Evaluating the Packing Process in Food Industry

Using Fuzzy $\widetilde{\overline{X}}$ and $\widetilde{S}$ Control Charts

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Abstract

The fuzzy set theory addresses the development of concepts and techniques for dealing with uncertainty or impression conditions. If the collected data from a process include vagueness due to human subjectively or measurement system, fuzzy control charts are available tools for monitoring and evaluating the process. The main contribution of fuzzy control charts is to provide flexibility to the control limits. When sample mean is too close to the control limits and the used measurement system is not so sensitive, the decision may be faulty. In this paper, the fuzzy standard deviation is firstly introduced to obtain fuzzy $\widetilde{\overline{X}}$ and $\widetilde{S}$ control charts and then these fuzzy control charts are employed in food industry to monitor if the processes are under control or not. Additionally, the fuzzy $\widetilde{\overline{X}}$ and $\widetilde{S}$ control charts are developed for the case that the population parameters (μ and σ) are known.

Keywords Fuzzy, fuzzy standard deviation, $\widetilde{\overline{X}}$ - $\widetilde{S}$ control chart, α-cut, α-level fuzzy midrange.

1. Introduction

Statistical process control (SPC) is a powerful and a widely used methodology to monitor and evaluate processes. In many applications on practice, the traditional control charts are used. The main aim of traditional control charts, named Shewhart control charts, is to detect whether or not a process is "out of control". The control charts contain three elements: the centerline (CL), the upper control limit (UCL) and the lower control limit (LCL). CL represents the average value of the quality characteristic, UCL and LCL present the $\pm 3\sigma_{\bar{x}}$ distance from the CL, where $\sigma_{\bar{x}}$ is the standard deviation of sample means. Also, the warning limits keep on $\pm 2\sigma_{\bar{x}}$ from the centre line in control charts. These control limits are represented as in Figure 1.
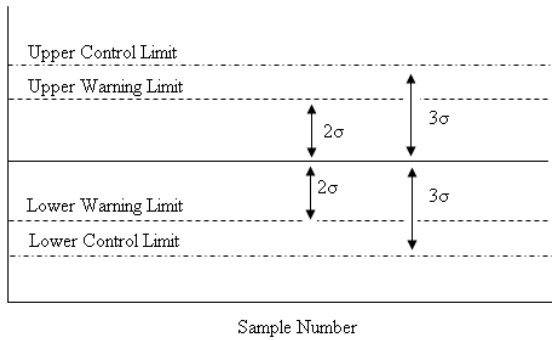
Fig. 1. A traditional control chart with its warning and control limits.

Traditional control charts require crisp values from a process. But in many cases data include fuzziness. While collecting data from a process, this uncertainty or vagueness exist inherently on the process and on the measurement system which include operators and gauges. If data contain some vagueness, the fuzzy control charts are useful tools for evaluating fuzzy data. The fuzzy set theory can be adopted on the fuzzy control charts.

Zadeh[1] first introduced the concepts of fuzzy sets and fuzzy logic. Fuzzy logic is a form of multi-valued logic derived from the fuzzy set theory to deal with reasoning that is approximate rather than precise. Attempts to handle the uncertainty, imprecision and subjectiveness inherent in the human decision making process can be carried out basically along the lines of the probability theory and the fuzzy set theory. Imprecisely defined "classes" play an important role in human thinking, particularly in the domains of patter recognition, communication of information, and abstraction[1].

The aim of this study is to give the calculation methodology of fuzzy standard deviation for obtaining the fuzzy $\widetilde{\overline{X}} - \widetilde{S}$ control charts. Additionally the theoretical structure of fuzzy $\widetilde{\overline{X}}$ and $\widetilde{S}$ control charts is developed for the case that the population parameters, μ and σ are known.

In the application section, we show how to control the packing process of biscuits with fuzzy $\widetilde{\overline{X}} - \widetilde{S}$ control charts. Thus, the false alarms being early or late are avoided and unnecessary time consumptions are prevented in packing process.

The rest of the paper is organized in the following order: A literature review on fuzzy control charts is given in Section 2. The calculation of fuzzy standard deviation is presented in Section 3. Fuzzy $\widetilde{\overline{X}} - \widetilde{S}$ control charts with and without the known population parameters, μ and σ are introduced in Section 4. The

application to the packing process of biscuits with fuzzy $\widetilde{\overline{X}} - \widetilde{S}$ control charts are presented in Section 5. In Section 6, the conclusions are given.

2. Literature Review on Fuzzy Control Charts

The fuzzy control charts were firstly presented by Raz and Wang[2] and Wang and Raz[3]. They proposed two approaches: probabilistic approach and membership approach. In probabilistic approach, the observations are converted from linguistic terms to their crisp values, and the control limits are calculated for variables. In membership approach, the fuzzy subsets corresponding to the observations in a sample are combined into a single fuzzy subset that corresponds to the sample average according to the rules of fuzzy arithmetic. Kanagawa et al[4] proposed control charts using linguistic terms as labeled fuzzy data from a stand point different to that of Wang and Raz[3]. Their control charts are aimed at directly controlling the underlying probability distribution of the linguistic data. El-Shal and Morris[5] made an investigation into the use of fuzzy logic to modify SPC rules, with the aim of reducing the generation of false alarm and also improving the detection and detection-speed of real faults. Rowlands and Wang[6] introduced a method of fuzzy SPC evaluation and control (FSEC) which combines traditional statistical process control methodology with an intelligent system approach. Fuzzy logic is employed to evaluate SPC zone rules which lead to the special membership functions and fuzzy if-then rules. FSEC is based on a multiple-input/ single output system. Gülbay et al[7] proposed α-cut control charts for attributes data to regulate the tightness of the inspection for attribute with triangular fuzzy numbers. In their approaches the tightness of the inspection can be defined by selecting a suitable α-cut. The α-cut approach provides the ability of detecting out of control points and effectiveness of fuzzy control charts. Cheng[8] proposed the method of constructing a fuzzy control chart for a process with fuzzy outcomes. Fuzzy outcomes are described in fuzzy numbers based on experts' quality ratings. Two fuzzy control charts are constructed to directly monitor the fuzzy outcomes in order to establish whether or not the process is in control. Gülbay and Kahraman[9,10] proposed an alternative approach to fuzzy control chart: direct fuzzy approach. They used a direct fuzzy approach to fuzzy control chart for attributes under vague data using the probabilities of fuzzy events. Direct fuzzy approach does not require the use of defuzziffication and compare the linguistic data in fuzzy space without making any transformation. Faraz and Moghadam[11] introduced a fuzzy chart for controlling the process mean. They designed the fuzzy chart that had a warning line besides

upper control limit. Erginel[12] showed the theoretical structure of fuzzy individual and moving range control charts with α-cuts by using α-level fuzzy median transformation techniques. Sentürk and Erginel[13] presented the theoretical structure of fuzzy $\bar{\bar{X}} - \tilde{R}$ and $\bar{\bar{X}} - \tilde{S}$ control charts with α-cuts by using α-level fuzzy midrange transformation techniques. They used the triangular fuzzy membership functions to obtain the fuzzy $\bar{\bar{X}} - \tilde{R}$ and $\bar{\bar{X}} - \tilde{S}$ control charts. Sentürk[14] introduced a fuzzy structure of regression control charts named α-level fuzzy midrange for α-cut fuzzy $\bar{\bar{X}}$-regression control chart. Sentürk et al[15] developed fuzzy $\tilde{u}$ control charts. Kahraman et al[16] comparatively analyzed fuzzy statistical process control techniques in production system. Kaya and Kahraman[17] used the fuzzy set theory to add more information and flexibility to process capability analyses. For this aim, linguistic definition of the quality characteristic measurements were converted to fuzzy numbers and fuzzy control charts are derived for fuzzy measurements of the related quality characteristic. Fuzzy control charts were used to increase the accuracy of PCA by determining whether or not the process is in statistical control. Also, fuzzy set theory is used for multi-criteria decision making applications, like Kahraman and Tolga[18], and Pahlavani[19].

3. Calculation of fuzzy standard deviation

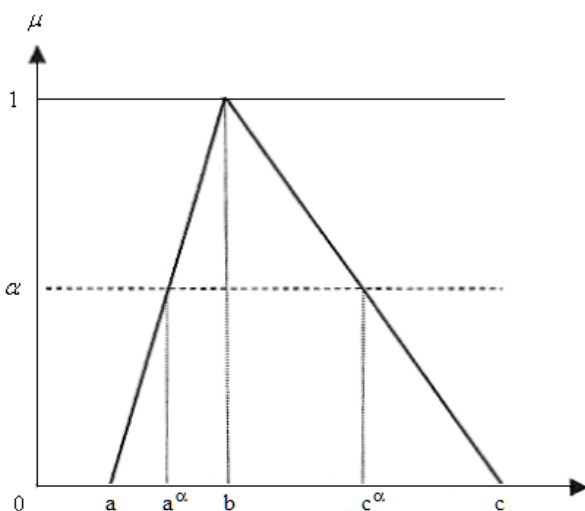A measurement of a certain characteristic is represented by a triangular fuzzy number (a, b, c) as shown in Figure 2[9].

Fig. 2. Representation of a sample by triangular fuzzy numbers.

In this study, triangular fuzzy number is represented by (X_a, X_b, X_c) for each fuzzy observation from process. Fuzzy arithmetic mean of fuzzy samples is also represented by $(\bar{X}_a, \bar{X}_b, \bar{X}_c)$, and fuzzy overall mean is denoted as $(\bar{\bar{X}}_a, \bar{\bar{X}}_b, \bar{\bar{X}}_c)$. Their formulations are given as follows:

$$\bar{X}_{a_j} = \frac{\sum_{i=1}^{n} X_{a_{ji}}}{n} \tag{1}$$

$$\bar{X}_{b_j} = \frac{\sum_{i=1}^{n} X_{b_{ji}}}{n} \tag{2}$$

$$\bar{X}_{c_j} = \frac{\sum_{i=1}^{n} X_{c_{ji}}}{n} \tag{3}$$

$$\bar{\bar{X}}_a = \frac{\sum_{j=1}^{m} \bar{X}_{a_j}}{m} \tag{4}$$

$$\bar{\bar{X}}_b = \frac{\sum_{j=1}^{m} \bar{X}_{b_j}}{m} \tag{5}$$

$$\bar{\bar{X}}_c = \frac{\sum_{j=1}^{m} \bar{X}_{c_j}}{m} \tag{6}$$

$$(\bar{\bar{X}}_a, \bar{\bar{X}}_b, \bar{\bar{X}}_c) = \left(\frac{\sum_{j=1}^{m} \bar{X}_{a_j}}{m}, \frac{\sum_{j=1}^{m} \bar{X}_{b_j}}{m}, \frac{\sum_{j=1}^{m} \bar{X}_{c_j}}{m} \right) \tag{7}$$

for $i = 1, 2, \cdots, n$, $j = 1, 2, \cdots, m$ where n is the size of fuzzy sample and m is the number of fuzzy samples.

The triangular fuzzy standard deviation is represented as (S_a, S_b, S_c) for each sample and the fuzzy average standard deviation is denoted as $(\bar{S}_a, \bar{S}_b, \bar{S}_c)$. The theoretical structure of fuzzy $\bar{\bar{X}}$ control chart and fuzzy $\tilde{S}$ control chart was first time

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presented by Sentürk and Erginel[13]. They give the formulation of standard deviation of *j*th sample ($\widetilde{S}_j$), and the average fuzzy standard deviation ($\widetilde{\overline{S}}$) by Eq. (8) and Eq. (9), respectively:

$$\widetilde{S}_j = \sqrt{\frac{\sum_{i=1}^{n}\left[(X_a,\ X_b,\ X_c)_{ij} - (\overline{X}_a,\ \overline{X}_b,\ \overline{X}_c)_j\right]^2}{n-1}} \qquad (8)$$

$$\widetilde{\overline{S}} = \left(\overline{S}_a,\ \overline{S}_b,\ \overline{S}_c\right) = \left(\frac{\sum_{j=1}^{m}S_{aj}}{m},\ \frac{\sum_{j=1}^{m}S_{bj}}{m},\ \frac{\sum_{j=1}^{m}S_{cj}}{m}\right) \qquad (9)$$

In the following we define the steps of another way of calculating standard deviation since the formulations given in Eqs. 8-9 are somewhat complex.

Step 1:
The fuzzy mean ($\overline{X}_a,\ \overline{X}_b,\ \overline{X}_c$) is calculated for each sample with sample size *n* by using Eqs. 1-3.

Step 2:
If there is no intersection between a TFN of the considered sample and the TFN of that sample's mean as in Figure 3, the distance ($d_{min}, d_{mod}, d_{max}$) between the fuzzy number ($X_a,\ X_b,\ X_c$) and the fuzzy mean ($\overline{X}_a,\ \overline{X}_b,\ \overline{X}_c$) are calculated as in Eqs. 10-12:

$$d_{min} = (X_a - \overline{X}_c) \qquad (10)$$

$$d_{mod} = (X_b - \overline{X}_b) \qquad (11)$$

$$d_{max} = (X_c - \overline{X}_a) \qquad (12)$$

where d_{min} is the least distance between the fuzzy numbers; d_{mod} is the distance between the mod of fuzzy number and the mod of fuzzy mean; and d_{max} is the largest distance between the fuzzy number and the fuzzy mean.
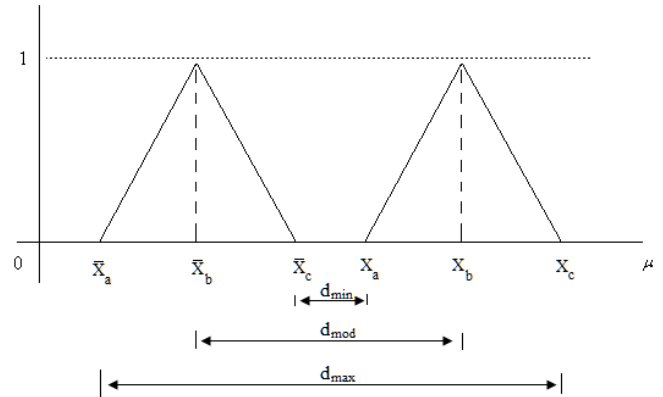

Fig. 3. The distances between fuzzy number and fuzzy mean : the fuzzy number is exactly on the right hand side of fuzzy average.

If the fuzzy number is exactly on the left hand side of the fuzzy average as in Figure 4, thus there is no any intersection between two membership functions, the distance ($d_{min}, d_{mod}, d_{max}$) between the fuzzy number ($X_a,\ X_b,\ X_c$) and the fuzzy average ($\overline{X}_a,\ \overline{X}_b,\ \overline{X}_c$) is calculated by Eqs. 13-15.

$$d_{min} = (\overline{X}_a - X_c) \qquad (13)$$

$$d_{mod} = (\overline{X}_b - X_b) \qquad (14)$$

$$d_{max} = (\overline{X}_c - X_a) \qquad (15)$$
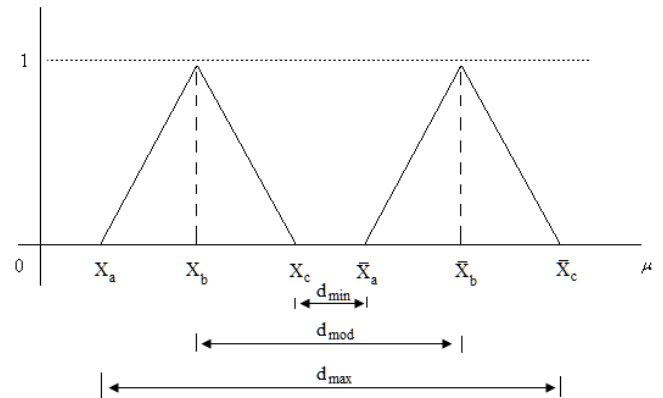
Fig. 4. The distances between fuzzy number and fuzzy mean : the fuzzy number is exactly on the left hand side of fuzzy average.

If there is an intersection (small or large, the size is not important) between membership functions of fuzzy numbers as in Figure 5, the distance ($d_{min}, d_{mod}, d_{max}$) between the fuzzy number ($X_a,\ X_b,\ X_c$) and the fuzzy average ($\overline{X}_a,\ \overline{X}_b,\ \overline{X}_c$) is calculated by Eqs. 16-18.

$$d_{\min} = 0 \qquad (16)$$

$$d_{\mathrm{mod}} = (X_b - \overline{X}_b) \qquad (17)$$

$$d_{\max} = \max\left\{(X_c - \overline{X}_a), (\overline{X}_c - X_a)\right\} \qquad (18)$$

where $d_{\min} = 0$.

There may be an intersection between the membership function of a fuzzy value in the sample and the membership function of the fuzzy sample mean. In this case, the distance of these values is equal to zero. This value indicates the minimum distance. Additionally, $d_{\max}$ represents the maximum distance between the fuzzy value in the sample and the fuzzy sample mean. So, the maximum of $(X_c - \overline{X}_a)$ and $(\overline{X}_c - X_a)$ should be chosen.
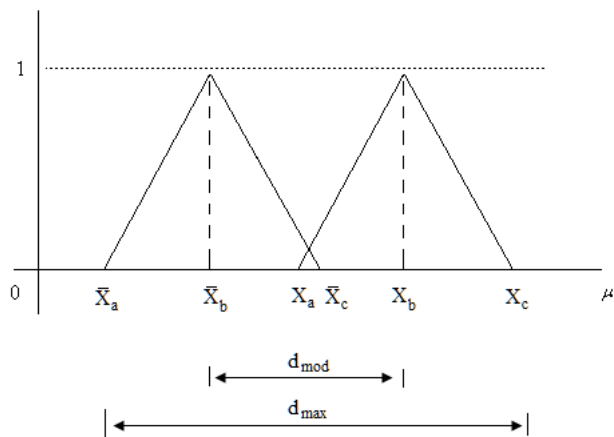

Fig. 5. The mod and the largest distance of the fuzzy number value and the fuzzy mean.

Step 3: $(d_{\min}, d_{\mathrm{mod}}, d_{\max})$ are calculated for each value within the sample, being one of the three cases in Step2. Using these values, the fuzzy standard deviation is calculated as follows:

$$S_{a,j} = \sqrt{\frac{\sum_{i=1}^{n} d_{\min,i}^2}{n-1}} \qquad (19)$$

$$S_{b,j} = \sqrt{\frac{\sum_{i=1}^{n} d_{\mathrm{mod},i}^2}{n-1}} \qquad (20)$$

$$S_{c,j} = \sqrt{\frac{\sum_{i=1}^{n} d_{\max,i}^2}{n-1}} \qquad (21)$$

where $i = 1, 2, \ldots, n$ and $j = 1, 2, \cdots, m$. The average fuzzy standard deviation $(\overline{S}_a, \overline{S}_b, \overline{S}_c)$ is calculated as follows:

$$\overline{S}_a = \frac{\sum_{j=1}^{m} S_{a,j}}{m} \qquad (22)$$

$$\overline{S}_b = \frac{\sum_{j=1}^{m} S_{b,j}}{m} \qquad (23)$$

$$\overline{S}_c = \frac{\sum_{j=1}^{m} S_{c,j}}{m} \qquad (24)$$

After obtaining the fuzzy overall mean $(\overline{\overline{X}}_a, \overline{\overline{X}}_b, \overline{\overline{X}}_c)$ and the average fuzzy standard deviation $(\overline{S}_a, \overline{S}_b, \overline{S}_c)$, the theoretical structure of fuzzy $\overline{\overline{X}}$ and $\widetilde{S}$ control charts are given in the following section.

4. Fuzzy $\overline{\overline{X}}$ *and* $\widetilde{S}$ **control charts**

4.1. *μ and σ are not known*

The packing process of biscuits can be monitored and controlled by $\overline{\overline{X}}$ and $\widetilde{S}$ control charts since the weight of biscuits is a quantitative random variable and a standard deviation considers all values in a sample.

Upper and lower limits are the two rigid lines in traditional control charts with no flexibility. The fuzzy set theory is an excellent approach to provide flexibility on control limits. So, the fuzzy control charts can be used instead of the traditional control charts. In this application, the packing process is controlled by fuzzy $\overline{\overline{X}} - \widetilde{S}$ control charts and the sample size is 5.

The traditional $\overline{X}$ control limits are given as follows:

$$UCL_{\overline{X}} = \overline{\overline{X}} + A_3 \overline{S} \qquad (25)$$

$$CL_{\overline{X}} = \overline{\overline{X}} \tag{26}$$

$$LCL_{\overline{X}} = \overline{\overline{X}} - A_3\overline{S} \tag{27}$$

where, A_3 is a control chart coefficient[20]. $\overline{S}$ is calculated by the following equations:

$$S_j = \sqrt{\frac{\sum_{i=1}^{n}(X_{ij} - \overline{X}_j)^2}{n-1}} \tag{28}$$

$$\overline{S} = \frac{\sum_{j=1}^{m} S_j}{m} \tag{29}$$

where S_j is the standard deviation of sample j and $\overline{S}$ is the average of S_j's, n is the sample size and m is the number of samples.

The theoretical structure of fuzzy $\widetilde{\overline{X}} - \widetilde{S}$ control charts with α-cuts by using α-level fuzzy midrange transformation technique was introduced by Şentürk and Erginel[13]. The proposed calculation method of fuzzy standard deviation in Section 3 is incorporated into control charts in the following subsections.

4.1.1. Fuzzy $\widetilde{\overline{X}}$ control chart

The control limits of fuzzy $\widetilde{\overline{X}}$ control chart with standard deviation $\widetilde{S}$ are obtained as follows:

$$U\widetilde{CL}_{\overline{x}} = C\widetilde{L}_{\overline{X}} + A_3\overline{S} = (\overline{\overline{X}}_a, \overline{\overline{X}}_b, \overline{\overline{X}}_c) + A_3(\overline{S}_a, \overline{S}_b, \overline{S}_c)$$
$$= (\overline{\overline{X}}_a + A_3\overline{S}_a, \overline{\overline{X}}_b + A_3\overline{S}_b, \overline{\overline{X}}_c + A_3\overline{S}_c) \tag{30}$$

$$C\widetilde{L}_{\overline{X}} = (\overline{\overline{X}}_a, \overline{\overline{X}}_b, \overline{\overline{X}}_c) \tag{31}$$

$$L\widetilde{CL}_{\overline{x}} = C\widetilde{L}_{\overline{X}} - A_3\overline{S} = (\overline{\overline{X}}_a, \overline{\overline{X}}_b, \overline{\overline{X}}_c) - A_3(\overline{S}_a, \overline{S}_b, \overline{S}_c)$$
$$= (\overline{\overline{X}}_a - A_3\overline{S}_c, \overline{\overline{X}}_b - A_3\overline{S}_b, \overline{\overline{X}}_c - A_3\overline{S}_a) \tag{32}$$

4.1.2. α-cut fuzzy $\widetilde{\overline{X}}$ control chart

α-cut approximation is widely used in the fuzzy set theory. α-cut fuzzy $\widetilde{\overline{X}}$ control chart limits with standard deviation are obtained as follows:

$$U\widetilde{CL}_{\overline{x}}{}^{\alpha} = (\overline{\overline{X}}_a^{\alpha}, \overline{\overline{X}}_b^{\alpha}, \overline{\overline{X}}_c^{\alpha}) + A_3(\overline{S}_a^{\alpha}, \overline{S}_b, \overline{S}_c^{\alpha})$$
$$= (\overline{\overline{X}}_a^{\alpha} + A_3\overline{S}_a^{\alpha}, \overline{\overline{X}}_b + A_3\overline{S}_b, \overline{\overline{X}}_c^{\alpha} + A_3\overline{S}_c^{\alpha}) \tag{33}$$

$$C\widetilde{L}_{\overline{x}}{}^{\alpha} = (\overline{\overline{X}}_a^{\alpha}, \overline{\overline{X}}_b, \overline{\overline{X}}_c^{\alpha}) \tag{34}$$

$$L\widetilde{CL}_{\overline{x}}{}^{\alpha} = (\overline{\overline{X}}_a^{\alpha}, \overline{\overline{X}}_b, \overline{\overline{X}}_c^{\alpha}) - A_3(\overline{S}_a^{\alpha}, \overline{S}_b, \overline{S}_c^{\alpha})$$
$$= (\overline{\overline{X}}_a^{\alpha} - A_3\overline{S}_c^{\alpha}, \overline{\overline{X}}_b - A_3\overline{S}_b, \overline{\overline{X}}_c^{\alpha} - A_3\overline{S}_a^{\alpha}) \tag{35}$$

where;

$$\overline{\overline{X}}_a^{\alpha} = \overline{\overline{X}}_a + \alpha(\overline{\overline{X}}_b - \overline{\overline{X}}_a) \tag{36}$$

$$\overline{\overline{X}}_c^{\alpha} = \overline{\overline{X}}_c - \alpha(\overline{\overline{X}}_c - \overline{\overline{X}}_b) \tag{37}$$

and

$$\overline{S}_a^{\alpha} = \overline{S}_a + \alpha(\overline{S}_b - \overline{S}_a) \tag{38}$$

$$\overline{S}_c^{\alpha} = \overline{S}_c - \alpha(\overline{S}_c - \overline{S}_b) \tag{39}$$

4.1.3 α-level fuzzy midrange for α-cut fuzzy $\widetilde{\overline{X}}$ control chart based on standard deviation

The control limits and center line for α-cut fuzzy $\widetilde{\overline{X}}$ control chart based on standard deviation using α-level fuzzy midrange are:

$$UCL_{mr-\overline{X}}^{\alpha} = CL_{mr-\widetilde{\overline{X}}}^{\alpha} + A_3\left(\frac{\overline{S}_a^{\alpha} + \overline{S}_c^{\alpha}}{2}\right) \tag{40}$$

$$CL_{mr-\overline{X}}^{\alpha} = f_{mr-\overline{X}}^{\alpha}(C\widetilde{L}) = \frac{\overline{\overline{X}}_a^{\alpha} + \overline{\overline{X}}_c^{\alpha}}{2} \tag{41}$$

$$LCL_{mr-\overline{X}}^{\alpha} = CL_{mr-\overline{X}}^{\alpha} - A_3\left(\frac{\overline{S}_a^{\alpha} + \overline{S}_c^{\alpha}}{2}\right) \tag{42}$$

The definition of α-level fuzzy midrange of the average of sample j for fuzzy $\widetilde{\overline{X}}$ control chart is

$$S_{mr-\overline{x},j}^{\alpha} = \frac{(\overline{X}_{a_j} + \overline{X}_{c_j}) + \alpha[(\overline{X}_{b_j} - \overline{X}_{a_j}) - (\overline{X}_{c_j} - \overline{X}_{b_j})]}{2} \tag{43}$$

The condition of process control for each sample is defined as:

$$Pr\,ocess\ control =$$
$$\begin{Bmatrix} under\ control \quad , for\ LCL^{\alpha}_{mr-\overline{X}} \le S^{\alpha}_{mr-\overline{X},j} \le UCL^{\alpha}_{mr-\overline{X}} \\ out-of\ control\,, for\ otherwise \end{Bmatrix}$$
(44)

After obtaining α-cut fuzzy $\widetilde{\overline{X}}$ control chart with standard deviation using α-level fuzzy midrange, fuzzy $\widetilde{S}$ control chart are presented as follows.

4.1.4 *Fuzzy $\widetilde{S}$ control chart*

The traditional S control chart is given by the following equations:

$$UCL_s = B_4\overline{S} \tag{45}$$

$$CL_s = \overline{S} \tag{46}$$

$$LCL_s = B_3\overline{S} \tag{47}$$

where B_3 and B_4 are control chart coefficients[18].

Fuzzy $\widetilde{S}$ control chart limits are obtained fuzzy average of standard deviation as follows:

$$U\widetilde{C}L_S = B_4(\overline{S}_a,\ \overline{S}_b,\ \overline{S}_c) \tag{48}$$

$$\widetilde{CL}_S = (\overline{S}_a,\ \overline{S}_b,\ \overline{S}_c) \tag{49}$$

$$L\widetilde{C}L_s = B_3(\overline{S}_a,\ \overline{S}_b,\ \overline{S}_c) \tag{50}$$

4.1.5 *α-cut fuzzy $\widetilde{S}$ control chart*

When integrating α-cut approximation to the fuzzy $\widetilde{S}$ control chart, the control limits of α-cut fuzzy $\widetilde{S}$ control chart are obtained as follows:

$$U\widetilde{C}L_S^{\alpha} = B_4(\overline{S}_a^{\alpha},\ \overline{S}_b,\ \overline{S}_c^{\alpha}) \tag{51}$$

$$\widetilde{CL}_S^{\alpha} = (\overline{S}_a^{\alpha},\ \overline{S}_b,\ \overline{S}_c^{\alpha}) \tag{52}$$

$$L\widetilde{C}R_S^{\alpha} = B_3(\overline{S}_a^{\alpha},\ \overline{S}_b,\ \overline{S}_c^{\alpha}) \tag{53}$$

where $(\overline{S}_a^{\alpha},\ \overline{S}_c^{\alpha})$ are calculated by using Eqs. 38-39.

4.1.6. *α-level fuzzy midrange for α-cut fuzzy $\widetilde{S}$ control chart*

The control limits of α-level fuzzy midrange for α-cut fuzzy $\widetilde{S}$ control chart are obtained by the following equations:

$$UCL^{\alpha}_{mr-s} = B_4\ f^{\alpha}_{mr-S}(\widetilde{CL}) \tag{54}$$

$$CL^{\alpha}_{mr-S} = f^{\alpha}_{mr-S}(\widetilde{CL}) = \frac{\overline{S}_a^{\alpha} + \overline{S}_c^{\alpha}}{2} \tag{55}$$

$$LCL^{\alpha}_{mr-S} = B_3\ f^{\alpha}_{mr-S}(\widetilde{CL}) \tag{56}$$

The α-level fuzzy midrange of standard deviation of sample j for fuzzy $\widetilde{S}$ control chart is

$$S^{\alpha}_{mr-S,j} = \frac{(S_{aj} + S_{cj}) + \alpha[(S_{bj} - S_{aj}) - (S_{cj} - S_{bj})]}{2} \tag{57}$$

The condition of process control for each sample is defined as

$$Pr\,ocess\ control =$$
$$\begin{Bmatrix} under\ control \quad , for\ LCL^{\alpha}_{mr-S} \le S^{\alpha}_{mr-S,j} \le UCL^{\alpha}_{mr-S} \\ out-of\ control\,, for\ otherwise \end{Bmatrix}$$
(58)

4.2. *Fuzzy $\widetilde{\overline{X}}$ control chart when μ and σ are known*

If the population parameters μ and σ are known from past data, these parameters can be used to construct the traditional $\overline{X}$ control charts as follows[21]:

$$UCL_{\overline{x}} = \mu + A\sigma \tag{59}$$

$$CL_{\overline{x}} = \mu \tag{60}$$

$$OCL_{\overline{x}} = \mu - A\sigma \tag{61}$$

In fuzzy case, fuzzy $\widetilde{\overline{X}}$ control limits are obtained as in the following:

$$U\widetilde{C}L_{\overline{x}} = (\mu_a,\ \mu_b,\ \mu_c) + A(\sigma_a,\ \sigma_b,\ \sigma_c)$$
$$= (\mu_a + A\sigma_a,\ \mu_b + A\sigma_b,\ \mu_c + A\sigma_c) \tag{62}$$

$$\widetilde{CL}_{\overline{X}} = (\mu_a,\ \mu_b,\ \mu_c) \tag{63}$$

$$L\widetilde{CL}_{\overline{x}} = (\mu_a, \mu_b, \mu_c) - A(\sigma_a, \sigma_b, \sigma_c)$$
$$= (\mu_a - A\sigma_c, \mu_b - A\sigma_b, \mu_c - A\sigma_a) \quad (64)$$

where (μ_a, μ_b, μ_c) is the fuzzy mean of a population; $(\sigma_a, \sigma_b, \sigma_c)$ is the fuzzy standard deviation of a population, and A is the constant based on sample size n.

4.2.1. α-cut fuzzy $\widetilde{\overline{X}}$ control chart when μ and σ are known

α-cut approximation is combined with fuzzy $\widetilde{\overline{X}}$ control chart as follows:

$$U\widetilde{CL}_{\overline{x}}^{\alpha} = (\mu_a^{\alpha}, \mu_b, \mu_c^{\alpha}) + A(\sigma_a^{\alpha}, \sigma_b, \sigma_c^{\alpha})$$
$$= (\mu_a^{\alpha} + A\sigma_a^{\alpha}, \mu_b + A\sigma_b, \mu_c^{\alpha} + A\sigma_c^{\alpha}) \quad (65)$$

$$\widetilde{CL}_{\overline{x}}^{\alpha} = (\mu_a^{\alpha}, \mu_b, \mu_c^{\alpha}) \quad (66)$$

$$L\widetilde{CL}_{\overline{x}}^{\alpha} = (\mu_a^{\alpha}, \mu_b, \mu_c^{\alpha}) - A(\sigma_a^{\alpha}, \sigma_b, \sigma_c^{\alpha})$$
$$= (\mu_a^{\alpha} - A\sigma_c^{\alpha}, \mu_b - A\sigma_b, \mu_c^{\alpha} - A\sigma_a^{\alpha}) \quad (67)$$

where

$$\mu_a^{\alpha} = \mu_a + \alpha(\mu_b - \mu_a) \quad (68)$$

$$\mu_c^{\alpha} = \mu_c - \alpha(\mu_c - \mu_b) \quad (69)$$

and

$$\sigma_a^{\alpha} = \sigma_a + \alpha(\sigma_b - \sigma_a) \quad (70)$$

$$\sigma_c^{\alpha} = \sigma_c - \alpha(\sigma_c - \sigma_b) \quad (71)$$

4.2.2. . α-level fuzzy midrange for α-cut fuzzy $\widetilde{\overline{X}}$ control chart when μ and σ are known

By using α-level fuzzy midrange, the control limits and center line for α-cut fuzzy $\widetilde{\overline{X}}$ control chart are obtained as follows:

$$UCL_{mr-\overline{X}}^{\alpha} = CL_{mr-\overline{X}}^{\alpha} + A(\frac{\sigma_a^{\alpha} + \sigma_c^{\alpha}}{2}) \quad (72)$$

$$CL_{mr-\overline{X}}^{\alpha} = f_{mr-\overline{X}}^{\alpha}(\widetilde{CL}) = \frac{\mu_a^{\alpha} + \mu_c^{\alpha}}{2} \quad (73)$$

$$LCL_{mr-\overline{X}}^{\alpha} = CL_{mr-\overline{X}}^{\alpha} - A(\frac{\sigma_a^{\alpha} + \sigma_c^{\alpha}}{2}) \quad (74)$$

The α-level fuzzy midrange of sample j for fuzzy $\widetilde{\overline{X}}$ control chart is

$$S_{mr-\overline{x},j}^{\alpha} = \frac{(\overline{X}_{a_j} + \overline{X}_{c_j}) + \alpha[(\overline{X}_{b_j} - \overline{X}_{a_j}) - (\overline{X}_{c_j} - \overline{X}_{b_j})]}{2}$$
$$(75)$$

The condition of process control for each sample is defined as:

$$\text{Pr}\,ocess\ control =$$
$$\begin{cases} under\ control & , for\ LCL_{mr-\overline{X}}^{\alpha} \le S_{mr-\overline{x},j}^{\alpha} \le UCL_{mr-\overline{X}}^{\alpha} \\ out-of\ control & , for\ otherwise \end{cases}$$
$$(76)$$

5. Application

The application of the developed fuzzy control charts was applied in a biscuit factory in Istanbul Turkey. The factory is one of the largest four in Turkey. Biscuits are packed with a nominal weight of 100 gr. The packing process of biscuits was controlled with fuzzy control charts. Fuzzy data were collected as fuzzy numbers; the sample size was five and the total sample number was twelve. All the fuzzy observations of the first sample are given in Table 1. The fuzzy averages and the fuzzy standard deviations for all the samples, fuzzy overall means and averages of standard deviations are given in Table 1.

Table 1 The fuzzy parameters of the weights of biscuit packets

A.   Sample means and distances

Sample	X_a	X_b	X_c	$d_{\min}$	d_{mod}	$d_{\max}$
S1	$\overline{X}_{a,1}=97.54$	$\overline{X}_{b,1}=99.66$	$\overline{X}_{c,1}=100.56$	$S_{a,1}=0.68$	$S_{b,1}=2.11$	$S_{c,1}=4.62$
S2	$\overline{X}_{a,2}=98.64$	$\overline{X}_{b,2}=100.40$	$\overline{X}_{c,2}=101.34$	$S_{a,2}=0.02$	$S_{b,2}=1.41$	$S_{c,2}=3.11$
S3	$\overline{X}_{a,3}=97.8$	$\overline{X}_{b,3}=101.40$	$\overline{X}_{c,3}=102.58$	$S_{a,3}=0.16$	$S_{b,3}=1.74$	$S_{c,3}=7.24$
S4	$\overline{X}_{a,4}=96.52$	$\overline{X}_{b,4}=98.52$	$\overline{X}_{c,4}=99.48$	$S_{a,4}=0.06$	$S_{b,4}=1.51$	$S_{c,4}=4.38$
S5	$\overline{X}_{a,5}=99.68$	$\overline{X}_{b,5}=102.66$	$\overline{X}_{c,5}=103.50$	$S_{a,5}=0.00$	$S_{b,5}=0.53$	$S_{c,5}=5.25$
S6	$\overline{X}_{a,6}=100.00$	$\overline{X}_{b,6}=100.40$	$\overline{X}_{c,6}=101.98$	$S_{a,6}=0.00$	$S_{b,6}=0.59$	$S_{c,6}=2.94$
S7	$\overline{X}_{a,7}=100.00$	$\overline{X}_{b,7}=100.58$	$\overline{X}_{c,7}=102.22$	$S_{a,7}=0.59$	$S_{b,7}=1.45$	$S_{c,7}=4.28$
S8	$\overline{X}_{a,8}=98.64$	$\overline{X}_{b,8}=99.58$	$\overline{X}_{c,8}=101.00$	$S_{a,8}=0.00$	$S_{b,8}=0.69$	$S_{c,8}=3.72$
S9	$\overline{X}_{a,9}=99.74$	$\overline{X}_{b,9}=100.98$	$\overline{X}_{c,9}=101.94$	$S_{a,9}=0.50$	$S_{b,9}=2.15$	$S_{c,9}=4.38$
S10	$\overline{X}_{a,10}=96.28$	$\overline{X}_{b,10}=98.16$	$\overline{X}_{c,10}=99.38$	$S_{a,10}=0.00$	$S_{b,10}=0.48$	$S_{c,10}=4.16$
S11	$\overline{X}_{a,11}=97.14$	$\overline{X}_{b,11}=98.50$	$\overline{X}_{c,11}=99.36$	$S_{a,11}=0.12$	$S_{b,11}=1.32$	$S_{c,11}=3.53$
S12	$\overline{X}_{a,12}=99.48$	$\overline{X}_{b,12}=100.14$	$\overline{X}_{c,12}=101.46$	$S_{a,12}=0.07$	$S_{b,12}=1.24$	$S_{c,12}=3.17$
Overall mean	$\overline{\overline{X}}_a=98.46$	$\overline{\overline{X}}_b=100.08$	$\overline{\overline{X}}_c=101.23$	$\overline{S}_a=0.18$	$\overline{S}_b=1.27$	$\overline{S}_c=4.23$

B.   Fuzzy values of Sample 1

Sample	X_a	X_b	X_c	$d_{\min}$	d_{mod}	$d_{\max}$
S1-1	95.8	96.2	98.9	1.36	3.46	4.76
S1-2	98.1	101.7	101.9	0.00	2.04	4.36
S1-3	100.4	100.9	101.2	0.16	1.24	3.66
S1-4	96.2	100	100.3	0.00	0.34	4.36
S1-5	97.2	99.5	100.5	0.00	0.16	3.36

Application of Fuzzy $\widetilde{\overline{X}}$ control chart with standard deviation

After calculating the fuzzy parameters of the weights of biscuit packets, the limits of fuzzy $\widetilde{\overline{X}}$ control chart with standard deviation have been obtained as follows:

$$
\begin{aligned}
U\widetilde{C}L_{\bar{x}} &= (\overline{\overline{X}}_a + A_3\overline{S}_a,\ \overline{\overline{X}}_b + A_3\overline{S}_b,\ \overline{\overline{X}}_c + A_3\overline{S}_c) \\
&= 98.46 + (1.427)\,0.18,\ 100.8 + (1.427)\,1.27, \\
&\quad 101.23 + (1.427)\,4.23 \\
&= (98.71,\ 102.61, 107.26)
\end{aligned} \tag{77}
$$

$$
\widetilde{CL}_{\overline{X}} = (\overline{\overline{X}}_a,\ \overline{\overline{X}}_b,\ \overline{\overline{X}}_c) = (98.46,\ 100.8,\ 101.23) \tag{78}
$$

$$
\begin{aligned}
L\widetilde{C}L_{\bar{x}} &= (\overline{\overline{X}}_a - A_3\overline{S}_c,\ \overline{\overline{X}}_b - A_3\overline{S}_b,\ \overline{\overline{X}}_c - A_3\overline{S}_a) \\
&= 98.46 - (1.427)\,4.23,\ 100.8 - (1.427)\,1.27, \\
&\quad 101.23 - (1.427)\,0.18 \\
&= (92.42,\ 98.98, 100.9)
\end{aligned} \tag{79}
$$

where $A_3 = 1.427$ for $n = 5$.

α-cut fuzzy $\widetilde{\overline{X}}$ control chart limits with standard deviation are obtained as follows:

$$
\begin{aligned}
U\widetilde{C}L_{\bar{x}}^{0.55} &= (\overline{\overline{X}}_a^{0.55} + A_3\overline{S}_a^{0.55},\ \overline{\overline{X}}_b + A_3\overline{S}_b, \\
&\quad \overline{\overline{X}}_c^{0.55} + A_3\overline{S}_c^{0.55}) \\
&= (99.75 + (1.427)0.78,\ 100.8 + (1.427)1.27, \\
&\quad 100.99 + (1.427)2.60) \\
&= (100.86, 102.61, 104.70)
\end{aligned} \tag{80}
$$

$$
\begin{aligned}
\widetilde{CL}_{\bar{x}}^{0.55} &= (\overline{\overline{X}}_a^{0.55},\ \overline{\overline{X}}_b,\ \overline{\overline{X}}_c^{0.55}) \\
&= (99.75,\ 100.8, 100.99)
\end{aligned} \tag{81}
$$

$$
\begin{aligned}
L\widetilde{C}L_{\bar{x}}^{0.55} &= (\overline{\overline{X}}_a^{0.55} - A_3\overline{S}_c^{0.55},\ \overline{\overline{X}}_b - A_3\overline{S}_b, \\
&\quad \overline{\overline{X}}_c^{0.55} - A_3\overline{S}_a^{0.55}) \\
&= (99.75 - (1.427)2.60,\ 100.8 - (1.427)1.27, \\
&\quad 100.99 - (1.427)0.78) \\
&= (96.04,\ 98.98, 99.88)
\end{aligned} \tag{82}
$$

where $\alpha = 0.55$. α-cut represents the cutting point of the membership. The values of α-cut are determined as 0.65 in metal processing in the literature. Due to the sensitivity of packing process of biscuits is less than metal processing, α-cut is selected less than 0.65 in this study.

$$
\begin{aligned}
\overline{\overline{X}}_a^{\alpha} &= \overline{\overline{X}}_a + \alpha(\overline{\overline{X}}_b - \overline{\overline{X}}_a) \\
&= 98.46 + 0.55(100.8 - 98.46) = 99.75
\end{aligned} \tag{83}
$$

$$
\begin{aligned}
\overline{\overline{X}}_c^{0.55} &= \overline{\overline{X}}_c - \alpha(\overline{\overline{X}}_c - \overline{\overline{X}}_b) \\
&= 101.23 - 0.55(101.23 - 100.8) = 100.99
\end{aligned} \tag{84}
$$

and

$$
\begin{aligned}
\overline{S}_a^{0.55} &= \overline{S}_a + \alpha(\overline{S}_b - \overline{S}_a) \\
&= 0.18 + 0.55(1.27 - 0.18) = 0.78
\end{aligned} \tag{85}
$$

$$
\begin{aligned}
\overline{S}_c^{0.55} &= \overline{S}_c - \alpha(\overline{S}_c - \overline{S}_b) \\
&= 4.23 - 0.55(4.23 - 1.27) = 2.60
\end{aligned} \tag{86}
$$

The control limits and center line for α-cut fuzzy $\widetilde{\overline{X}}$ control chart with standard deviation using α-level fuzzy midrange are

$$
\begin{aligned}
UCL_{mr-\overline{X}}^{\alpha} &= CL_{mr-\overline{\overline{X}}}^{\alpha} + A_3\left(\frac{\overline{S}_a^{\alpha} + \overline{S}_c^{\alpha}}{2}\right) \\
&= 100.37 + 1.427\left(\frac{0.78 + 2.60}{2}\right) = 102.78
\end{aligned} \tag{87}
$$

$$
\begin{aligned}
CL_{mr-\overline{X}}^{\alpha} &= f_{mr-\overline{X}}^{\alpha}(\widetilde{CL}) \\
&= \frac{\overline{\overline{X}}_a^{\alpha} + \overline{\overline{X}}_c^{\alpha}}{2} = \frac{99.75 + 100.99}{2} = 100.37
\end{aligned} \tag{88}
$$

$$
\begin{aligned}
LCL_{mr-\overline{X}}^{\alpha} &= CL_{mr-\overline{X}}^{\alpha} - A_3\left(\frac{\overline{S}_a^{\alpha} + \overline{S}_c^{\alpha}}{2}\right) \\
&= 100.37 - 1.427\left(\frac{0.78 + 2.60}{2}\right) = 97.95
\end{aligned} \tag{89}
$$

The α-level fuzzy midrange of sample j for fuzzy $\widetilde{\overline{X}}$ control chart, $S_{mr-\bar{x},j}^{\alpha}$ and the condition of process control for each sample are given in the second and third columns in Table 2.

Fuzzy $\widetilde{S}$ control chart

After evaluating the fuzzy averages with fuzzy $\widetilde{\overline{X}}$ control chart of the packing process of biscuits, the fuzzy standard deviation of the packing process of biscuits are given in the following:

$$
\begin{aligned}
U\widetilde{C}L_S &= B_4(\overline{S}_a,\ \overline{S}_b,\ \overline{S}_c) \\
&= 2.089\,(0.18, 1.27, 4.23) = (0.37, 2.65, 8.83)
\end{aligned} \tag{90}
$$

$$C\widetilde{L}_S = (\overline{S}_a, \overline{S}_b, \overline{S}_c) = (0.18, 1.27, 4.23) \qquad (91)$$

$$L\widetilde{CL}_s = B_3(\overline{S}_a, \overline{S}_b, \overline{S}_c)$$
$$= 0(0.18, 1.27, 4.23) = (0, 0, 0) \qquad (92)$$

where $B_4 = 2.089$ and $B_3 = 0$ for $n = 5$.

The control limits of α-cut fuzzy $\widetilde{S}$ control chart are obtained as follows:

$$U\widetilde{CL}_S^{0.55} = B_4(\overline{S}_a^{0.55}, \overline{S}_b, \overline{S}_c^{0.55})$$
$$= 2.089(0.78, 1.27, 2.60) = (1.63, 2.65, 5.43) \qquad (93)$$

$$C\widetilde{L}_S^{0.55} = (\overline{S}_a^{0.55}, \overline{S}_b, \overline{S}_c^{0.55})$$
$$= (0.78, 1.27, 2.60) \qquad (94)$$

$$L\widetilde{CR}_S^{0.55} = B_3(\overline{S}_a^{0.55}, \overline{S}_b, \overline{S}_c^{0.55})$$
$$= 0(0.78, 1.27, 2.60) = (0, 0, 0) \qquad (95)$$

The control limits of α-level fuzzy midrange for α-cut fuzzy $\widetilde{S}$ control chart are given in the following:

$$UCL_{mr-s}^{\alpha} = B_4 \, f_{mr-S}^{\alpha}(C\widetilde{L})$$
$$= 2.089(1.69) = 3.53 \qquad (96)$$

$$CL_{mr-S}^{\alpha} = f_{mr-S}^{\alpha}(C\widetilde{L})$$
$$= \frac{\overline{S}_a^{\alpha} + \overline{S}_c^{\alpha}}{2} = \frac{0.78 + 2.60}{2} = 1.69 \qquad (97)$$

$$LCL_{mr-S}^{\alpha} = B_3 \, f_{mr-S}^{\alpha}(C\widetilde{L})$$
$$= 0(1.69) = 0 \qquad (98)$$

As it is seen from Table 2, α-level fuzzy midranges of averages for all samples are "under control" according to the fuzzy $\overline{\overline{X}}$ control chart. Also, α-level fuzzy midranges of standard deviation for all samples are "under control" according to the fuzzy $\widetilde{S}$ control chart. So, it can be said that the packing process of biscuits are "under control". These control charts can be used for monitoring to this process.

Table 2 Control limits using α-level fuzzy midrange for α-cut fuzzy $\overline{\overline{X}}$ and $\widetilde{S}$ control charts

Sample no	$S_{mr-\overline{X},j}^{\alpha}$	$97.95 \le S_{mr-\overline{X},j}^{\alpha} \le 102.78$	$S_{mr-S,j}^{\alpha}$	$0 \le S_{mr-S,j}^{\alpha} \le 3.53$
1	99.4	Under control	2.35	Under control
2	100.2	Under control	1.48	Under control
3	100.9	Under control	2.62	Under control
4	98.3	Under control	1.83	Under control
5	102.2	Under control	1.47	Under control
6	100.7	Under control	0.98	Under control
7	100.8	Under control	1.89	Under control
8	99.7	Under control	1.21	Under control
9	100.9	Under control	2.28	Under control
10	98.0	Under control	1.20	Under control
11	98.4	Under control	1.55	Under control
12	100.3	Under control	1.41	Under control

6. Conclusion

Control charts in SPC have very common usage in practice. But if there is uncertainty or impression in data, the process should be controlled with the fuzzy sets. Fuzzy control charts provide flexibility to upper and lower limits.

There are two contribution of this paper:

1. A new way of calculating fuzzy standard deviation has been proposed in Section 3. So, the difficulty of calculating fuzzy standard deviation with fuzzy numbers is removed and it can be easily used in fuzzy $\widetilde{\overline{X}}$ and $\widetilde{S}$ control charts. Also, the application of fuzzy $\widetilde{\overline{X}}$ and $\widetilde{S}$ control charts has been illustrated in a food industry for evaluating the packing process of biscuits.

2. The theoretical structure of fuzzy $\widetilde{\overline{X}}$ and $\widetilde{S}$ control charts has been proposed for the case that the population parameters (μ and σ) are known.

For further research, other membership functions like trapezoidal or L-R type can be used in the developed control charts.

References

[1]. Zadeh, L.A. (1965).Fuzzy Sets. *Information and Control,* 8, 338-353.

[2]. Raz, T., Wang, J.H. (1990).Probabilistic and memberships approaches in the construction of control chart for linguistic data. *Production Planning and Control*, 1, 147.

[3]. Wang, J.H., Raz, T. (1990).On the construction of control charts using linguistic variables. *Intelligent Journal of Production Research*, 28, 477-487.

[4]. Kanagawa, A., Tamaki, F., Ohta, H. (1993). Control charts for process average and variability based on linguistic data. *Intelligent Journal of Production Research*, 31 (4), 913-922.

[5]. El-Shal, S.M., Morris A.S. (2000). A fuzzy rule-based algorithm to improve the performance of statistical process control in quality systems. *Journal of Intelligent Fuzzy Systems*, 9, 207-223.

[6]. Rowlands, H., Wang, L.R. (2000). An approach of fuzzy logic evaluation and control in SPC. *Quality Reliability Engineering Intelligent*, 16, 91-98.

[7]. Gülbay, M., Kahraman, C., Ruan, D. (2004). α-cut Fuzzy control charts for linguistic data. *International Journal of Intelligent Systems*, 19, 1173-1196.

[8]. Cheng, C.B., (2005). Fuzzy process control: construction of control charts with fuzzy numbers. *Fuzzy sets and Systems*, 154, 287-303.

[9]. Gülbay, M., Kahraman, C. (2006a). Development of fuzzy process control charts and fuzzy unnatural pattern analyses. *Computational Statistics and Data Analysis*, 51, 434-451.

[10]. Gülbay, M., Kahraman, C. (2006b).An alternative approach to fuzzy control charts: Direct fuzzy approach. *Information Sciences*, 77 (6), 1463-1480.

[11]. Faraz ,A., Moghadam, M.B. (2007). Fuzzy control chart a beter alternative for Shewhart Average Chart. *Quality & Quantity,* 41, 375-385.

[12]. Erginel, N. (2008). Fuzzy individual and moving range control charts with α-cuts. *Journal of Intelligent & Fuzzy Systems*, 19, 373-383.

[13]. Sentürk, S.,Erginel, N. (2009). Development of fuzzy $\widetilde{\overline{X}} - \widetilde{R}$ and $\widetilde{\overline{X}} - \widetilde{S}$ control charts using α -cuts *Information Sciences*, 179, 1542-1551.

[14]. Sentürk, S. (2010). Fuzzy Regression Control Chart Based on α-cut Approximation International Journal of Computational Intelligence Systems, Vol.3, No. 1, 123-140.

[15]. Sentürk S., Erginel N., Kaya,İ, Kahraman,C. Design of Fuzzy $\tilde{u}$ Control Chart, Journal of Multiple Valued- Logic, and Soft Computing (2010), pp. 1-15, (In press)

[16]. Kahraman,C. Gülbay M., Erginel N., Şentürk S. Fuzzy Statistical Process Control Techniques in Production Systems Production Engineering and Management Under Fuzziness, (Eds Cengiz Kahraman and Mesut Yavuz), Chapter 18, Springer-Verlag Berlin Heidelberg, 2010.

[17]. Kaya, İ., Kahraman, C., (2011). Process capability analyses based on fuzzy measurements and fuzzy control charts, *Expert Systems with Applications*, 38(4), 3172-3184.

[18]. Kahraman, C., Tolga . A.Ç. (2009). An Alternative Ranking Approach and Its Usage in Multi-Criteria Decision-Making, International Journal of Computational Intelligence Systems, Vol.2, No. 3, 219-235.

[19]. Pahlavani. A. (2010). A New Fuzzy MADM Approach and its Application to Project Selection Problem International Journal of Computational Intelligence Systems, Vol.3, No. 1, 103-114.

[20]. Kolarik, W.J. (1995).*Creating Quality- Concepts. Systems. Strategies and Tools*. McGraw Hill. 305.

[21]. Montgomery, D.C. (1991).Introduction to Statistical Quality Control. John Wiley & Sons.Inc. 351.