	0.007638	0.002903	0.005696	0.005189	0.007624	0.00382	0.003101	0.004519	0.006854	0.005343
5	0.004101	0.004443	0.004387	0.009361	0.005126	0.008124	0.004918	0.004533	0.011976	0.005165	0.009927	0.002874	0.005755	0.005469	0.007563	0.00385	0.003738	0.004456	0.006986	0.005252
6	0.003925	0.004701	0.004549	0.008652	0.005041	0.008133	0.004907	0.004613	0.011924	0.004984	0.010327	0.002891	0.005481	0.005419	0.007787	0.003841	0.003709	0.004504	0.006988	0.005287
7	0.003979	0.004691	0.004654	0.00857	0.005099	0.008346	0.00484	0.004707	0.012232	0.005093	0.01051	0.002973	0.005409	0.005236	0.007945	0.00392	0.003774	0.004517	0.007164	0.005423
8	0.003893	0.004864	0.004306	0.008931	0.00529	0.008759	0.004752	0.004806	0.012053	0.005161	0.011645	0.002987	0.005612	0.005217	0.008789	0.00379	0.003818	0.004069	0.006762	0.005588
9	0.004105	0.004673	0.00441	0.008963	0.005361	0.009599	0.005177	0.004748	0.011501	0.005252	0.011916	0.002951	0.005204	0.005182	0.009201	0.003707	0.004167	0.004691	0.006629	0.005654
10	0.00386	0.00479	0.004094	0.009114	0.005113	0.009727	0.005049	0.004668	0.011201	0.005342	0.011766	0.002987	0.005335	0.005184	0.007702	0.003506	0.003954	0.004825	0.006723	0.005563
11	0.004185	0.005214	0.004605	0.009295	0.005747	0.010497	0.005686	0.004853	0.016956	0.005836	0.013535	0.003077	0.005514	0.005153	0.009439	0.00353	0.004302	0.004777	0.007409	0.005438
12	0.004208	0.005204	0.003927	0.009494	0.005626	0.010013	0.005754	0.004877	0.016902	0.005706	0.013313	0.003208	0.005609	0.005159	0.009888	0.003681	0.004321	0.004648	0.006755	0.005407
13	0.004064	0.005303	0.00402	0.009557	0.005195	0.010232	0.005437	0.00474	0.016961	0.005807	0.01346	0.003156	0.005682	0.005092	0.010067	0.003772	0.004441	0.004744	0.006701	0.005321
14	0.004366	0.00514	0.003909	0.009648	0.005375	0.009697	0.00541	0.005447	0.016554	0.005878	0.013841	0.003088	0.005559	0.005241	0.010268	0.003846	0.004328	0.004861	0.006075	0.005342
15	0.004462	0.005373	0.003845	0.010921	0.005414	0.009593	0.005319	0.005213	0.016952	0.008499	0.013778	0.003025	0.005684	0.005086	0.010284	0.003225	0.004301	0.00485	0.006021	0.005287
16	0.004586	0.005525	0.003925	0.011008	0.005559	0.009866	0.005333	0.005227	0.016715	0.008677	0.01416	0.0031	0.005767	0.005199	0.010576	0.003124	0.004218	0.004988	0.006178	0.005428
17	0.004626	0.006502	0.004142	0.011228	0.006211	0.009575	0.005638	0.005082	0.016659	0.00836	0.014548	0.003142	0.006364	0.005139	0.013222	0.003215	0.004798	0.004883	0.006972	0.00586

Implied Volatility

	AKBNK.IS	ARCLK.IS	EKGYO.IS	EREGL.IS	GARAN.IS	HALKB.IS	ISCTR.IS	KCHOL.IS	KRDMD.IS	PETKM.IS	PGSUS.IS	SAHOL.IS	SISE.IS	TCELL.IS	THYAO.IS	TOASO.IS	TTKOM.IS	TUPRS.IS	VAKBN.IS	YKBNK.IS
1	0.067856	0.016055	0.019817	0.020679	0.016469	0.020998	0.017901	0.015049	0.019817	0.311898	0.020563	0.01636	0.081494	0.020092	0.020863	0.017324	0.01577	0.012701	0.021625	0.242989
2	0.0909	0.021436	0.020938	0.024722	0.023203	0.022754	0.023192	0.020145	0.020383	0.148099	0.022876	0.020756	0.052154	0.021865	0.021428	0.022429	0.018138	0.022315	0.019503	0.200727
3	0.123115	0.020528	0.020174	0.021678	0.022798	0.022505	0.02166	0.019353	0.020409	0.272052	0.023255	0.020083	0.08114	0.020879	0.021839	0.021095	0.018244	0.020985	0.021893	0.294235
4	0.068841	0.019981	0.020385	0.022494	0.022576	0.022426	0.021745	0.017397	0.020353	0.146001	0.022536	0.019783	0.056492	0.018814	0.020308	0.02341	0.017479	0.025237	0.021085	0.157235
5	0.112706	0.019277	0.021239	0.021162	0.021761	0.021595	0.02129	0.016491	0.020421	0.2314	0.022125	0.019635	0.020856	0.017313	0.02082	0.021686	0.016747	0.021409	0.020525	0.217282
6	0.070225	0.020933	0.023614	0.022237	0.02343	0.022845	0.022263	0.017782	0.02253	0.121002	0.023096	0.020726	0.02346	0.020781	0.021402	0.024159	0.01769	0.026167	0.021582	0.197625
7	0.080717	0.020296	0.02239	0.022889	0.022725	0.023763	0.022026	0.017376	0.022453	0.185054	0.023556	0.020218	0.021301	0.019451	0.021817	0.023796	0.017459	0.023839	0.022097	0.226427
8	0.098489	0.020253	0.02475	0.024393	0.023038	0.023492	0.022882	0.017044	0.023886	0.227913	0.022704	0.020118	0.020759	0.020413	0.022195	0.024223	0.017906	0.025687	0.021092	0.311879
9	0.123946	0.019352	0.024768	0.023985	0.022427	0.022766	0.022022	0.016641	0.022463	0.300009	0.022691	0.019176	0.019205	0.019021	0.022764	0.022796	0.017552	0.022522	0.022168	0.418967
10	0.094064	0.020278	0.026991	0.025604	0.023673	0.025358	0.023851	0.017446	0.023922	0.180414	0.023489	0.020562	0.022162	0.020711	0.024215	0.024894	0.019576	0.02598	0.024431	0.173369
11	0.130625	0.019656	0.025191	0.024027	0.023568	0.026077	0.022988	0.017343	0.025376	0.268095	0.023544	0.020158	0.021326	0.019599	0.02471	0.023646	0.019755	0.02486	0.024156	0.299506
12	0.078696	0.013273	0.027567	0.025912	0.02577	0.027868	0.026002	0.013829	0.025064	0.137858	0.024697	0.021644	0.022557	0.022598	0.026015	0.013696	0.020818	0.004255	0.025344	0.144278
13	0.085963	0.02076	0.027485	0.004671	0.013421	0.026914	0.011251	0.010849	0.025953	0.186014	0.024337	0.010172	0.02186	0.021261	0.026758	0.02417	0.020412	0.036963	0.024667	0.163678
14	0.058213	0.021207	0.00825	0.005295	0.027387	0.017585	0.027082	0.01784	0.027187	0.069816	0.027863	0.020462	0.013246	0.024507	0.030973	0.027985	0.021383	0.027942	0.021414	0.11105
15	0.078775	0.023662	0.028295	0.026298	0.025494	0.008091	0.026598	0.017436	0.029121	0.14748	0.030633	0.021229	0.024514	0.024346	0.031903	0.025971	0.023665	0.026772	0.018611	0.184186
16	0.084227	0.026385	0.033084	0.031053	0.030897	0.026176	0.031595	0.022788	0.031688	0.14975	0.033909	0.024649	0.029251	0.029335	0.034448	0.032061	0.026283	0.031822	0.026399	0.026974
17	0.130546	0.025127	0.028168	0.029488	0.027635	0.023097	0.028173	0.021951	0.030579	0.166248	0.03446	0.020651	0.027181	0.025592	0.034356	0.028848	0.024492	0.028349	0.024301	0.026577

Minimum Variance Sample Weights

	AKBNK.IS	ARCLK.IS	EKGYO.IS	EREGL.IS	GARAN.IS	HALKB.IS	ISCTR.IS	KCHOL.IS	KRDMD.IS	PETKM.IS	PGSUS.IS	SAHOL.IS	SISE.IS	TCELL.IS	THYAO.IS	TOASO.IS	TTKOM.IS	TUPRS.IS	VAKBN.IS	YKBNK.IS
1	0.04	0.048	0.128	0.01	0.012	0.078	0.018	0.002	0.026	0.142	0.018	0.072	0.066	0.084	0.016	0.052	0	0.184	0.004	0
2	0.04	0.048	0.128	0.01	0.012	0.078	0.018	0.002	0.026	0.142	0.018	0.072	0.066	0.084	0.016	0.052	0	0.184	0.004	0
3	0.04	0.048	0.128	0.01	0.012	0.078	0.018	0.002	0.026	0.142	0.018	0.072	0.066	0.084	0.016	0.052	0	0.184	0.004	0
4	0.04	0.048	0.128	0.01	0.012	0.078	0.018	0.002	0.026	0.142	0.018	0.072	0.066	0.084	0.016	0.052	0	0.184	0.004	0
5	0.04	0.048	0.128	0.01	0.012	0.078	0.018	0.002	0.026	0.142	0.018	0.072	0.066	0.084	0.016	0.052	0	0.184	0.004	0
6	0.04	0.048	0.128	0.01	0.012	0.078	0.018	0.002	0.026	0.142	0.018	0.072	0.066	0.084	0.016	0.052	0	0.184	0.004	0
7	0.04	0.048	0.128	0.01	0.012	0.078	0.018	0.002	0.026	0.142	0.018	0.072	0.066	0.084	0.016	0.052	0	0.184	0.004	0
8	0.04	0.048	0.128	0.01	0.012	0.078	0.018	0.002	0.026	0.142	0.018	0.072	0.066	0.084	0.016	0.052	0	0.184	0.004	0
9	0.04	0.048	0.128	0.01	0.012	0.078	0.018	0.002	0.026	0.142	0.018	0.072	0.066	0.084	0.016	0.052	0	0.184	0.004	0
10	0	0.152	0.034	0	0.172	0.014	0.11	0.002	0.012	0.03	0.01	0.038	0	0	0.022	0.108	0.082	0.196	0.016	0.002
11	0	0.152	0.034	0	0.172	0.014	0.11	0.002	0.012	0.03	0.01	0.038	0	0	0.022	0.108	0.082	0.196	0.016	0.002
12	0	0.152	0.034	0	0.172	0.014	0.11	0.002	0.012	0.03	0.01	0.038	0	0	0.022	0.108	0.082	0.196	0.016	0.002
13	0	0.152	0.034	0	0.172	0.014	0.11	0.002	0.012	0.03	0.01	0.038	0	0	0.022	0.108	0.082	0.196	0.016	0.002
14	0	0.152	0.034	0	0.172	0.014	0.11	0.002	0.012	0.03	0.01	0.038	0	0	0.022	0.108	0.082	0.196	0.016	0.002
15	0	0.152	0.034	0	0.172	0.014	0.11	0.002	0.012	0.03	0.01	0.038	0	0	0.022	0.108	0.082	0.196	0.016	0.002
16	0	0.152	0.034	0	0.172	0.014	0.11	0.002	0.012	0.03	0.01	0.038	0	0	0.022	0.108	0.082	0.196	0.016	0.002
17	0	0.106	0.028	0.054	0.008	0.002	0.098	0.006	0.026	0.014	0.002	0.044	0.182	0.094	0.02	0.162	0.006	0.086	0.048	0.014
18	0	0.106	0.028	0.054	0.008	0.002	0.098	0.006	0.026	0.014	0.002	0.044	0.182	0.094	0.02	0.162	0.006	0.086	0.048	0.014

Mean Variance Sample Weights

	AKBNK.IS	ARCLK.IS	EKGYO.IS	EREGL.IS	GARAN.IS	HALKB.IS	ISCTR.IS	KCHOL.IS	KRDMD.IS	PETKM.IS	PGSUS.IS	SAHOL.IS	SISE.IS	TCELL.IS	THYAO.IS	TOASO.IS	TTKOM.IS	T
1	0.004	0.128	0.026	0.02	0	0.01	0.012	0.112	0.014	0.174	0.01	0.01	0.182	0.006	0.002	0.102	0.092	
2	0.004	0.128	0.026	0.02	0	0.01	0.012	0.112	0.014	0.174	0.01	0.01	0.182	0.006	0.002	0.102	0.092	
3	0.004	0.128	0.026	0.02	0	0.01	0.012	0.112	0.014	0.174	0.01	0.01	0.182	0.006	0.002	0.102	0.092	
4	0.004	0.128	0.026	0.02	0	0.01	0.012	0.112	0.014	0.174	0.01	0.01	0.182	0.006	0.002	0.102	0.092	
5	0.004	0.128	0.026	0.02	0	0.01	0.012	0.112	0.014	0.174	0.01	0.01	0.182	0.006	0.002	0.102	0.092	
6	0.004	0.128	0.026	0.02	0	0.01	0.012	0.112	0.014	0.174	0.01	0.01	0.182	0.006	0.002	0.102	0.092	
7	0.004	0.128	0.026	0.02	0	0.01	0.012	0.112	0.014	0.174	0.01	0.01	0.182	0.006	0.002	0.102	0.092	
8	0.138	0.16	0.056	0.024	0.016	0.014	0.026	0.016	0.012	0.05	0	0.02	0.08	0.028	0.022	0.098	0.01	
9	0.138	0.16	0.056	0.024	0.016	0.014	0.026	0.016	0.012	0.05	0	0.02	0.08	0.028	0.022	0.098	0.01	
10	0.138	0.16	0.056	0.024	0.016	0.014	0.026	0.016	0.012	0.05	0	0.02	0.08	0.028	0.022	0.098	0.01	
11	0.138	0.16	0.056	0.024	0.016	0.014	0.026	0.016	0.012	0.05	0	0.02	0.08	0.028	0.022	0.098	0.01	
12	0.138	0.16	0.056	0.024	0.016	0.014	0.026	0.016	0.012	0.05	0	0.02	0.08	0.028	0.022	0.098	0.01	
13	0.138	0.16	0.056	0.024	0.016	0.014	0.026	0.016	0.012	0.05	0	0.02	0.08	0.028	0.022	0.098	0.01	
14	0.138	0.16	0.056	0.024	0.016	0.014	0.026	0.016	0.012	0.05	0	0.02	0.08	0.028	0.022	0.098	0.01	
15	0.018	0.1	0.036	0.13	0.002	0.016	0.038	0	0.048	0.062	0.014	0.076	0.124	0.048	0.002	0.052	0	
16	0.018	0.1	0.036	0.13	0.002	0.016	0.038	0	0.048	0.062	0.014	0.076	0.124	0.048	0.002	0.052	0	
17	0.018	0.1	0.036	0.13	0.002	0.016	0.038	0	0.048	0.062	0.014	0.076	0.124	0.048	0.002	0.052	0	
18	0.018	0.1	0.036	0.13	0.002	0.016	0.038	0	0.048	0.062	0.014	0.076	0.124	0.048	0.002	0.052	0	

Implied Minimum Variance Weights

	AKBNK.IS	ARCLK.IS	EKGYO.IS	EREGL.IS	GARAN.IS	HALKB.IS	ISCTR.IS	KCHOL.IS	KRDMD.IS	PETKM.IS	PGSUS.IS	SAHOL.IS	SISE.IS	TCELL.IS	THYAO.IS	TOASO.IS	TTKOM.IS	TUPRS.IS	VAKBN.IS	YKBNK.IS
1	0.02	0.084	0.092	0.034	0.022	0.042	0.082	0.03	0.084	0	0.07	0.124	0.008	0.01	0.03	0	0.052	0.188	0.028	0
2	0.006	0.102	0.002	0.174	0.024	0.036	0	0.014	0.158	0.002	0.188	0.032	0.004	0.008	0.13	0.002	0.02	0.026	0.07	0.002
3	0.008	0.178	0.002	0.056	0.032	0.062	0.024	0.104	0.09	0.002	0.012	0.032	0.022	0.018	0.132	0.044	0.048	0.098	0.032	0.004
4	0.02	0.098	0.018	0.148	0.006	0.01	0.044	0.058	0.112	0.03	0.048	0.022	0.004	0.008	0.126	0.078	0.01	0.14	0.018	0.002
5	0.004	0.114	0	0.06	0.04	0.01	0.002	0.098	0.004	0.008	0.088	0.07	0.148	0.002	0.106	0.08	0.036	0.124	0.006	0
6	0.006	0.176	0.04	0.056	0.012	0.032	0.006	0.06	0.012	0.004	0.02	0.006	0.174	0.032	0.036	0.124	0.014	0.092	0.098	0
7	0.008	0.096	0.112	0.01	0.02	0.044	0.002	0.004	0.026	0.01	0.144	0.066	0.126	0.152	0.006	0.018	0.034	0.114	0.004	0.004
8	0.008	0.164	0.026	0.022	0.034	0.194	0.01	0.058	0	0.01	0.012	0	0.002	0.026	0.12	0.1	0.018	0.178	0.016	0.002
9	0	0.154	0.084	0.038	0.022	0.008	0.012	0.048	0.096	0	0.08	0.056	0.12	0.022	0.03	0.01	0.028	0.084	0.1	0.008
10	0.002	0.168	0.03	0.01	0.002	0.008	0.048	0.046	0.01	0	0.136	0.026	0.002	0.12	0.012	0.098	0.084	0.088	0.104	0.006
11	0.018	0.194	0.014	0.07	0.112	0.004	0.016	0.122	0.006	0	0.04	0	0.064	0.004	0.036	0.056	0.086	0.158	0	0
12	0.016	0.176	0.012	0.086	0.01	0.062	0.016	0	0.014	0	0.134	0.066	0.122	0.09	0	0	0.006	0.188	0.002	0
13	0	0.046	0.1	0.146	0.004	0.048	0.164	0.05	0.038	0	0.066	0.006	0.05	0.002	0.064	0.05	0.056	0.076	0.034	0
14	0.006	0.184	0.194	0.074	0.01	0.074	0.01	0.016	0.054	0.01	0.044	0.038	0.11	0.004	0.036	0.04	0.022	0.024	0	0.05
15	0.008	0.178	0.094	0.012	0.006	0.146	0.078	0.008	0.018	0.014	0.002	0.068	0.038	0.012	0.014	0.058	0.072	0.158	0.008	0.008
16	0	0.158	0.186	0.148	0.006	0.002	0	0.004	0	0.002	0.012	0.14	0.022	0.082	0.052	0.034	0.064	0.07	0.006	0.012
17	0	0.18	0.142	0.028	0.004	0	0	0.008	0.022	0.004	0.034	0.08	0.07	0.018	0.106	0.132	0.028	0.104	0.002	0.038

Implied Mean Variance Weights

	AKBNK.IS	ARCLK.IS	EKGYO.IS	EREGL.IS	GARAN.IS	HALKB.IS	ISCTR.IS	KCHOL.IS	KRDMD.IS	PETKM.IS	PGSUS.IS	SAHOL.IS	SISE.IS	TCELL.IS	THYAO.IS	TOASO.IS	TTKOM.IS	TUPRS.IS	VAKBN.IS	YKBNK.IS
1	0.01	0.126	0.188	0.032	0.096	0.004	0.012	0.024	0.018	0	0.162	0.068	0	0.004	0.01	0.028	0.024	0.17	0.022	0.002
2	0.016	0.152	0.044	0.056	0.002	0.034	0.032	0.086	0.084	0.002	0.086	0.004	0.008	0.042	0.168	0.038	0.008	0.092	0.046	0
3	0	0.09	0.056	0.048	0.012	0.07	0.002	0.072	0.168	0.008	0.036	0.052	0.022	0.014	0.078	0.096	0.02	0.144	0	0.012

4	0.01	0.086	0.03	0.12	0.002	0.032	0.01	0.016	0.084	0.006	0.012	0.098	0.062	0.006	0.032	0.146	0	0.178	0.062	0.008
5	0	0.16	0.11	0.05	0.04	0	0.008	0.012	0.004	0.018	0.002	0.048	0.156	0.002	0.02	0.074	0.054	0.17	0.072	0
6	0	0.14	0.046	0.092	0.002	0.032	0	0.058	0.026	0.018	0.01	0.002	0.17	0.096	0	0.076	0	0.19	0.042	0
7	0.006	0.036	0.142	0.14	0.024	0.04	0	0.032	0.102	0.036	0.04	0.036	0.022	0	0.02	0.126	0.002	0.182	0.014	0
8	0	0.06	0.048	0.018	0.014	0.006	0.006	0.172	0.022	0.01	0.052	0.038	0.11	0.12	0.022	0.012	0.088	0.168	0.026	0.008
9	0.008	0.118	0.012	0.118	0.014	0	0.02	0.058	0.038	0.006	0.002	0.148	0.052	0.002	0.148	0.088	0.046	0.116	0	0.006
10	0	0.196	0.112	0.006	0.002	0.026	0.016	0.008	0.044	0.028	0	0.064	0.052	0.096	0.11	0.032	0.096	0.09	0.022	0
11	0.002	0.136	0.152	0.006	0.002	0.072	0.006	0.088	0.022	0.014	0.002	0.03	0.006	0.148	0.014	0.08	0	0.172	0.046	0.002
12	0.006	0.16	0.018	0.008	0.008	0.096	0.126	0.002	0.12	0	0.016	0	0.028	0.062	0.03	0.094	0.022	0.174	0.03	0
13	0	0.18	0.028	0.184	0.004	0.024	0.002	0.02	0.006	0	0.02	0.064	0.046	0.022	0.026	0.16	0.002	0.122	0.088	0.002
14	0.004	0.174	0.104	0.104	0.012	0.006	0.002	0.002	0.046	0.066	0.004	0.028	0.144	0.062	0.106	0.032	0.016	0.086	0.002	0
15	0.008	0.122	0.092	0.11	0.022	0.018	0.02	0.086	0.056	0.008	0.004	0.016	0.164	0.046	0.136	0.018	0	0.034	0.032	0.008
16	0	0.134	0.026	0.18	0.078	0.018	0.004	0	0.008	0.072	0.02	0.004	0.164	0.014	0.012	0.02	0.004	0.186	0.022	0.034
17	0.006	0.148	0.098	0.118	0.002	0.004	0.026	0	0.074	0.02	0.01	0.004	0.188	0.03	0.004	0.05	0.04	0.138	0.018	0.022

Stock Returns

	AKBNK.IS	ARCLK.IS	EKGYO.IS	EREGL.IS	GARAN.IS	HALKB.IS	ISCTR.IS	KCHOL.IS	KRDMD.IS	PETKM.IS	PGSUS.IS	SAHOL.IS	SISE.IS	TCELL.IS	THYAO.IS	TOASO.IS	TTKOM.IS	TUPRS.IS	VAKBN.IS	YKBNK.IS
1	0.114889	0.106287	0.017241	0.271493	0.151871	0.135969	0.145241	0.133755	0.137931	0.142032	0.110028	0.101336	0.069544	0.036759	0.107861	0.144583	0.081356	0.146638	0.132463	0.134211
2	0.002103	0.03038	0.033898	-0.01843	0.001043	0.078098	1.23E-12	-0.03952	0.136364	0.126273	0.14931	0.011342	0.111459	-0.02015	0.194719	-0.00947	0.010972	0.061453	0.056013	0.020882

3	0.028332	0.067158	-0.03607	0.103286	0.019792	0.068285	0.062767	0.008728	0.166667	0.095841	0.024563	0.021495	-0.02335	0.029126	0.111878	-0.01365	-0.03256	0.065263	0.024195	0.020455
4	0.067347	-0.0023	0.081633	0.112057	0.076609	0.147529	0.016107	0.012361	0.154286	0.061056	0.283964	-0.01098	0.073701	0.110535	0.096894	0.069204	0.161859	0.071146	0.088099	0.011136
5	-0.01147	-0.07692	-0.07547	0.048469	0.018975	-0.01789	-0.01057	0.101343	0.178218	-0.02955	0.163485	-0.01943	-0.01333	0.027216	0.105323	0.022654	-0.00966	0.082103	0.022727	0.070485
6	-0.09091	-0.05333	-0.08844	-0.05961	-0.0987	-0.18219	-0.09479	-0.09368	-0.04622	-0.04167	-0.11127	-0.05472	-0.07658	-0.032548	-0.10348	-0.02278	-0.05432	-0.036658	-0.12917	-0.11111
7	0.064894	-0.08803	-0.01493	0.151358	0.077479	-0.08746	0.053097	0.037309	0.066079	0.110368	0.201445	0.051896	0.085366	0.116627	0.186286	2.62E-11	-0.05007	0.122533	0.009569	0.064815
8	-0.1019	-0.04392	-0.06818	-0.03258	-0.07862	-0.19711	-0.13585	-0.02771	-0.10744	-0.01807	-0.11423	-0.0351	-0.05843	0.073394	0.181118	0.010363	-0.11783	-0.11209	-0.08531	-0.09783
9	0.095662	0.08632	0.142276	0.163763	0.115505	0.213964	0.12966	0.120679	0.486111	0.191718	0.27451	0.093412	0.121718	0.080615	0.279772	0.058333	0.13181	0.002475	0.169257	0.045783
10	0.108629	-0.11431	-0.05694	-0.00699	0.146455	-0.05937	0.15495	-0.00974	0.05296	0.021879	0.056805	0.031475	0.06383	0.008398	0.050351	-0.03392	-0.01553	-0.05021	0.11226	0.073733
11	-0.02656	-0.00577	-0.00377	0.141709	-0.04557	-0.0503	-0.0559	-0.01694	0.017751	0.016373	-0.00728	-0.06364	-0.044	-0.0173	0.15716	-0.04765	0.031546	0.012998	-0.0239	0.021459
12	-0.023	0.010372	-0.04167	-0.07746	-0.06735	-0.07061	-0.06053	-0.09172	0.151163	0.008674	0.00564	-0.02235	-0.079498	-0.01239	0.002098	0.00078	0.01682	0.173931	-0.11429	-0.06723
13	-0.11414	0.012885	-0.03953	-0.03053	-0.08411	-0.07598	-0.06072	-0.12366	0.027778	-0.11794	-0.2092	-0.02926	-0.12597	-0.07855	-0.12559	-0.0747	-0.05564	-0.0452	-0.08295	-0.1036
14	-0.05083	-0.08075	-0.00028	0.269688	-0.03257	-0.07134	-0.03247	0.018222	0.169533	-0.2726	-0.05532	-0.03015	-0.05775	-0.1404	-0.00359	-0.0008	-0.05573	0.025121	-0.08375	0.01005
15	-0.06351	-0.08243	-0.01932	-0.09172	-0.05948	-0.03646	-0.04362	0.017895	-0.07353	0.027601	-0.07583	-0.05359	0.062814	0.066756	0.18679	-0.0376	-0.15852	0.017908	-0.10238	-0.04266

16	-0.03856	-0.17311	-0.133	0.082353	-0.15155	-0.06351	-0.10877	-0.0218	-0.0068	0.009091	0.17303	-0.03851	0.182033	0.06327	0.276219	-0.04406	-0.03808	-0.0037	-0.1609	-0.14516
17	-0.20332	0.052339	0.045455	0.072464	-0.16315	-0.06848	-0.21457	0.076923	-0.03881	0.240991	-0.17452	-0.16137	0.232	-0.18083	-0.05787	-0.00435	-0.2375	0.096654	-0.13537	-0.20755

Portfolio Returns

	Benchmark	Min Variance Sample	Min Variance Implied	Mean Variance Sample	Mean Variance Implied
1	0.070191242	0.107393572	0.145086254	0.115881769	0.122721231
2	0.026406287	0.056521286	0.08146239	0.052488688	0.06906799
3	0.029742096	0.041507411	0.061214952	0.028828373	0.061824585
4	0.071019142	0.07663783	0.087008918	0.066020471	0.073914554
5	0.025455685	0.006966634	0.039114166	0.005090235	0.000109779
6	-0.068761233	-0.047393694	-0.052516352	-0.055017004	-0.050528035
7	0.070840387	0.058860439	0.068539675	0.03973587	0.065957056
8	-0.052765488	-0.059218694	-0.055007928	-0.060430459	-0.049888599
9	0.110924887	0.123446213	0.127098307	0.095150802	0.138715839
10	0.030736938	0.013403174	0.002854448	-0.001898175	-0.014846726
11	0.000501091	-0.014410655	0.004689785	-0.006807013	-0.010660567
12	-0.043842573	0.015496351	-0.012867585	0.026542719	-0.003506796
13	-0.094201326	-0.054039354	-0.068134952	-0.064311053	-0.050716558
14	-0.025839785	-0.031394355	-0.005246908	-0.032808342	-0.014668342
15	-0.044841628	-0.050921863	-0.053061319	-0.0236396	-0.026623009
16	0.011400978	-0.074736066	-0.028869442	-0.002191415	0.005769515
17	-0.042194082	0.004488678	0.000655979	0.03193089	0.047487321

R CODES USED IN THE STUDY

```
if(!require(dplyr)) install.packages("dplyr"); library(dplyr)
if(!require(readr)) install.packages("readr"); library(readr)
if(!require(quantmod)) install.packages("quantmod"); library(quantmod)
if(!require(PortfolioAnalytics)) install.packages("PortfolioAnalytics"); library(PortfolioAnalytics)
if(!require(ROI)) install.packages("ROI"); library(ROI)
if(!require(ROI.plugin.glpk)) install.packages("ROI.plugin.glpk"); library(ROI.plugin.glpk)
if(!require(ROI.plugin.quadprog)) install.packages("ROI.plugin.quadprog"); library(ROI.plugin.quadprog)
if(!require(ROI.plugin.symphony)) install.packages("ROI.plugin.symphony"); library(ROI.plugin.symphony)
if(!require(RQuantLib)) install.packages("RQuantLib"); library(RQuantLib)
if(!require(bsts)) install.packages("bsts"); library(bsts)
if(!require(MASS)) install.packages("MASS"); library(MASS)
if(!require(lubridate)) install.packages("lubridate"); library(lubridate)
if(!require(derivmkts)) install.packages("derivmkts"); library(derivmkts)
if(!require(ggplot2)) install.packages("ggplot2"); library(ggplot2)
if(!require(reshape2)) install.packages("reshape2"); library(reshape2)
if(!require(plm)) install.packages("plm"); library(plm)
if(!require(broom)) install.packages("broom"); library(broom)
stocks <- read_csv("Stocks.csv")
index <- read_csv("Index.csv")
# stocks[, 2] <- stocks[, 2]/1000
options <- read_csv("Month-End Options.csv")
dates <- read_csv("Dates.csv")
dates <- as.matrix(dates)
divYield <- read_csv("Dividend Yield.csv")
getOption <- function(stock) {
singleStock <- filteredStocks %>% dplyr::select(stock)
  singleOption <- filteredOptions %>% filter(UNDERLYING == stock)
  callNumber <- which(abs(singleOption$`strike price` - as.numeric(singleStock)) == min(abs(singleOption$`strike
price` - as.numeric(singleStock))))
  callNumber <- callNumber[1]
  finalOption <- singleOption[callNumber, ]
  return(finalOption)
}
uniqueStocks <- colnames(unique(stocks)[-1])
finalOptions <- list()
```

```
for (i in 1:nrow(dates)) {
  tradeDate <- dates[i, 1]
  expiryDate <- dates[i, 2]
  filteredStocks <- stocks %>% filter(Date == tradeDate)
  filteredOptions <- options %>% filter(`TRADE DATE` == tradeDate &
                                        `End of Month Date` == expiryDate &
                                        `C/P` == "C")
      tempOptions <- c()
  for (j in 1:length(uniqueStocks)) {
    tempOptions <- rbind(tempOptions, getOption(uniqueStocks[j]))
  }
  finalOptions[[i]] <- tempOptions
}
spotPrices <- c()
for (i in 1:nrow(dates)) {
  tradeDate <- dates[i, 1]
  filteredStocks <- stocks %>% filter(Date == tradeDate)
  spotPrices <- rbind(spotPrices, filteredStocks)
}
for (i in 1:nrow(dates)) {
  finalOptions[[i]] <- cbind(finalOptions[[i]], t(spotPrices[i, -1]))
  finalOptions[[i]] <- cbind(finalOptions[[i]], t(divYield[i, -1]))
  finalOptions[[i]] <- cbind(finalOptions[[i]],
                             (as.numeric(finalOptions[[i]]$`End of Month Date`-finalOptions[[i]]$`TRADE DATE`)/365))
  colnames(finalOptions[[i]])[27:29] <- c("Spot", "DivYield", "Maturity")
}
finalIndexOption <- list()
for (i in 1:nrow(dates)) {
  tradeDate <- dates[i, 1]
  expiryDate <- dates[i, 2]
  filteredStocks <- index %>% filter(Date == tradeDate)
  filteredOptions <- options %>% filter(`TRADE DATE` == tradeDate &
                                        `End of Month Date` == expiryDate &
                                        `C/P` == "C")
  tempOptions <- getOption("D_XU030D")
  finalIndexOption[[i]] <- tempOptions
}
```

```
spotPrices <- c()
for (i in 1:nrow(dates)) {
  tradeDate <- dates[i, 1]
  filteredStocks <- index %>% filter(Date == tradeDate)
  spotPrices <- rbind(spotPrices, filteredStocks)
}
for (i in 1:nrow(dates)) {
 finalIndexOption[[i]] <- cbind(finalIndexOption[[i]], t(spotPrices[i, -1]))
  finalIndexOption[[i]] <- cbind(finalIndexOption[[i]], 0)
  finalIndexOption[[i]] <- cbind(finalIndexOption[[i]],
                           (as.numeric(finalIndexOption[[i]]$`End of Month Date`-finalIndexOption[[i]]$`TRADE DATE`)/365))
  colnames(finalIndexOption[[i]])[27:29] <- c("Spot", "DivYield", "Maturity")
}

finalOptionsOutput <- c()
finalOptionsIndexOutput <- c()
for (i in 1:nrow(dates)) {
  finalOptionsOutput <- rbind(finalOptionsOutput, finalOptions[[i]])
  finalOptionsIndexOutput <- rbind(finalOptionsIndexOutput, finalIndexOption[[i]])
}
write.csv(finalOptionsOutput, "Stock Options Used.csv")
write.csv(finalOptionsIndexOutput, "Index Options Used.csv")
start <- as.Date("2014-03-01")
end <- as.Date("2018-08-31")
returns <- read.csv("Total Returns.csv")
returnDates <- as.Date(returns[, 1])
returns <- returns[, -1]
returns <- xts(returns, order.by = returnDates)
index(returns) <- LastDayInMonth(as.Date(as.yearmon(index(returns), "%mm/%Y")))
returns <- returns/100
write.csv(returns, "Returns.csv")
rf <- 0.126411475409836/12
sampleReturns <- last(returns, 17)
stockMeans <- sapply(sampleReturns, mean) * 12
stockStdDev <- sapply(sampleReturns, sd) * 12^0.5
stockSharpe <- (stockMeans - rf*12)/(stockStdDev)
```

```
write.csv(rbind(stockMeans, stockStdDev, stockSharpe), "Stock Performance.csv")
weights <- read_csv("Weights.csv")[, -1]
historicWeightsOptim <- function(x, startDate, sampleOption, portOption, endDate, rf, i) {
  sampleData <- window(x, start = startDate, end = endDate)
  sampleVol <- apply(sampleData, 2, var)
  sampleStdDev <- apply(sampleData, 2, sd)*(12)^0.5
  # colnames(sampleOption)[7:9] <- c("Spot", "divYield", "maturity")
  # colnames(portOption)[7:9] <- c("Spot", "divYield", "maturity")
  sampleImpVol <- c()
  for (j in 1:nrow(sampleOption)) {
    sampleImpVol <- c(sampleImpVol,
                      bscallimpvol(price=sampleOption$`SETTLEMENT PRICE`[j],
                                   s=sampleOption$Spot[j],
                                   k=sampleOption$`strike price`[j],
                                   d=sampleOption$DivYield[j],
                                   r=rf*12,
                                   tt=sampleOption$Maturity[j]))
  }
  portImpVol <- bscallimpvol(price=portOption$`SETTLEMENT PRICE`,
                             s=portOption$Spot,
                             k=portOption$`strike price`,
                             d=portOption$DivYield,
                             r=rf*12,
                             tt=portOption$Maturity)

  sampleImpVol <- sampleImpVol/12
  sampleImpSD <- sampleImpVol ^ 0.5
  sampleCor <- cor(sampleData)
  portWeights <- weights[i, ]
  samplePortVar <- as.matrix(portWeights) %*% diag(sampleImpSD) %*% sampleCor %*% diag(sampleImpSD)
%*% t(as.matrix(portWeights))
  L <- matrix(-1/20, 20, 20)
  diag(L) <- 1
  alphaWnum <- portImpVol - samplePortVar
  alphaWden <- as.matrix(portWeights) %*% diag(sampleImpSD) %*% (L - sampleCor) %*% diag(sampleImpSD)
%*% t(as.matrix(portWeights))
  alphaW <- c(alphaWnum/alphaWden)
```

```
alphaW <- max(alphaW, -1)
impCor <- sampleCor + alphaW * (matrix(1, 20, 20) - sampleCor)
impCov <- diag(sampleImpSD) %*% sampleCor %*% diag(sampleImpSD)
port <- portfolio.spec(assets = colnames((sampleData)))
port <- add.constraint(portfolio = port,
                       type = "full_investment")
port <- add.constraint(portfolio = port,
                       type = "long_only")
port <- add.constraint(portfolio = port,
                       type = "box",
                       min = 0.0,
                       max = 0.2)

minVar.port <- add.objective(portfolio = port,
                             type = "risk",
                             name = "StdDev")
meanVar.port <- add.objective(portfolio = minVar.port,
                              type = "return",
                              name  = "mean")
num_assets = ncol(x)
momentargs = list()
momentargs$mu = colMeans(x)
momentargs$sigma = impCov
momentargs$m3 = matrix(0, nrow = num_assets, ncol = num_assets ^ 2)
momentargs$m4 = matrix(0, nrow = num_assets, ncol = num_assets ^ 3)
minVarSample.opt <- optimize.portfolio(R = sampleData,
                                       portfolio = minVar.port,
                                       optimize_method = "random",
                                       momentargs = momentargs,
                                       trace = TRUE)
meanVarSample.opt <- optimize.portfolio(R = sampleData,
                                        portfolio = meanVar.port,
                                        optimize_method = "random",
                                        momentargs = momentargs,
                                        trace = TRUE)
return(list(minVarWeights = extractWeights(minVarSample.opt),
       meanVarWeights = extractWeights(meanVarSample.opt),
```

```
        covar = impCov,
        var = sampleImpVol,
        impCor = impCor,
        portImpVol = portImpVol,
        samplePortVar = samplePortVar,
        alphaW = alphaW,
        impVol = sampleImpVol
        ))
}
minVarWeights <- c()
meanVarWeights <- c()
coVars <- list()
variance <- c()
impCor <- list()
impVol <- list()
for (i in 1:nrow(dates)) {
  startDate <- as.Date(dates[i, 1])
  startDate <- startDate %m-% months(36)
  endDate <- as.Date(dates[i, 1])
  sampleOptimisation <- historicWeightsOptim(x = returns,
                                             startDate = startDate,
                                             endDate = endDate,
                                             rf = rf,
                                             i = i,
                                             sampleOption = finalOptions[[i]],
                                             portOption = finalIndexOption[[i]])
  minVarWeights <- rbind(minVarWeights,
                         sampleOptimisation$minVarWeights)
  meanVarWeights <- rbind(meanVarWeights,
                          sampleOptimisation$meanVarWeights)
  coVars[[i]] <- sampleOptimisation$covar
  variance <- rbind(variance, sampleOptimisation$var)
  write.csv(coVars[[i]], paste0("Implied Covariance ", i, ".csv"))
  impCor[[i]] <- sampleOptimisation$impCor
  impVol[[i]] <- sampleOptimisation$impVol
  write.csv(impCor[[i]], paste0("Implied Correlation ", i, ".csv"))
}
```

```
dates <- as.Date(as.matrix(dates[, 1]))
minVarWeights <- xts(minVarWeights,
                     order.by = dates,
                     frequency = "months")
meanVarWeights <- xts(meanVarWeights,
                     order.by = dates,
                     frequency = "months")
write.csv(minVarWeights, "Implied MinVar Weights.csv")
write.csv(meanVarWeights, "Implied MeanVar Weights.csv")

benchmark <- read_csv("Benchmark.csv")
benchmarkVals <- as.matrix(benchmark[,2])
benchmarkDates <- as.Date(as.matrix(benchmark[,1]))
benchmark <- xts(benchmarkVals, benchmarkDates)
benchmarkReturns <- Return.calculate(benchmark)[-1, ]
rf <- 0.126411475409836/12
port <- portfolio.spec(assets = colnames((returns)))
port <- add.constraint(portfolio = port,
                       type = "full_investment")
port <- add.constraint(portfolio = port,
                       type = "long_only")

port <- add.constraint(portfolio = port,
                       type = "box",
                       min = 0.0,
                       max = 0.2)
minVar.port <- add.objective(portfolio = port,
                             type = "risk",
                             name = "StdDev")
meanVar.port <- add.objective(portfolio = minVar.port,
                              type = "return",
                              name  = "mean")
minVarSample.opt <- optimize.portfolio.rebalancing(R = returns,
                                                   portfolio = minVar.port,
                                                   optimize_method = "random",
                                                   trace = TRUE,
                                                   rebalance_on = "months",
```

```r
                                              training_period = 36,
                                              rolling_window = 36)
meanVarSample.opt <- optimize.portfolio.rebalancing(R = returns,
                                               portfolio = meanVar.port,
                                               optimize_method = "random",
                                               trace = TRUE,
                                               rebalance_on = "months",
                                               training_period = 36,
                                               rolling_window = 36)
test <- optimize.portfolio(R = returns,
                        portfolio = meanVar.port,
                        optimize_method = "random",
                        trace = TRUE)
chart.Weights(minVarSample.opt, main = "Min Var Sample")
chart.Weights(meanVarSample.opt, main = "Mean Var Sample")

retMinVarSample <- Return.portfolio(R = returns,
                               weights = extractWeights(minVarSample.opt))
retMeanVarSample <- Return.portfolio(R = returns,
                                weights = extractWeights(meanVarSample.opt))
retMinVarImplied <- Return.portfolio(R = returns,
                                weights = minVarWeights)
retMeanVarImplied <- Return.portfolio(R = returns,
                                 weights = meanVarWeights)
portImpliedRet <- cbind(retMinVarImplied, retMeanVarImplied)
portRetAll <- cbind(benchmarkReturns,
                  retMinVarSample,
                  retMinVarImplied,
                  retMeanVarSample,
                  retMeanVarImplied)
colnames(portRetAll) <- c("Benchmark",
                       "Min Variance Sample",
                       "Min Variance Implied",
                       "Mean Variance Sample",
                       "Mean Variance Implied")
write.csv(table.AnnualizedReturns(portRetAll, Rf = rf), "Port Performance.csv")
write.csv(portRetAll, "portRetAll.csv")
```

```
chart.CumReturns(portRetAll, main = "Cumulative Returns", legend.loc = "bottom")
chart.Drawdown(portRetAll, main = "Drawdown", legend.loc = "bottom")
minVarSampleWeights <- extractWeights(minVarSample.opt)
meanVarSampleWeights <- extractWeights(meanVarSample.opt)
write.csv(minVarSampleWeights, "Min Var Sample Weights.csv")
write.csv(meanVarSampleWeights, "Mean Var Sample Weights.csv")
covarCalc <- function(x, startDate, endDate, impliedCor, i) {
  sampleData <- window(x, start = startDate, end = endDate)
  sampleCov <- cov(sampleData)
  sampleCor <- cor(sampleData)
  melted_cormat <- melt(sampleCor)
  melted_imp_cormat <- melt(impliedCor)
  # colnames(melted_cormat) <- c("Stock1", "Stock2")
  heatmap <- ggplot(data = melted_cormat, aes(Var1, Var2, fill = value))+
    geom_tile(color = "white")+
    scale_fill_gradient2(low = "blue", high = "red", mid = "white",
                         midpoint = 0, limit = c(-1,1), space = "Lab",
                         name="Correlation") +
    theme_minimal()+
    theme(axis.text.x = element_text(angle = 90, vjust = 1,
                                     size = 8, hjust = 1))+
    coord_fixed() +
    theme(axis.text.y = element_text(size = 8))
  heatmap2 <- ggplot(data = melted_imp_cormat, aes(Var1, Var2, fill = value))+
    geom_tile(color = "white")+
    scale_fill_gradient2(low = "blue", high = "red", mid = "white",
                         midpoint = 0, limit = c(-1,1), space = "Lab",
                         name="Correlation") +
    theme_minimal()+
    theme(axis.text.x = element_text(angle = 90, vjust = 1,
                                     size = 8, hjust = 1))+
    coord_fixed() +
    theme(axis.text.y = element_text(size = 8))
  return(list(sampleCov = sampleCov,
              sampleVar = diag(sampleCov),
              sampleCor = sampleCor,
              heatmap = heatmap,
```

```
              heatmap2 = heatmap2))
}
sampleCov <- list()
sampleVar <- c()
sampleCor <- list()
heatmap <- list()
impHeatmap <- list()
for (i in 1:length(dates)) {
  startDate <- dates[i]
  startDate <- startDate %m-% months(36)
  endDate <- dates[i]
  calc <- covarCalc(returns, startDate, endDate, impCor, i)
  sampleCov[[i]] <- calc$sampleCov
  sampleVar <- rbind(sampleVar, calc$sampleVar)
  sampleCor[[i]] <- calc$sampleCor
  heatmap[[i]] <- calc$heatmap
  impHeatmap[[i]] <- calc$heatmap2
  ggsave(paste0("Heatmap ", i, ".png"), heatmap[[i]])
  ggsave(paste0("Implied Heatmap ", i, ".png"), impHeatmap[[i]])
  write.csv(sampleCov[[i]], paste0("Sample Covariance ", i, ".csv"))
  write.csv(sampleCor[[i]], paste0("Correlation ", i, " .csv"))
}
colnames(variance) <- colnames(sampleVar)
write.csv(variance, "Implied Volatility.csv")
write.csv(sampleVar, "Sample Volatility.csv")
```