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RESEARCH ARTICLE /**ARAŞTIRMA MAKALESİ**

ON THE LEVEL DENSITY PARAMETERS OF SOME SUPERDEFORMED LIGHT NUCLEI IN THE MASS REGION OF 40 ≤ A ≤ 70

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ABSTRACT

The nuclear level densities are extremely important for a wide variety of phenomena, ranging from nuclear astrophysics to radiochemical applications for steward-ship science. The nuclear level density is also an important physical quantity both from the fundamental point of view as well as in understanding the particle and gamma ray emission in various reactions. In superdeformed light and heavy nucleus, the gamma-ray energies drop with decreasing spin in a very regular fashion. The nuclear level density parameter itself changes with excitation energy depending on both shell effect in the single-particle model and different excitation modes in the collective models.

In this study, the energy level density parameters of some superdeformed light nucleus (^{40}Ca, ^{57}Co, ^{59}Ni, ^{64}Cu, ^{68}Zn) are determined by using energy spectrum of the interest nucleus for different band. In calculation of energy-level density parameters dependent upon excitation energy of nuclei studied, a model was considered which relies on the fact that energy levels of superdeformed light nuclei, just like those of superdeformed heavy nuclei, are equidistant and which relies on collective motions of their nucleons. The present calculation results have been compared with the corresponding experimental and theoretical results and found to be well in agreement.

Keywords : Energy level density parameters, Collective excitation bands, Superdeformed light nuclei.

40 ≤ A ≤ 70 BÖLGESİNDEKİ BAZI SÜPERDEFORME HAFİF ÇEKİRDEKLERİN ENERJİ SEVİYE YOĞUNLUK PARAMETRELERİ

ÖZ

Nükleer astrofizikten radyokimyasal uygulamalara kadar sıralama ve doğa olaylarının geniş bir çeşitliliği için nükleer seviye yoğunlukları son derece önemlidir. Aynı zamanda nükleer enerji seviye yoğunluğu hem parçacığın anlaşılması hem de çeşitli reaksiyonlarda gamma ışını yayınlanması için önemli bir fiziksel niceliktir. Hafif ve ağır deforme çekirdeklerde gamma enerjileri düzenli bir biçimde spin azalmasıyla birlikte düşmektedir. Nükleer seviye yoğunluk parametresi, hem kollektif modelde farklı uyarılma modlarına hem de tek-parçacık modelinde shell etkisine bağlı olarak uyarılma enerjisi ile birlikte değişmektedir.

Bu çalışmada, bazı süperdeforme hafif çekirdeklerin (^{40}Ca, ^{57}Co, ^{59}Ni, ^{64}Cu, ^{68}Zn) seviye yoğunluk parametreleri, her çekirdeğin farklı bandları için, enerji spektrumlarından yararlanılarak hesaplanmıştır. İncelenen çekirdeklerin uyarılma enerjisine bağlı olarak enerji seviye yoğunluk parametreleri hesaplanırken, süperdeforme hafif çekirdeklerinin de ağır deforme çekirdekler gibi, enerji seviyelerinin eş-aralıklı olmasını ve nükleonların kollektif hareketlerini temel alan bir model göz önüne alınmıştır. Elde edilen parametre sonuçları diğer çalışmaların deneysel ve teorik değerleri ile karşılaştırılmış ve uyum içinde olduğu belirlenmiştir.

Anahtar Kelimeler: Süperdeforme hafif çekirdekler, Enerji seviye yoğunluk parametresi, Kollektif uyarılma bandları.

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1. INTRODUCTION

Nuclear level density is interesting both from a purely theoretical point of view (the problem of a quantum many-body system with contınuum excitation energy), as well as from a perspective of applications (e.g., an essential ingredient of statistical models of nuclear reactions) (Pezer et al., 2003). Besides, the nuclear level density, which is a basic ingredient in statistical analysis of nuclear reactions, has been the subject of many investigations at low excitation energy where the level density is obtained directly by counting low-lying levels (Bohr and Mottelson, 1969). However, at increasing excitation energy, the level density becomes large and individual levels are often not resolved in experiment (Gilbert and Cameron, 1965).

Main parameter of theory, which is related to the nuclear level density-the level density parameter, is obtained from nuclear resonances and from the analysis of evaporation spectra using the independent particle model prediction for the level density (Shlomo and Natowitz, 1990). The first attempt, the simplest expression for the nuclear level density has been obtained a long time ago by Bethe (1936,1937) who utilized the assumption that an energy independent density of single particle states and later modified by Bloch (1954) who developed general methods to deal with the mathematical problem. There are, however, a number of shortcomings in this approach. For example, the lack of coupling to the collective part of the nuclear spectrum leads to an energy-independent level density parameter. Recently there has been considerable theoretical activity in the determination of the nuclear many-body density of states, taking into account shell, pairing, and deformation effects (Gilbert and Cameron, 1965; Woosley, 1980), finite size effects (Yen and Miller, 1992), and thermal and quantal effects (Puddu et al., 1990; Puddu et al., 1991), as well as improvements in the determination of the spin cut-off factors (Woosley, 1980) and different collective modes in the excitation of a nuclear matter (Ignatyuk et al., 1979; Ahmadov et al., 2002; Okuducu, Ahmadov, 2003).

2. CALCULATION METHOD FOR LEVEL DENSITY PARAMETER

Above-mentioned theory gives also dependence of nuclear level density on the total angular momentum I of the nucleus. The most used theoretical formula for the observable nuclear level densities ρ in the calculation of level density parameters a can be written as (Gilbert and Cameron, 1965 ; Bethe, 1936),

$$\rho(U,J)=\frac{2J+1}{24\sqrt{2}\sigma^{3}a^{1/4}U^{5/4}}\exp\left[-\left(J+\frac{1}{2}\right)^{2}/2\sigma^{2}\right]\exp\left(2\sqrt{aU}\right) \quad \textbf{(1)}$$

In this relation, J is the angular momentum of the level with any excitation energy U, σ is called as "spin cut-off parameter" which characterizes the distribution of the level density in spin. The parameters a and σ, which are related to the density of single-particle states $g(\varepsilon_F)$ at the Fermi energy ε_F, can be defined respectively as,

$$a=\frac{\pi^{2}}{6}g\left(\varepsilon_F\right) \quad \textbf{(2)}$$

$$\sigma^{2}=g\left(\varepsilon_F\right)\left\langle m^{2}\right\rangle t \quad \textbf{(3)}$$

Here, $<m^2>$ is the mean square magnetic quantum number that is the average of the square of z-projection of individual particle angular momentum j and t is the nuclear thermodynamic temperature of an excited nucleus in the Fermi-gas model. These factors given in Eqs. (2) and (3) are expressed as follows:

$$g\left(\varepsilon_F\right)=\frac{2}{3}\frac{A}{\varepsilon_F}\ , \quad \left\langle m^{2}\right\rangle=0.146A^{2/3}\ , \quad t^{2}=\frac{U}{a} \quad \textbf{(4)}$$

where, A is the mass number of a nucleus. The experimental observations cannot distinguish the different values of J. Therefore, it is more useful to obtain the observable level density which has form (Gilbert and Cameron, 1965 ; Bethe, 1936),

$$\rho(U)=\sum_{J}\rho(U,J)=\frac{\pi^{2}}{12}\frac{\exp\left(2\sqrt{aU}\right)}{a^{1/4}U^{5/4}}\frac{1}{\sqrt{2\pi}\sigma} \quad \textbf{(5)}$$

Hence, substituting the Eqs. (2-4) into Eq. (5) one finds the observable level density as,

$$\rho(U)=\frac{a}{12\sqrt{2}\times0.298A^{1/3}\left(aU\right)^{3/2}}\exp\left(2\sqrt{aU}\right) \quad \textbf{(6)}$$

The nuclear level density parameter of the Bethe theory is well established in a number of studies (Gilbert and Cameron, 1965 ; Baba, 1970) on the s-wave neutron resonance for different mass nuclei. However, this theory does not take into account the collective effects of the nuclear particles in the excitation of the nuclei for determining of the nuclear level density. On the other hand, the measured magnetic and quadrupole moments of the nuclei deviate considerably from the ones calculated using the single-particle shell model in which the closed shells forming the nuclear core play no part. In other words, the excited states and the magnetic and quadrupole moments are the results of collective motion of many nucleons, not just of those nucleons that are outside the closed shell. The existence of collective energy level bands of rotational and vibrational types can now easily be identified from nuclear spectra data (Nuclear Structure and Decay Data, 2001) of many deformed nuclei. In some studies such as in Refs. (Rohr, 1984; Ignatyuk et al., 1979), the contribution of collective motion of nucleons on the energy level density has been considered.

However, these studies naturally involve messy equations and make the model complex for calculation of the nuclear level density parameters of deformed nuclei.

Many superdeformed light nuclei, especially with mass ranging from A=30 to A=70, have stable deformation in their ground-states. Such these nuclei may rotate due to interactions with an external incident particle or emitting the particle. Rotational energy of an axially symmetric deformed even-even nucleus is given as (Bohr and Mottelson, 1969),

$$E_{rot}(I,K)=\frac{\hbar^2}{2}\left[\frac{I(I+1)}{J_0}+\left(\frac{1}{J_3}-\frac{1}{J_0}\right)K^2\right] \qquad \textbf{(7)}$$

where I and K are the total angular momentum and its projection on the axis of symmetry, respectively, of a nucleus; J_3 and J_0 are the principal moments of inertia about a symmetry axis and an arbitrary axis perpendicular to the symmetry axis, respectively. Authors of Ref. (Davidson, 1968) have used the hydrodynamic moments of inertia restricting the deformed nuclear surface by a quadrupole term only. A further assumption of this model is that these nuclei are, on the average, symmetric: that is $J_3=0$. Therefore, Eq.(7) will be meaningful only if the value of K is taken identically zero. Then we come to the following rotational energy equation:

$$E_{rot}=\frac{\hbar^2}{2J_0}I(I+1),\quad K{=}0 \qquad \textbf{(8)}$$

Note here that, the above expression is in good agreement with the observed low-lying energy levels of the even–even deformed nuclei, which is the values of angular momentum I, I = 0, 2, 4, 6,…. As mentioned above the energy level sequence in such a case is called as ground-state rotational band having positive-parity.

The so-called β and γ excited bands introduced in Ref. (Davidson, 1968) are also well identified the observed energy levels of the collective nature in the large deformed even–even nuclei. The β band is associated with vibrations that preserve the axis of symmetry and therefore is K = 0 band with the level sequence given by Eq. (8) and the band head $h\omega_\beta$. The γ band is associated with the vibrations not preserving the symmetry axis and having the levels given by Eq. (7).

The spin sequence of γ band with K=2 is I=2+,3+,4+,5+…In such a case, the rotational band with a given K value and spin sequence $I=K=\Omega, K+1, K+2,\cdots$, where K and Ω are the projections of the total angular momentum and odd nucleon angular momentum, respectively on the nuclear symmetry axis, has level sequence and spacing which are given by (Nilsson, 1955),

$$\Delta E(I,K)=E_{IK}-E_{KK}=\frac{\hbar^2}{2J_0}\left[I(I+1)-K(K+1)\right] \qquad \textbf{(9)}$$

A good example of this simple level structure given by Eq (9) is to be found in the odd-neutron nucleus ${}^{59}_{28}Ni_{31}$. In the deformed axially symmetric odd-odd nuclei quantum number K is also determined as $K=\left|\Omega_p \pm \Omega_n\right|$, where Ω_p and Ω_n are projections of proton and neutron angular momentums, respectively on the symmetry axis. The ground state spins of these nuclei are determined with the same coupling rules. Each band with a given K is built upon the proton and neutron intrinsic states of the Nilsson model (Nilsson, 1955). Some superdeformed light nuclei, such as even-even, odd-odd and odd-A nuclei, with their different corresponding bands for which the level density parameters are estimated in the present work are listed in Table.

Table 1. The calculated and compiled values of the nuclear level density parameters for some superdeformed light nuclei

Nucleus	**Baba** a**,** $\mathbf{MeV^{-1}}$	**Gilbert-Cameron** a**,** $\mathbf{MeV^{-1}}$	**BSFG Model** a**,** $\mathbf{MeV^{-1}}$	**Calculated** a_0**,** $\mathbf{MeV^{-1}}$	**Corresponding bands**
${}^{40}_{20}Ca$	5,24	5,44	3,6	5,31	β-vibrational band
${}^{57}_{27}Co$	5,70	5,95	-	5,51	Octupol Band
${}^{59}_{28}Ni$	6,97	5,97	5,07	6,62	Octupol band
${}^{64}_{29}Cu$	8,78	8,09	6,55	8,12	γ-vibrational positive parity
${}^{68}_{30}\mathrm{Zn}$	9,13	9,75	7,28	8,74	γ-vibrational positive parity

The nuclear energy level density depending on the excitation energy, U taking into account different excitation modes can be written in the following form,

$$\rho(U)=\sum_i a_i\rho_i(U) \qquad \textbf{(10)}$$

where $\rho_i(U)$ is the partial energy level density for i th excitation band and a_i is the weighting coefficient satisfying the condition $\sum_i a_i=1$. As shown in this work, to derive the universal expression for ρ_i we follow the work in Ref. (Bohr and Kalckar, 1937) expressing the excitation energy U by a number of many different combinations of the unit energy and use a simple expression for the energy level density which considered the collective excitation modes. Here we remain the important properties of observed energy spectrum of nuclei considered. These properties can approximately be verified for the energies of the collective rotational and vibrational bands in the even-even and of the coupled state bands in the odd-odd and odd-*A* superdeformed light nuclei as being the ratios given by,

$$R_1:R_2:R_3:R_4:\ldots=1:r:2r:3r:\ldots \qquad \textbf{(11)}$$

Here, R_1, R_2, R_3, R_4,... are the ratios of sequential level energies to the appropriate energy unit of a corresponding band. In our present study the nuclear level density formula introduced depending on the excitation energy U and energy unit ε_o for i th excitation band can be represented as (Ahmadov et al., 2002; Okuducu, Ahmadov, 2003),

$$\rho_i(U,\varepsilon_{oi})\cong\frac{\pi^2 a_{oi}}{24\sqrt{3}(a_{oi}U)^{3/2}}\exp\left(2\sqrt{a_{oi}U}\right) \qquad \textbf{(12)}$$

which are fairly simple and contains only one parameter a_{oi} defined as,

$$a_{oi}=\frac{\pi^2}{6\varepsilon_{oi}} \qquad \textbf{(13)}$$

and represents a collective level density parameter corresponding to the i th band with the unit energy ε_{oi}. The unit energy ε_{oi} is the energy difference of the low-lying energy levels. For example, the unit energies are $\varepsilon_{0GS}=E(2^+)$, $\varepsilon_{0\beta}=E(2^+)-E(0^+)$ and $\varepsilon_{0oct}=E(3^-)-E(1^-)$ for ground-state, β and octupole bands, respectively. Since the β and octupole bands have not ground-state energy levels, we have used suitable unit energy E(2^+)- E(0^+) for β band and E(3^-) - E(1^-) for octupole band. In the odd-odd and odd-*A* nuclei it has been shown that the unit energy is either energy of first excited state (for ground state bands) or the energy separation between the second and first excited states (for excited bands) of the corresponding band with given the projection of the total angular momentum K. As mentioned before, these band energies clearly should, at least approximately, satisfy Eq.(11).

3. RESULTS AND DISCUSSION

Now, it can be compared the observable level density expression of Eq. (6) and Eq. (12) which have similar dependence on the energy, even they have been obtained from different approaches. Eq. (6) obtained from Bethe theory has been based on a single-particle nuclear model whereas Eq. (12) has been extracted from symmetry properties of the nuclear spectra data expressed by Eq.(11). So, our approach which are succesfully used earlier for the classification of the level density parameter using Ref. (Ahmadov et al., 2002; Okuducu and Ahmadov, 2003) in this study takes into consideration different collective-excitation modes of light deformed nuclei considered in the nuclear level density parameter calculation. That is to say the investigated of the parameters a_{oi} in our prescription has been made simply from nuclear spectra data obtained from in Ref. (Nuclear Structure and Decay Data, 2001) by the use of Eq. (13).

In the present study we have seen that the energy levels of different excitation bands (in particular, the bands given in Table) for even-even, odd-odd and odd-*A* superdeformed light nuclei also approximately satisfies Eq. (11). Thus, Eq. (13) can be applied for estimation of the corresponding level density parameters. The calculated values of the level density parameters due to different excitation bands for some superdeformed light nuclei have been shown in Table. In figure, we illustrate the comparison of the single-particle level density parameters a versus the mass number with our calculated values of a_0 for corresponding to the different bands in the region of some deformed light nuclei. The demonstrated values of the parameters a were compiled by Refs. (Gilbert and Cameron, 1965; Baba, 1970; Belgya et al., 2005) for s-wave neutron resonances near the neutron binding energy. From figure, it is clear that the values of the level density parameters a_0 calculated by Eq. (13) for corresponding to the different bands are in good agreement with the compiled values of the parameters a.

In the light of database above, we can conclude that the nuclear level density parameters of superdeformed light nuclei, with mass ranging from A= 40 to A= 70, may identified different excitation bands (octupole, β, γ), just like those in the region of large deformations. In other words, no dominant band alone is responsible for identification of level density parameters a for even-even, odd-odd and odd-*A* nuclei of the region of interest, and the nuclear level density for such nuclei involves combination of the partial level densities corresponding to the

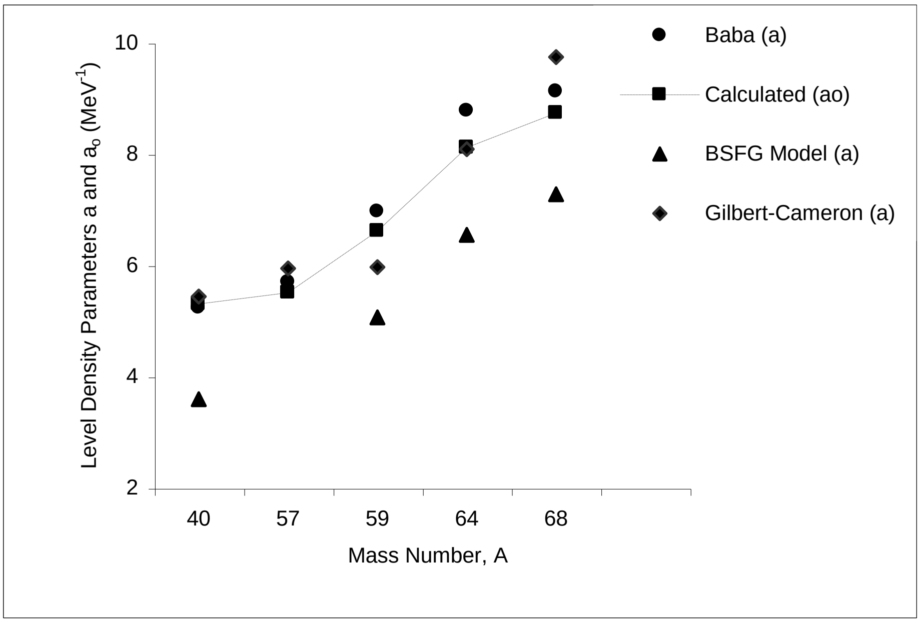

Figure. Mass dependence of nuclear level density parameters a_0 and a for some superdeformed light nuclei. The values of parameters a_0 are calculated for different bands, and the theoretical results for BSFG Model are taken from Ref.(Belgya et al., 2005)

different bands, which is given by Eq. (10). The energy level population at any excitation near the neutron binding energy may clearly have different character such as collective rotational, collective vibrational, intrinsic and so on. This property of nuclear excitations, as it is well established especially for superdeformed light nuclei, changes from nucleus to nucleus. Consequently, we remark that the present level density parameter calculations based on the properties of measured nuclear low-lying level spectra should prove a productive area of study that should override the inherent experimental difficulties involved, at least in the region of superdeformed light nuclei.

4. REFERENCES

Ahmadov, H., Zorba, I., Yilmaz, M., Gonul, B. (2002). *Nucl. Phys.* 706(A), 313.

Baba, H. (1970). *Nucl. Phys.* 159(A), 625.

Belgya, T., Bersillon, O., Capote, R., Fukahori, T., Zhigang, G., Goriely, S., Herman, M., Ignatyuk, A.V., Kailas, S., Koning, A., Obloz-hinsky, P., Plujko, V. and Young, P.(2005). Handbook for Calculations of Nuclear Reaction Data: Reference Input Parameter Library,http://www-nds.iaea.org/RIPL-2/, IAEA, Vienna.(BSFG Model).

Bethe, H. A. (1936). *Phys. Rev*. 50, 332.

Bethe, H.A. (1937). *Mod. Phys. Rev*. 9(2), 69.

Bloch, C. (1954). *Phys. Rev*. 93, 1094.

Bohr, N., Kalckar, F. (1937). *Math.- Fys. Medd*. 14, 10.

Bohr, A. and Mottelson, B. (1969). Nuclear Structure, Volume I, Benjamin, Reading, M.A.

Davidson, J. P. (1968). Collective models of the Nucleus, Academic Press, NY.

Gilbert, A. and Cameron, A.G.W. (1965). *Can. J. Phys*. 43, 1446.

Ignatyuk, A.V., Istekov, K. K., Smirenkin G.N. (1979). *Sov. J. Nucl. Phys*. 29, 450.

Nilsson, S. G. (1955). Kgl. Danske. Videnskab. Selskab. *Mat. Fys. Medd*. 29, 16.

Nuclear Structure and Decay Data. (2001). National Nuclar Data Center, Brookhaven National Laboratory, ENSDF (Evaluated Nuclear Structure Data File), Upton, NY.

Okuducu, Ş., Ahmadov, H. (2003). *Phys. Lett*. 565(B), 102

Pezer, R., Ventura, A., Vretenar, D. (2003). *Nucl. Phys*. 717(A), 21.

Puddu, G., Bortignon, P. and Broglia, R. A. (1990). *Phys. Rev*. 42, 1830.

Puddu, G., Bortignon, P. and Broglia, R. A. (1991). *Ann. Phys*. 206, 409.

Rohr, G. (1984). Z. *Phys*. 318(A), 299

Shlomo, S. and Natowitz, J.B. (1990). *Phys. Lett.* 187(B).

Woosley, S. E. (1980). In Theory and Applications of Moment Methods in Many Fermion Systems, edited by B. J. Dalton, S. M. Grimes, J. P. Vary, and S. A. Williams, Plenum, New York, p.61.

Yen, G. D. and Miller H. G. (1992).*Mod. Phys. Lett.* 7(A), 1503.

Savaş Sönmezoğlu, 1980 yılında Kahramanmaraş ilinde doğdu. İlkokul ve ortaokul eğitimini Kahramanmaraş'ta tamamladıktan sonra, Ankara Atatürk Anadolu Lisesi'nden 1998 yılında mezun oldu. Aynı yıl kazandığı Karadeniz Teknik Üniversitesi Eğitim Fakültesi Fizik Öğretmenliği Bölümü'nden 2003 yılında başarıyla mezun oldu. 2003 yılı Eylül ayında Gaziosmanpaşa Üniversitesi Fizik Bölümü Nükleer Fizik Anabilim Dalında yüksek lisans öğrenimine başladı ve 2006 yılında yüksek lisans öğrenimini tamamladı. 2006 yılında aynı üniversitenin Fizik Anabilim Dalında doktora öğrenimine başladı. Halen öğrencisi olduğu Fen Bilimleri Enstitüsünde 2004 yılından beri Araştırma Görevlisi olarak çalışmaktadır. Evli ve bir çocuk babasıdır.

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Şeref Okuducu, 1968 yılında Kahramanmaraş' da doğdu. İlk ve orta öğrenimini K.Maraş Yatılı İlköğretim Bölge Okulunda tamamladıktan sonra, Adana Düziçi Öğretmen Lisesi'nden 1987 yılında mezun oldu. Aynı yıl Orta Doğu Teknik Üniversitesi Eğitim Fakültesi Fizik Bölümü'nü kazandı. Bir yıl hazırlık okuduktan sonra, Fizik Bölümü'nden 1992 yılında mezun olup, aynı yıl Emniyet Genel Müdürlüğü'ne bağlı Ankara Polis Koleji'nde Fizik öğretmeni olarak göreve başladı. 1993 yılında Gazi Üniversitesi Fizik Bölümü Nükleer Fizik Anabilim Dalında Yüksek Lisans öğrenimine başladı ve 1995 yılında yüksek lisans öğrenimini tamamladı. 1996 yılında aynı üniversitenin Nükleer Fizik Anabilim Dalında doktora öğrenimine başladı ve 1999 yılında tamamladı. Kasım 2000 yılında Gaziosmanpaşa Üniversitesi Fen Edebiyat Fakültesi Fizik Bölümü Nükleer Fizik Anabilim Dalında Yardımcı Doçent olarak göreve başladı. 2004 yılında aynı üniversitede Doçentlik ünvanını almış ve 2006 yılından beri Gazi Üniversitesi, Fen Edebiyat Fakültesi, Fizik Bölümü, Nükleer Fizik Anabilim Dalında görevine devam etmektedir. Evli ve iki çocuk babasıdır.