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GRAPHICAL UNDERSTANDING IN MATHEMATICS EDUCATION: DERIVATIVE FUNCTIONS AND STUDENTS' DIFFICULTIES

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Abstract

Effective teaching is one of the most important factors to improve students' achievement. Therefore, many researchers in mathematics education are looking for several ways to improve the quality of learning. The purpose of this paper is to investigate an overview of specific difficulties based on the graph of derived function, another words, how students find the connections between the graph of derived function and the some properties of the original function.

The subjects were 102 high school students in grade 11. The study was conducted during the fall semester in two calculus classes. The data of the research has been collected from 5 diagnostic the graph of derived function involved the chance in slope, decreasing, increasing, a local maximum, a local minimum, the point of inflection.

The questionnaires were designed to assess how well the students had learned procedures the connections between the graph of derived function and the some properties of original function. Each student was asked explain how you arrived at your answers. While working on the answers of the students, not only the accuracy of the students' answers was examined but also the methods were being analysed even if they were incorrect.

The results indicated that the students find it difficult to make connections between the graph of derived function and the original function. Usually, they were to interpret the graph of derived function as the graph of function. The students did not use the mathematical language to describe the graph of derived function

Effective teaching in calculus course should be informal, intuitive and conceptually based on graphs and functions in order to improve the quality of learning and develop the understanding of calculus concepts.

1.Introduction

Derivative function is a new function which is formed from given function. The subject of derivation forms important chapter of analysis in university level as much as high school level. Additionally, the subject of derivation is mathematically cryptic subject. When conceptual structures, logical structure, abstract and symbolic character of derivation combine with deficiencies in conditions of education, complications in learning arise.

To gain theoretical thinking character of derivation encloses discussion activities graphically as researches, solutions of problems. Graphical discussion of derivation is needed thinking skill. Due to the usage of derivation concept in

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many disciplines, to know difficulties of students met in this level is important. These difficulties in understanding on this main subject were a studying topic for many researchers who are working on mathematics education.

Graph of derivative function was analysed. In this research graph of derivative function was analysed. Just because of the use of derivative function in various disciplines, the students have to be aware of these difficulties that they face in this level.

Visualisation in mathematics education, i.e. in other words, graphical interpretation became important increasingly. Until years of eighties, mathematics education was made operationally. At subsequently years, many mathematics

educators emphasized meanings of derivation in various disciplines and suggested a new method that used graphical operations (Decker, 1985, tall, 1997, Asial et al 1997).

Tall and Vinner (1981) defined some difficulties that students faced while they were using graphical methods in order to solve a function. In similar study, before giving the graph of derivative function Ferrini-Mundy and Graham (1994) investigated finding the equation of function given graphically and then defined difficulties.

Usage of graph in the subject of derivation have the students be concentrated on this subject of derivation and simplified examination of concerned problem solutions (Tall, 1986) To establish relationship between the graph and algebraic operations provides gaining power of knowledge which is obtained by the students (Dreyfus and Halevi, 1990).

Heid (1988) investigated that the college students' difficulties on understanding the subject of derivation, the mistakes made by students and effects of computer usage on this subject. In addition to this subject, to simplify understanding the derivation conceptually, Heid indicated the importance of computer usage and drawing graph. Mathematics is not a discipline demerged different subjects, operations and rules, it is a consecutive discipline abided to basic principles and concepts. When the basic concepts of analysis of derivation have not been understood by students thoroughly, the students will be ignorant of applications, formulas and their meanings. Thus, learning abided memorization of rule and definition gets difficult and doesn't contain real life applications (Orhun, 2012).

In similar study, Orton (1983) showed that many students in mathematics department have many basic errors in the subject of derivation and derivation can not be understood conceptually.

The students understand the subject of derivation algebraically. One another purpose of this research is to understand and to deepen the epistemology of subject of derivation.

By means of the strategies developed, learning of the students will be increased.

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2.Method

The subjects were 102 high school students in grade 12. The study was conducted during the fall semester in two calculus classes. The data of the research have been collected from 5 diagnostic the graph of derived function involved the chance in slope, decreasing, increasing, a local maximum, a local minimum, the point of inflection.
The questionnaires were designed to assess how well the students had learned procedures the connections between the graph of derived function and the some properties of original function. Each student was asked to explain how they did find the answer. The results that taken from the students' attempts were analysed. While working on the answers of the students, not only the accuracy of the students' answers was examined but also the methods were being analysed even if they were incorrect.

3.Findings

The results indicated that the students find it difficult to make connections between the graph of derived function and the original function. Usually, they interpreted the graph of derived function as the graph of function. The students did not use the mathematical language to describe the graph of derived function
It was determined that conceptual and practical errors which arised from this study, resulted from skill to understand basic principles, skill to think on the graph, skill to argue from graph and lack of relation in relationships which problems contained.
The questions that were asked to the students have been exemplified below.

Research Question 1.

The derived function graph is given in figure 1.

Find the generally the equation of function y=f(x).

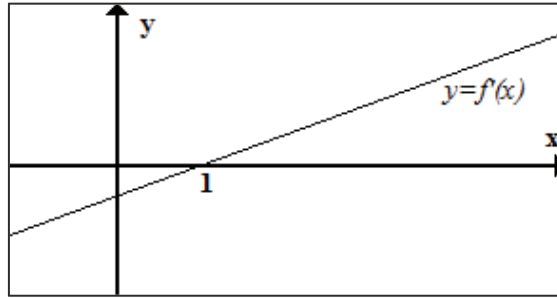Figure 1.Graph of derived function

The aim of this question is to build relation between derivative function and degrees of original function. Almost every single student knows that algebraically the degree lessens one in operation of derivation. Contrarily, the students have difficulties finding the degree of original function on the given graph of derivation.
The answers were as follows:
Generally given answers were to form symmetric line for graph of function unless making declaration (42%).
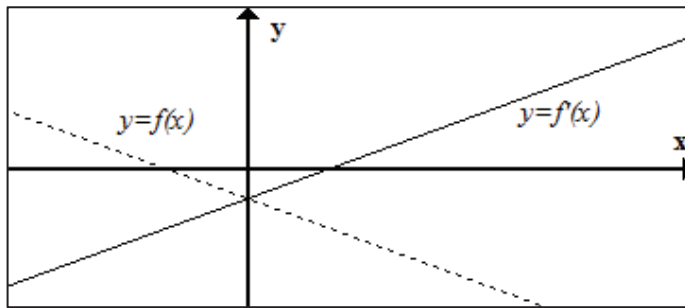

Figure 2. Graphs of derived function and original function

Another answer is derivative function is a linear function, base function can not be found (10%).
The equation of derivation must be given. In this position the equation of function could be everything (15%).

Research Question 2

The derived function graph is given in figure 3. Find the points of local maximum and local minimum of original function.
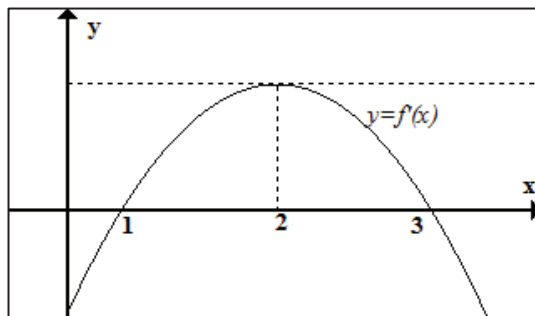

Figure 3. Graph of derived function

The aim of this question is to apply the definitions of points of local maximums and local minimums on derivative function. However the majority of the students (72%) answered this question that there is a local maximum at the point of x=2 and there isn't any local point of minimum. Here, derivation of graphically representation of derivative function could be converted to properties of function. This graph was generally perceived as the graph of function.

Research Question 3

The derived function graph is given in figure 4. . Find increasing and decreasing intervals of the original function.
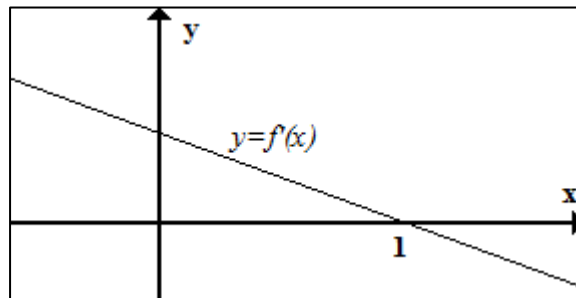
Figure 4. Graph of derived function

The aim of this question is to specify on which intervals derivative function takes positive or negative values and to apply the condition of occurring to decrease and to increase of original function. The answer of majority of students to this question (53%) is that function is descending at any point. The other answer (39%) is that due to derivative function is increasing, main function is decreasing.

Research Question 4

The derived function graph is given in figure 5. Is there any inflection point of function at the point of x=1 Explain.
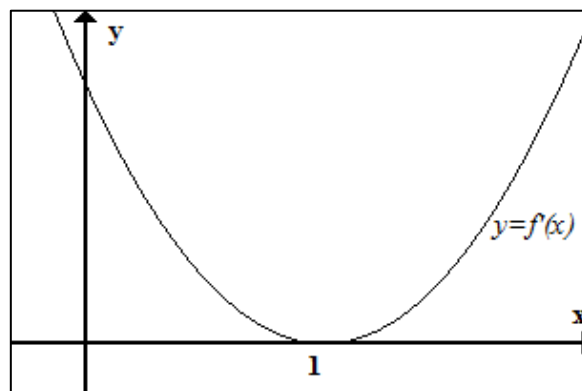
Figure 5. Graph of derivative function
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The answer of majority of students to this question (67%) is that there is inflection point, or there isn't inflection point. There is no explaining.

4.Conclusions and Suggestions

According to results produced, the students were not succesful in analysing derivative function. This case could be the result of traditional teaching method.
The derivation concept is derivative operation for students. The students haven't interpreted the graph of derivative function.
The graphs convey many concepts and collect much information. Then, the graphical interpretation and construction include the understanding connection among various mathematical topics. All of them, first approximation to calculus is not only theoretically, compounding with real life, giving place to numerical and graphical discussion.

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