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Continuity of selectorial maps

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ABSTRACT: In this paper the selectorial map notion is introduced and the continuity property of selectorial map is studied. The necessary and sufficient conditions for continuity of selectorial maps are obtained.

Key Words: Set valued map, Continuous selector, Selectorial map.

Contents

1. Introduction

The existence of the continuous selectors of a set valued map has been studied in many papers (see, e.g. [2]- [9] and references therein). The most famous continuous selection theorem is due to Michael where it is proved that a lower semicontinuous and convex closed valued map has a continuous selector (see, [9]).

In [1], the space of non-empty compact convex subsets of $\mathbb{R}^n$ is extended and in the extended space the algebraic structure and norm are defined (Section 2).

In Section 3, using this extension, the continuity of a map with values in the extended space is defined (Definition 3.1) and the sufficient condition for continuity of these maps is obtained (Theorem 3.1). Examples, illustrating continuity of the maps with values in the extended space are given.

In Section 4 the notion of a selectorial map with values in the extended space is introduced (Definition 4.1) and the proposition characterizing selectorial maps is proved (Proposition 4.1). The necessary and sufficient conditions for continuity of selectorial maps are proved (Theorem 4.1).

For $a \in \mathbb{R}^n$ and $r > 0$ we set

$$B_n(a,r) = \{x \in \mathbb{R}^n :\| x - a \|< r\}, \quad B_n = \{x \in \mathbb{R}^n :\| x \|< 1\}$$

where $\| \cdot \|$ means the Euclidean norm.

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The family of all non-empty compact and convex subsets of $\mathbb{R}^n$ is denoted by $conv\left(\mathbb{R}^n\right)$. The Hausdorff distance between by sets $E \in conv\left(\mathbb{R}^n\right)$ and $G \in conv\left(\mathbb{R}^n\right)$ is denoted by the symbol $h(E,G)$ and is defined as

$$h(E,G) = \max\left\{\max_{x\in E} d(x,G),\ \max_{y\in G} d(y,E)\right\}$$

where $d(x,G) = \min_{y\in G} \parallel x-y \parallel$. It is known that; $\left(conv\left(\mathbb{R}^n\right), h\left(\cdot,\cdot\right)\right)$ is a metric space (see, e.g. [8]).

Now let us define continuity of a set valued map $F(\cdot) : D \to conv\left(\mathbb{R}^n\right)$ at point x_0 where $x_0 \in D \subset \mathbb{R}^m$.

Definition 1.1 *Let $D \subset \mathbb{R}^m$, $F(\cdot) : D \to conv\left(\mathbb{R}^n\right)$ be a set valued map and $x_0 \in D$. The set valued map $F(\cdot)$ is said to be continuous at x_0 if for every $\varepsilon > 0$ there exists $\delta(\varepsilon, x_0) > 0$ such that for each $x \in B_m\left(x_0, \delta\left(\varepsilon, x_0\right)\right) \bigcap D$ the inequality $h\left(F(x), F(x_0)\right) < \varepsilon$ holds.*

2. Normed Space $(Conv(\mathbb{R}^n))^2$

For given $A \subset \mathbb{R}^n$ and $B \subset \mathbb{R}^n$ and $\lambda \in \mathbb{R}$ we denote

$$A + B = \{a + b : a \in A, b \in B\}$$

and

$$\lambda A = \{\lambda a : a \in A\}.$$

Note that these addition and scalar multiplication operations do not specify an algebraic structure in $\left(conv\left(\mathbb{R}^n\right), h\left(\cdot,\cdot\right)\right)$. Unfortunately it is not possible to define an effective algebraic structure in the metric space $\left(conv\left(\mathbb{R}^n\right), h\left(\cdot,\cdot\right)\right)$ and to turn it into the linear space. In [1] the space $\left(conv\left(\mathbb{R}^n\right), h\left(\cdot,\cdot\right)\right)$ is extended and in the extended space an algebraic structure is defined. Thus, the extended space turns out a linear space. Now let us introduce this construction.

Let

$$(Conv(\mathbb{R}^n))^2 = conv(\mathbb{R}^n) \times conv(\mathbb{R}^n).$$

On $(Conv(\mathbb{R}^n))^2$ the equivalence relation $\sim$ is defined as follows:

Definition 2.1 *[1] Let $(A,E) \in (Conv(\mathbb{R}^n))^2$ and $(C,D) \in (Conv(\mathbb{R}^n))^2$. We put*

$$(A,E) \sim (C,D) \quad \Leftrightarrow \quad A + D = E + C.$$

The equivalence class containing (A,B) is denoted by $(A,B)_{eq}$. The space $B(conv(\mathbb{R}^n))$ is taken to be quotient space $(Conv(\mathbb{R}^n))^2/\sim$ where addition and scalar multiplication operations in $B(conv(\mathbb{R}^n))$ are defined by the following way:

$$(A,B)_{eq} + (C,D)_{eq} = (A+C, B+D)_{eq},$$

$$\alpha\left(A,B\right)_{eq} = \begin{cases} \left(\alpha A, \alpha B\right)_{eq}, & \text{if} \quad \alpha \geq 0, \\ \left(\left|\alpha\right| B, \left|\alpha\right| A\right)_{eq}, & \text{if} \quad \alpha < 0. \end{cases}$$

Thus, with the addition and scalar multiplication operations defined above, $B(conv(\mathbb{R}^n))$ becomes a real linear space (see, [1]).

$(A,0)_{eq}$ is the equivalence class

$$\{(A+D,D) : D \in conv(\mathbb{R}^n)\}.$$

The zero element of $B(conv(\mathbb{R}^n))$ is the equivalence class

$$\{(D,D) : D \in conv(\mathbb{R}^n)\}$$

which will be denoted by $(0,0)_{eq}$.

Let us define a metric D_H on $B(conv(\mathbb{R}^n))$ is defined by

$$D_H((A,B)_{eq},(C,D)_{eq}) = h(A+D,B+C)$$

where $h(A+D,B+C)$ is the Hausdorff distance between the sets $A+D$ and $B+C$.

The relation

$$\|(A,B)_{eq}\|_{conv} = D_H((A,B)_{eq},(0,0)_{eq}) = h(A,B)$$

defines a norm on $B(conv(\mathbb{R}^n))$.

It is not difficult to verify that

$$D_H((A,B)_{eq},(C,D)_{eq}) = \|(A,B)_{eq} - (C,D)_{eq}\|_{conv}.$$

So, the space $(B(conv(\mathbb{R}^n)), \|\cdot\|_{conv})$ is a normed space.

3. The Map $F(\cdot) : A \to B(conv(\mathbb{R}^n))$

Let $A \subset \mathbb{R}^m$, $F(\cdot) : A \to B(conv(\mathbb{R}^n))$. Then there exist set valued maps $F_1(\cdot) : A \to conv(\mathbb{R}^n)$ and $F_2(\cdot) : A \to conv(\mathbb{R}^n)$ such that $F(x) = (F_1(x), F_2(x))_{eq}$ for all $x \in A$.

Since $B(Conv(\mathbb{R}^n))$ is a normed linear space then it is possible to define the continuity notion for the map $F(\cdot) : A \to B(conv(\mathbb{R}^n))$.

Definition 3.1 *Let $A \subset \mathbb{R}^m$, $F(\cdot) = (F_1(\cdot), F_2(\cdot))_{eq} : A \to B(conv(\mathbb{R}^n))$, $x_0 \in A$. The map $F(\cdot)$ is said to be continuous at x_0 if for every $\varepsilon > 0$ there exists $\delta = \delta(\varepsilon, x_0) > 0$ such that for each $x \in B_n(x_0,\delta) \cap A$ the inequality*

$$\|F(x) - F(x_0)\|_{conv} < \varepsilon$$

holds.

Note that the map $F(\cdot) = (F_1(\cdot), F_2(\cdot))_{eq} : A \to B(conv(\mathbb{R}^n))$ is said to be continuous on the set A iff it is continuous at every $x \in A$.

The following proposition characterizes the continuity of the map $F(\cdot) : A \to B(conv(\mathbb{R}^n))$.

Proposition 3.1 *Let $A \subset \mathbb{R}^m$, $x_0 \in A$, $F(\cdot) = (F_1(\cdot), F_2(\cdot))_{eq} : A \to B(conv(\mathbb{R}^n))$. The map $F(\cdot)$ is continuous at x_0 if and only if for every $\varepsilon > 0$ there exists $\delta = \delta(\varepsilon, x_0) > 0$ such that for each $x \in B_n(x_0, \delta) \cap A$ the inequality*

$$h(F_1(x) + F_2(x_0), F_2(x) + F_1(x_0)) < \varepsilon$$

holds.

Proof: In the consequence of definitions of addition, multiplication and norm at $B(conv(\mathbb{R}^n))$, it follows

$$\begin{aligned} \| F(x) - F(x_0) \|_{conv} &= \| (F_1(x), F_2(x))_{eq} - (F_1(x_0), F_2(x_0))_{eq} \|_{conv} \\ &= \| (F_1(x), F_2(x))_{eq} + (F_2(x_0), F_1(x_0))_{eq} \|_{conv} \\ &= \| (F_1(x) + F_2(x_0), F_2(x) + F_1(x_0))_{eq} \|_{conv} \\ &= h(F_1(x) + F_2(x_0), F_2(x) + F_1(x_0)) \qquad (3.1) \end{aligned}$$

The proof follows from (3.1) and the definition of continuity. □

The following theorem gives us a sufficient condition for continuity of the map $F(\cdot) = (F_1(\cdot), F_2(\cdot))_{eq} : A \to B(conv(\mathbb{R}^n))$.

Theorem 3.1 *Let $A \subset \mathbb{R}^m$, $x_0 \in A$, $F(\cdot) = (F_1(\cdot), F_2(\cdot))_{eq} : A \to B(conv(\mathbb{R}^n))$, the set valued maps $F_1(\cdot) : A \to conv(\mathbb{R}^n)$, $F_2(\cdot) : A \to conv(\mathbb{R}^n)$ are continuous at $x_0 \in A$. Then the map $F(\cdot)$ is continuous at $x_0 \in A$.*

Proof: Since set valued maps $F_1(\cdot) : A \to conv(\mathbb{R}^n)$, $F_2(\cdot) : A \to conv(\mathbb{R}^n)$ are continuous at $x_0 \in A$, then for every $\varepsilon > 0$ there exist $\delta_1 = \delta_1(\varepsilon, x_0)$ and $\delta_2 = \delta_2(\varepsilon, x_0)$ such that for every $x \in B_m(x_0, \delta_1) \cap A$ and $x \in B_m(x_0, \delta_2) \cap A$ the inequalities

$$h(F_1(x), F_1(x_0)) < \frac{\varepsilon}{2}\ , \quad h(F_2(x), F_2(x_0)) < \frac{\varepsilon}{2} \qquad (3.2)$$

hold respectively. Let $\delta_* = min\{\delta_1, \delta_2\}$. It follows from definition of Hausdorff distance and (3.2) that for every $x \in B_m(x_0, \delta_*) \cap A$ the inclusions

$$F_1(x) \subset F_1(x_0) + \frac{\varepsilon}{2} B_n\ , \quad F_1(x_0) \subset F_1(x) + \frac{\varepsilon}{2} B_n \qquad (3.3)$$

$$F_2(x) \subset F_2(x_0) + \frac{\varepsilon}{2} B_n\ , \quad F_2(x_0) \subset F_2(x) + \frac{\varepsilon}{2} B_n \qquad (3.4)$$

are satisfied. We get from (3.3) and (3.4) that for every $x \in B_m(x_0, \delta_*) \cap A$ the inclusions

$$F_1(x) + F_2(x_0) \subset F_2(x) + F_1(x_0) + \varepsilon B_n \qquad (3.5)$$

$$F_2(x) + F_1(x_0) \subset F_1(x) + F_2(x_0) + \varepsilon B_n \qquad (3.6)$$

hold. From (3.5) and (3.6) we obtain

$$h(F_1(x) + F_2(x_0), F_1(x_0) + F_2(x)) \le \varepsilon \qquad (3.7)$$

for every $x \in B_m(x_0, \delta_*) \cap A$. Consequently, from Proposition 3.1 and (3.7) we have that the map $F(\cdot) = (F_1(\cdot), F_2(\cdot))_{eq}$ is continuous at $x_0 \in A$. $\square$

The inverse of this theorem is not valid. That is, the continuity of the map $F(\cdot) = (F_1(\cdot), F_2(\cdot))_{eq} : A \to B(conv(\mathbb{R}^n))$ at $x_0 \in A$, does not imply the continuity of the set valued maps $F_1(\cdot) : A \to conv(\mathbb{R}^n)$ and $F_2(\cdot) : A \to conv(\mathbb{R}^n)$ at $x_0 \in A$.

Example 3.1 *Let* $F(\cdot) = (F_1(\cdot), F_2(\cdot))_{eq} : \mathbb{R} \to B(conv(\mathbb{R}))$ *where*

$$F_1(x) = F_2(x) = \begin{cases} [-1,1], & if \quad x \ is \ \ rational, \\ [-2,2], & if \quad x \ is \ \ irratinal. \end{cases} \tag{3.8}$$

It is obvious that the set valued maps $F_1(\cdot) : \mathbb{R} \to conv(\mathbb{R})$ *and* $F_2(\cdot) : \mathbb{R} \to conv(\mathbb{R})$ *are not continuous at any* $x_0 \in \mathbb{R}$. *Now let us choose an arbitrary* $x_0 \in \mathbb{R}$. *Then*

$$\begin{aligned} \| F(x) - F(x_0) \|_{conv} &= \| (F_1(x), F_2(x))_{eq} - (F_1(x_0), F_2(x_0))_{eq} \|_{conv} \\ &= \| (F_1(x), F_1(x))_{eq} - (F_1(x_0), F_1(x_0))_{eq} \|_{conv} \\ &= \| (F_1(x) + F_1(x_0), F_1(x) + F_1(x_0))_{eq} \|_{conv} \\ &= h(F_1(x) + F_1(x_0), F_1(x) + F_1(x_0)) = 0 \end{aligned}$$

and hence, the map $F(\cdot) = (F_1(\cdot), F_2(\cdot))_{eq} : \mathbb{R} \to B(conv(\mathbb{R}))$ *defined by (3.8) is continuous at* $x_0 \in \mathbb{R}$.

Now, let us give an example where the map $F(\cdot) = (F_1(\cdot), F_2(\cdot))_{eq} : \mathbb{R}^m \to B(conv(\mathbb{R}^n))$ is not continuous at given x_0.

Example 3.2 *Let* $F(\cdot) = (F_1(\cdot), F_2(\cdot))_{eq} : \mathbb{R} \to B(conv(\mathbb{R}))$ *where*

$$F_1(x) = [-2,2], \quad F_2(x) = \begin{cases} [-1,1], & if \quad x \ is \ \ rational, \\ [-2,2], & if \quad x \ is \ \ irrational. \end{cases} \tag{3.9}$$

We will show that the map $F(\cdot) = (F_1(\cdot), F_2(\cdot))_{eq}$ *is not continuous anywhere.*

Let us choose an arbitrary $x_0 \in \mathbb{R}$. *Assume that* x_0 *is rational and* $\varepsilon_* = \frac{1}{2}$. *Then for every irrational* $x_* \in R$ *we get*

$$\begin{aligned} \| F(x_*) - F(x_0) \|_{conv} &= \| (F_1(x_*), F_2(x_*))_{eq} - (F_1(x_0), F_2(x_0))_{eq} \|_{conv} \\ &= \| (F_1(x_*), F_2(x_*))_{eq} + (F_2(x_0), F_1(x_0))_{eq} \|_{conv} \\ &= \| (F_1(x_*) + F_2(x_0), F_2(x_*) + F_1(x_0))_{eq} \|_{conv} \\ &= h(F_1(x_*) + F_2(x_0), F_2(x_*) + F_1(x_0)) \\ &= h([-2,2] + [-1,1], [-2,2] + [-2,2]) \\ &= h([-3,3], [-4,4]) = 1 > \frac{1}{2} = \varepsilon_* \end{aligned}$$

and consequently, the map $F(\cdot) = (F_1(\cdot), F_2(\cdot))_{eq}$ *defined by (3.9) is not continuous at rational* $x_0 \in \mathbb{R}$.

Now let x_0 be irrational and $\varepsilon_* = \frac{1}{2}$. Then for every rational x_* we have

$$\begin{aligned} \| F(x_*) - F(x_0) \|_{conv} &= \| (F_1(x_*), F_2(x_*))_{eq} - (F_1(x_0), F_2(x_0))_{eq} \|_{conv} \\ &= \| (F_1(x_*), F_2(x_*))_{eq} + (F_2(x_0), F_1(x_0))_{eq} \|_{conv} \\ &= \| (F_1(x_*) + F_2(x_0), F_2(x_*) + F_1(x_0))_{eq} \|_{conv} \\ &= h(F_1(x_*) + F_2(x_0), F_2(x_*) + F_1(x_0)) \\ &= h([-2,2] + [-2,2], [-1,1] + [-2,2]) \\ &= h([-4,4], [-3,3]) = 1 > \frac{1}{2} = \varepsilon_* \end{aligned}$$

and hence the map $F(\cdot) = (F_1(\cdot), F_2(\cdot))_{eq}$ defined by (3.9) is not continuous at irrational $x_0 \in \mathbb{R}$.

Thus we obtain that the map $F(\cdot) = (F_1(\cdot), F_2(\cdot))_{eq}$ defined by (3.9) is not continuous anywhere.

4. Selectorial Maps

In this section, we will introduce selectorial maps notion and investigate their continuity properties.

Definition 4.1 *Let $A \subset \mathbb{R}^m$, $F(\cdot) = (F_1(\cdot), F_2(\cdot))_{eq} : A \to B(conv(\mathbb{R}^n))$. If there exist functions $f_1(\cdot) : A \to \mathbb{R}^n$ and $f_2(\cdot) : A \to \mathbb{R}^n$ such that*

$$(F_1(x), F_2(x)) \sim (f_1(x), f_2(x)) \quad \textit{for} \quad \textit{all } x \in A$$

then the map $F(\cdot) = (F_1(\cdot), F_2(\cdot))_{eq}$ is said to be a selectorial map.

The following proposition characterizes the selectorial maps.

Proposition 4.1 *Let $A \subset \mathbb{R}^m$ and $F(\cdot) = (F_1(\cdot), F_2(\cdot))_{eq} : A \to B(conv(\mathbb{R}^n))$. The map $F(\cdot)$ is a selectorial map if and only if there exists a function $\varphi(\cdot) : A \to \mathbb{R}^n$ such that the equality*

$$F_2(x) = F_1(x) + \varphi(x) \quad \textit{for} \quad \textit{all } x \in A$$

holds.

Proof: Let $F(\cdot) = (F_1(\cdot), F_2(\cdot))_{eq} : A \to B(conv(\mathbb{R}^n))$ be a selectorial map. Then according to Definition 4.1, there exist functions $f_1(\cdot) : A \to conv(\mathbb{R}^n)$ and $f_2(\cdot) : A \to conv(\mathbb{R}^n)$ such that for all $x \in A$ the relation

$$(F_1(x), F_2(x)) \sim (f_1(x), f_2(x))$$

holds.

By virtue of Definition 2.1 we get

$$F_1(x) + f_2(x) = F_2(x) + f_1(x)$$

for every $x \in A$ and hence,

$$F_2(x) = F_1(x) + \varphi(x) \quad \text{for every } x \in A$$

where $\varphi(x) = f_2(x) - f_1(x)$.

Now, let

$$F_2(x) = F_1(x) + \varphi(x)$$

for all $x \in A$. Then

$$\begin{aligned} (F_1(x), F_2(x)) &\sim (F_1(x), F_1(x) + \varphi(x)) \\ &\sim (0, \varphi(x)) + (F_1(x), F_1(x)) \\ &\sim (0, \varphi(x)) \end{aligned}$$

and consequently

$$(F_1(x), F_2(x)) \sim (f_1(x), f_2(x))$$

for every $x \in A$ where $f_1(x) = 0$, $f_2(x) = \varphi(x)$ □

The following theorem is a continuity criterion for selectorial maps.

Theorem 4.1 *Let $A \subset \mathbb{R}^m$, $f_1(\cdot) : A \to \mathbb{R}^n$, $f_2(\cdot) : A \to \mathbb{R}^n$ be given functions. The selectorial map $F(\cdot) = (f_1(\cdot), f_2(\cdot))_{eq} : A \to B(conv(\mathbb{R}^n))$ is continuous at $x_0 \in A$ if and only if the function $\varphi(\cdot) = f_1(\cdot) - f_2(\cdot) : A \to \mathbb{R}^n$ is continuous at $x_0 \in A$.*

Proof: Let $F(\cdot) = (f_1(\cdot), f_2(\cdot))_{eq} : A \to B(conv(\mathbb{R}^n))$ be a continuous selectorial map at $x_0 \in A$. Then for every $\varepsilon > 0$ there exists $\delta = \delta(\varepsilon, x_0) > 0$ such that for each $x \in B_n(x_0, \delta) \cap A$ the inequality

$$\| F(x) - F(x_0) \|_{conv} < \varepsilon \tag{4.1}$$

holds. Since $F(\cdot) = (f_1(\cdot), f_2(\cdot))_{eq}$ then we get

$$\begin{aligned} \| F(x) - F(x_0) \|_{conv} &= \| (f_1(x), f_2(x))_{eq} - (f_1(x_0), f_2(x_0))_{eq} \|_{conv} \\ &= \| (f_1(x) + f_2(x_0), f_2(x) + f_1(x_0))_{eq} \|_{conv} \\ &= h(f_1(x) + f_2(x_0), f_2(x) + f_1(x_0)) \\ &= \|f_1(x) + f_2(x_0) - f_2(x) - f_1(x_0)\| \\ &= \|[f_1(x) - f_2(x)] - [f_1(x_0) - f_2(x_0)]\|. \end{aligned} \tag{4.2}$$

It follows from (4.1) and (4.2) that for every $\varepsilon > 0$ there exists $\delta(\varepsilon, x_0) > 0$ such that for each $x \in B_n(x_0, \delta) \cap A$ the inequality

$$\|[f_1(x) - f_2(x)] - [f_1(x_0) - f_2(x_0)]\| < \varepsilon$$

holds and hence, the function $\varphi(\cdot) = f_1(\cdot) - f_2(\cdot)$ is continuous at $x_0 \in A$.

Now, let the function $\varphi(\cdot) = f_1(\cdot) - f_2(\cdot)$ be continuous at $x_0 \in A$. Then for every $\varepsilon > 0$ there exists $\delta(\varepsilon, x_0) > 0$ such that for each $x \in B_n(x_0, \delta) \cap A$ the inequality

$$\|[f_1(x) - f_2(x)] - [f_1(x_0) - f_2(x_0)]\| < \varepsilon \tag{4.3}$$

holds.

From (4.2) and (4.3) we obtain that for every $\varepsilon > 0$ there exists $\delta(\varepsilon, x_0) > 0$ such that for each $x \in B_n(x_0, \delta) \cap A$ the inequality

$$\| F(x) - F(x_0) \|_{conv} < \varepsilon$$

holds and consequently, the selectorial map $F(\cdot) = (f_1(\cdot), f_2(\cdot))_{eq}$ is continuous at $x_0 \in A$. □

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