Long Memory Analysis of USD/TRL Exchange Rate

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Abstract**—**A time series has a long memory, in this case there is autocorrelation at long lags. If prices or rates display long memory, they show significant autocorrelation between observations widely separated in time. The characterization of real exchange rate series as random in nature has been questioned in recent times by the application of some new statistical tools. This paper analysis long memory of foreign exchange rate US Dollar (USD) against the New Turkish Lira (TRL). The KPSS statistic, the Modified R/S statistic and the modified variance V/S statistic are used to detect long memory property of the series. Application of these tests suggests that USD/TRL real exchange rate movement shows evidence of long memory.

Keywords**—**Long memory, Hurst exponent, KPSS statistic, Modified R/S statistic, V/S statistic.

I. Introduction

THE long memory of discrete time series has increased much attention in the recent literature. In this context, extensive mathematical research over the last years has focused on important issues such as stock exchange and foreign currency, in economics and finance.

The long memory, or long term dependence, property describes the high-order correlation structure of a series. If a series exhibits long memory, there is persistent temporal dependence even between distant observations. Such series are characterized by distinct but non-periodic cyclical patterns. The presence of long memory dynamics causes nonlinear dependence in the first moment of the distribution and hence a potentially predictable component in the series dynamics. Fractionally integrated processes can give rise to long memory.

A stationary stochastic process $\{Y_t\}$ is called a long memory process if there exist a real number H and a finite constant C such that the autocorrelation function $\rho(\tau)$ has the following rate of decay:

$$\rho(k) \sim C_{\tau}^{2H-2} \text{ as } \tau \to \infty \qquad (1)$$

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The parameter H, Hurst Exponent, display the long memory property of the time series. A long memory time series is said fractionally integrated, where the fractional degree of integration d is related to the parameter H as follows:

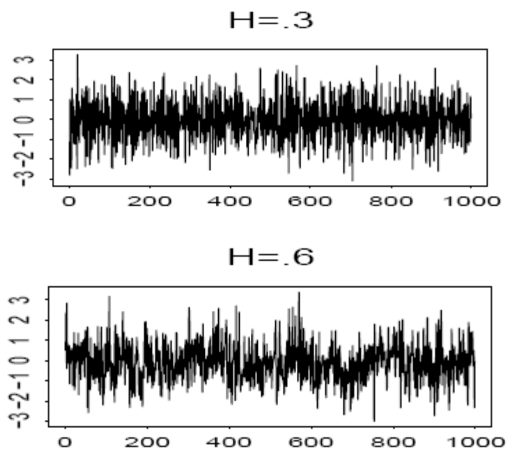
$$d = H - 1/2 \qquad (2)$$

The Hurst exponent takes values from 0 to 1 $(0 \le H \le 1)$. If $H = 0.5$, the series is a random walk (a Brownian time series). In a random walk there is no correlation between any element and a future element.

If $0.5 < H < 1$, the series indicates persistent behavior or long memory. If there is an increase from time step t_{i-1} to t_i there will be probably be an increase from t_i to t_{i+1}. on the other hand, the same is true for decreases. A decrease will tend to follow a decrease.

If $0 < H < 0.5$, the series is called anti-persistent. In this case, an increase will tend to be followed by a decrease or a decrease will be followed by an increase. This behavior is sometimes called mean reversion.

Brownian walks can be generated from a defined Hurst exponent. If the Hurst exponent is $0.5 < H < 1$, the random walk will be a long memory process. The time series like this is sometimes referred to as fractional Brownian motion.



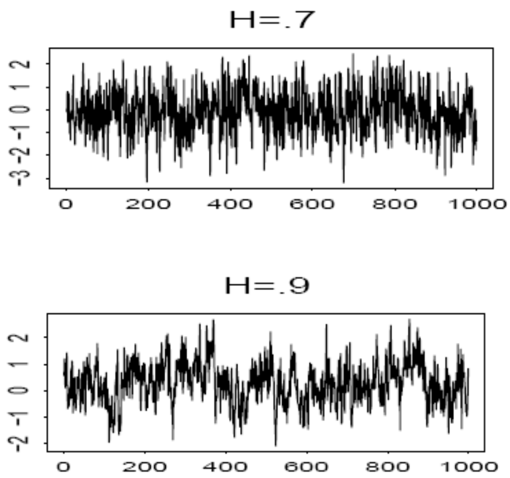


Fig. 1 Fractional motions at different Hurst Exponents respectively, H= 0.3, 0.6, 0.7, 0.9

As one compares the four plots on Fig. 1, it appears that, for larger values of H, the plots tend to indicate changing “level”, or the mean values, of the process at different time intervals.

II. Long Memory Analysis Of Real Exchange Rate Series

A. The Modified Rescaled Range (R/S) Analysis

The first test for long memory was used by the hydrologist Hurst (1951) for the design of an optimal reservoir for the Nile river, of where flow regimes were persistent. Hurst gave the following formula:

$$(R/S)_n = cn^H \tag{3}$$

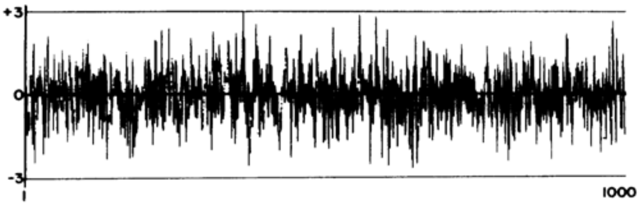
$(R/S)_n$ is the rescaled range statistic measured over a time index n, c is a constant and H the Hurst exponent. This shows the how the R/S statistic is scaling in time. The aim of the R/S statistic is to estimate the Hurst exponent which can characterize a series. Estimation of Hurst exponent can be done by transforming (3) to:

$$\log(R/S)_n = \log(c) + H\log(n) \tag{4}$$

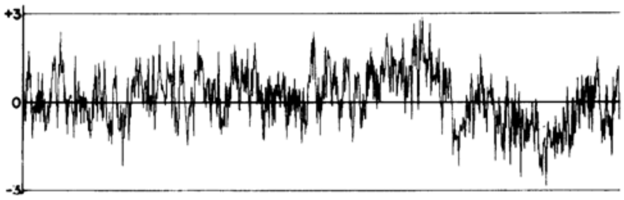
and H can be estimated as the slope of log/log plot of $(R/S)_n$ vs. *n*.

For a time series $\{X_t\}$ ($t = 1,\ldots,N$), the *R/S* statistic can be defined as the range of cumulative deviations from the mean of the series, rescaled by the standard deviation.

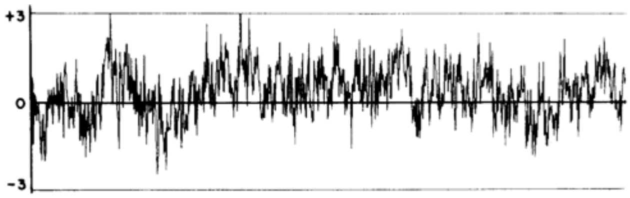
In the *R/S* sense, long memory or long term dependence may be described as extended periods of similar overall behavior that are of unequal duration (Mandelbrot, 1972). Within these periods, however, dependence need not exist. The nature of this type of dependency can be seen more clearly in Fig. 2.



(a) No persistent dependence (white noise)



(b) Moderate persistent dependence



(c) Strong persistent dependence

Fig. 2.(a)-(b)-(c) Three Series of 1000 Observations of zero mean and unit variance depicting different intensities of persistent dependence

A typical white noise process is displayed in Fig. 2(a). Visual inspection of this shape reveals that it is essentially featureless, especially with respect to “long fluctuations”. On the other hand, positive long term, or persistent, dependence is depicted in Fig. 2(b)-(c). In these figures, there are short-term random fluctuations, but “long fluctuations” of unequal duration are readily apparent. This persistence phenomenon is especially evident in Fig. 2(c).

The analytical procedure to estimate $(R/S)_n$ values can be described in the following steps:

Step1: The time period spanned by the time series of length *N* is divided into *m* contiguous subperiods of length *n* such that $m*n = N$. In each subperiod $X_{i,j}$, the elements have two subscripts. The first subscript ($i = 1,\ldots,n$) denotes the number of elements in each subperiod and the second one ($j = 1,\ldots,m$) denotes the subperiod index. For each subperiod *j* the *R/S* statistic is calculated as follows:

$$\left(\frac{R}{S}\right)_j = s_j^{-1}\left[\max_{1\le k\le n}\sum_{i=1}^{k}\left(X_{ij} - \bar{X}_j\right) - \min_{1\le k\le n}\sum_{i=1}^{k}\left(X_{ij} - \bar{X}_j\right)\right], \tag{5}$$

where s_j is the standard deviation for each subperiod.

In (4), the *k* deviations from the subperiod mean have zero mean, therefore the last value of the cumulative deviations for each subperiod will always be zero. Because of this, the maximum value of the cumulative deviations will be always be greater or equal to zero, while the minimum value will always be less or equal to zero. Thus the bracketed term in (5), range value, will be always non-negative.

Rescaling the range is crucial since it allows diverse phenomena and time periods to be compared, which means that *R/S* analysis can describe time series with no characteristic scale.

Step2: The $(R/S)_n$ is computed by the average of the $(R/S)_j$ values for all the *m* contiguous subperiods with length *n* as:

$$\left(\frac{R}{S}\right)_n = \frac{1}{m}\sum_{j=1}^{m}\left(\frac{R}{S}\right)_j . \tag{6}$$

Step 3: Eq. (5) computes the *R/S* value which corresponds to a certain time interval of length *n*. For applying Eq. (4), steps 1 and 2 are repeated by increasing *n* to the next integer value, until *n = N/2*, since at least two subperiods are needed, to avoid bias.

From these steps, it is obvious that the time dimension is included in the *R/S* analysis by examining whether the range of the cumulative deviations depends on the length of time used for the measurement. Once (6) is evaluated for different *n* periods, the Hurst exponent is estimated through an ordinary least square regression from (4).

Although Mandelbrot (1975) gave a formal justification for the use of this test, Lo(1991) showed that this statistic was not robust to short memory dependence and modified this statistic.

Lo defined modified R/S statistic as:

$$Q_T(q) = \frac{1}{\hat{\sigma}_T(q)}\left[\max_{1\le k\le T}\sum_{j=1}^{k}\left(X_j - \bar{X}_T\right) - \min_{1\le k\le T}\sum_{j=1}^{k}\left(X_j - \bar{X}_T\right)\right] \tag{7}$$

where,

$$\hat{\sigma}_T^2(q) = \hat{\gamma}_0 + 2\sum_{j=1}^{q}\left(1+\frac{j}{1+q}\right)\hat{\gamma}_j, \quad q < T \cdot \tag{8}$$

$\hat{\gamma}_0$ is the variance of the series and the sequence $\{\hat{\gamma}_j\}_{j=1}^{q}$ denotes the autocovariances of the series up to order q.

Lo shows that under certain conditions which place restrictions on the maximal moments, the degree of distributional heterogeneity and the maximal degree of dependence in $\{X_t\}$, the statistic $V_q = \tau^{-(1/2)}Q_\tau(q)$ converges to the range of a "Brownian bridge" on the unit interval.

The distribution of V_q is asymptotic to that of

$$W_1 = \max_{0\le t\le 1} W_0(t) - \min_{0\le t\le 1} W_0(t) \tag{9}$$

where W_0 is a standard Brownian bridge $W_0(t) = B(t) - tB(1)$, where B denotes standard Brownian motion. Since the distribution of the random variable W_1 is known,

$$P[W_1 \le x] = 1 - 2\sum_{n=1}^{\infty}\left(4x^2n^2 - 1\right)e^{-2x^2n^2} \tag{10}$$

it follows that

$$P\{W_1 \in [0.809, 1.862]\} = 0.95. \tag{11}$$

Lo uses the interval [0.809,1.862] as the %95 (asymptotic) acceptance region for testing the null hypothesis

H_0 = { no long-term dependence, i.e., *H* = 0.5}

against the composite alternative

H_1 = { there is long-term dependence, i.e., 0.5< *H* <1}.

The critical values of the test derived by the asymptotic cumulative distribution function are given in Table I.

TABLE I
ASYMPTOTIC CRITICAL VALUES OF THE MODIFIED R/S STATISTIC

Probability level	Critical value
0.5%	0.721
2.5%	0.809
5%	0.861
10%	0.927
90%	1.620
95%	1.747
97.5%	1.862
99.5%	2.098

The main advantage of this test is that it permits for formal statistical testing and is robust against serial correlation and some forms of non-stationarity.

The main disadvantage of the test is that unlike the classic *R/S* analysis is not able to specify the cycle length of the series tested.

There is a problem that is related to the sensitivity of the test to the truncation parameter *q*. If q = 0, Lo's statistic reduces to Hurst's R/S statistic. This statistic is highly sensitive to the order of truncation q but there is no a statistical criteria for choosing q in the framework of this statistic. If q is too small, this statistic does not account for the autocorrelation of the process, while if q is too large, it accounts for any form of autocorrelation and the power of this test tends to its size. Given that the power of a useful test should be greater than its size; this statistic is not very helpful. For that reason, Teverovsky et al. (1999) suggest to use this statistic with other tests.

Since there is no data driven guidance for the choice of this parameter, the default values for q = 2, 4, 6, 8, 10 are considered. At 5% significance level, the null hypothesis of no long memory process is rejected if the modified R/S statistic does not fall within the confidence interval [0.809, 1.862].

B. *The KPSS Statistic*

The KPSS statistic (Kwiatkowski, Phillips, Schmidt and Shin) is often used for testing the null hypothesis of stationarity against the alternative of unit root. This test has a power equivalent to modified *R/S* statistic against to long memory processes and can therefore be used to distinguish between short and long memory processes. This test is similar to Lo's modified *R/S* statistic in power and construction.

The two KPSS statistics η_t and η_μ are respectively based on the residuals of two regression models. In these regression models, *t* is an intercept and a trend, and μ is a constant.

The partial sums $S_t = \sum_{i=1}^{t} \hat{e}_i$ is denoted by S_t, where $\hat{e}_t$ are the residuals of these regressions, the KPSS statistic is defined by:

$$\eta = T^{-2} \sum S_t^2 / \hat{\sigma}_T^2(q) \tag{12}$$

$\hat{\sigma}_T^2(q)$ is the estimator of the variance of residuals defined in equation (8). The statistic η_t tests for trend-stationarity against a long memory alternative, while the statistic η_μ tests for stationarity against a long memory alternative.

Under the null hypothesis of I(0), this statistic asymptotically converges to a well defined random variable $U = \int_0^1 \left(W^0(t)\right)^2 dt$, where $W^0(t)$ is the Brownian bridge defined as $W(t) - tW(1)$, $W(t)$ being the standardized Wiener process.

This test is evaluated for lag orders 0, 2 and 4. Critical values are as follow:

TABLE II
KPSS TEST CRITICAL VALUES

	%10	%5	%1
Constant	0.347	0.463	0.739
Trend	0.119	0.146	0.216

C. *The Rescaled Variance (V/S) Statistic*

Giraitis *et al.* (2003) have proposed a centering of the KPSS statistic is based on the partial sum of the deviations from the mean. They called it rescaled variance test V/S as its expression given by:

$$V/S = \frac{1}{T^2 \hat{\sigma}_T^2(q)} \left[\sum_{k=1}^{T} \left(\sum_{j=1}^{K} (Y_j - \bar{Y}_T \right)^2 - \frac{1}{T} \left(\sum_{k=1}^{T} \sum_{J=1}^{k} (Y_j - \bar{Y}_T \right)^2 \right] \tag{13}$$

can be equivalently be rewritten as

$$V/S = T^{-1} \frac{\hat{V}(S_1, ..., S_T)}{\hat{\sigma}_T^2(q)} \tag{14}$$

where

$$S_k = \sum_{j=1}^{k} (Y_j - \bar{Y}_T) \tag{15}$$

are the partial sums of the observations and

$$\hat{V}(S_1, ..., S_T) = T^{-1} \sum_{1}^{T} \left(S_j - \bar{S}_T \right)^2$$

is their sample variance. The V/S statistic is the sample variance of the series of partial sums. The limiting distribution of this statistic is a Brownian bridge of which the distribution is related to the Kolmogorov statistic.

This statistic has uniformly higher power than the KPSS, and is less sensitive than the Lo's statistic to the choice of the order q. For $2 \le q \le 10$, the V/S statistic can appropriately detect the existence of long memory in the level series, although, like most tests and estimators, this test may wrongly detect the existence of long memory in series with shifts in the levels.

This test is evaluated for lag of orders q = 2, 4, 6, 8 and 10. The critical value for this test is 0.1869 at 5% significance level.

III. Estimating the Long Memory of Exchange Rate Series

In this paper, time series data consist of first differences of the natural logarithms of daily exchange rate prices of US Dollar to New Turkish Lira. The rates are determined from Central Bank of Turkey.

The data cover a two-year period, from 3rd of January 2005 to the 28th of December 2006, consisting of 2957 observations which one-hour exchange rate prices. This amount of data is relatively is small when compared to the time series used in the Natural Sciences, but large enough compared to other studies in Economics and Finance, for most of which data use are much smaller.

The descriptive statistics of USD/TRL exchange rate series is given in Table III.

TABLE III
DESCRIPTIVE STATISTICS OF EXCHANGE RATE SERIES

Minimum	-,02442
Maksimum	,02755
Mean	,0000087
Std. Deviation	,00401727
Skewness	,535
Kurtosis	6,615

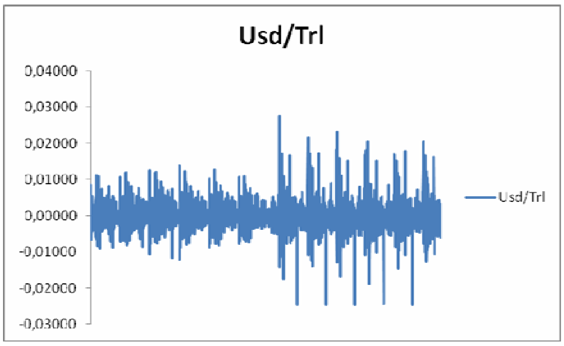


Fig. 3 Daily Exchange Rate Series Plot

TABLE IV
KPSS TEST RESULTS

KPSS Test
3.6785*
0.3624*

KPSS (1992) test is performed on the Dollar-based real exchange rate of New Turkish Lira. The test is evaluated for lag order of 0. Critical values are as follow: Constant: 0.347, 0.463, 0.739 and Trend: 0.119, 0.146, 0.216 for 10%, 5% and 1% significance level, respectively. * indicates significance at the 5% level.

TABLE V
MODIFIED RESCALED RANGE (R/S) TEST RESULTS

Lag order q	Test Statistic
2	3.4824 *
4	3.1356 *
6	2.9177 *
8	2.7455 *
10	2.6191 *

Modified R/S test is performed on the Dollar-based real exchange rate changes of New Turkish Lira. The test is evaluated for lag orders of q = 2, 4, 6, 8 and 10. At the 5% significance level, the null hypothesis of a short memory process is rejected if the modified R/S statistic does not fall within the confidence interval [0.809, 1.862]. * indicates significance at the 5% level.

TABLE VI
RESCALED VARIANCE (V/S) TEST RESULTS

Lag order q	Test Statistic
2	0.7230 *
4	0.5861 *
6	0.5075 *
8	0.4494 *
10	0.4089 *

The V/S test suggested by Giraitis *et. al.*(2003) is performed on the Dollar-based real exchange rate changes of New Turkish Lira. The test is evaluated for lag orders of q = 2, 4, 6, 8 and 10. The critical value is 0.1869 at 5% level. * indicates significance at 5% level.

IV. CONCLUSION

This study has examined the long memory behavior of real exchange rate of USD/TRL over the period of 2005-2006. It is tested for the presence of long memory or fractional dynamics of the series. The method was based on the most recent test of long memory of a time series. The results show that US Dollar against to New Turkish Lira real exchange rate series exhibit long memory by three of tests.

Although this paper reports the existence of exchange rate series, it leaves for future research the investigation of the types of underlying stochastic processes that may be useful in explaining the observed phenomenon.

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