Hacettepe Üniversitesi Eğitim Fakültesi Dergisi (H.U. Journal of Education) 32(1): 123-142 [2017]
doi: 10.16986/HUJE.2016018796

Students' Mathematization Process of the Concept of Slope within the Realistic Mathematics Education*

Eğim Kavramının Gerçekçi Matematik Eğitimi Yaklaşımı Altında Matematikleştirilme Süreci

Ömer DENİZ**, Tangül UYGUR-KABAEL***

• ***Received:*** *15.10.2015* • ***Accepted:*** *06.06.2016* • ***Published:*** *31.01.2017*

ABSTRACT: The aim of this study is to analyze eight grade students' mathematizing processes of the concept of slope in light of the teaching process designed based on Realistic Mathematics Education. Participants of this design based research study were chosen via purposeful sampling in accordance with results of an open-ended test intended for prerequisite information before design teaching-learning environment. Survey data was acquired from open-ended test, researcher log, individual and group study papers and clinical interviews. Contexts, which provide opportunities to reflect preliminary conceptions of slope to teaching process are revealed. Conceptualization process of slope as ratio started with the need to be base of informal linear constant. Then it develops with the need of measurement of slope. Students reinvent that the slope is also an algebraic ratio by reflecting geometric ratio representation to coordinate plane. Designed teaching in this study was revealed that mathematization is important in transition among representations. Development of right triangle model occurred in three phases: (i) a context-dependent interpretive tool, (ii) a physical tool used to calculate the slope, (iii) a cognitive tool that that is no longer needed to be represented physically.
Keywords: Slope, Mathematization, Realistic Mathematics Education

ÖZ: Bu çalışmada Gerçekçi Matematik Eğitimine dayalı olarak desenlenen öğretim sürecinde sekizinci sınıf öğrencilerinin eğim kavramını matematikleştirme süreçlerinin incelenmesi amaçlanmıştır. Desen tabanlı bu araştırmanın katılımcıları, ön koşul bilgilere yönelik olarak hazırlanan açık uçlu testin sonuçlarına göre amaçlı örneklem yolu ile seçilmiştir. Araştırmanın verileri, açık uçlu test, araştırmacı günlüğü, bireysel ve grup çalışma kâğıtları ile katılımcılarla öğretim boyunca birebir gerçekleştirilen üçer klinik görüşmeden elde edilmiştir. Eğimin başlangıç kavramsallaştırmalarının öğretim sürecine yansıtılmasına fırsat veren bağlamlar ortaya konmuştur. Eğimin bir oran olarak kavramsallaştırılması süreci informal olarak edinilen doğrusal sabitlik kavramsallaştırmasına dayanak yaratma gereksinimi ile başlamıştır. Ardından eğimi ölçme gereksinimi ile gelişim göstermiştir. Eğimin geometrik oran temsilinin koordinat düzleminde yansıtılması yoluyla aynı zamanda cebirsel bir oran oluşunun öğrenciler tarafından keşfedilmesi sağlanmıştır. Ortaya konan öğretim temsiller arası anlamlı geçişlerde matematikleştirmenin önemini ortaya koymuştur. Ayrıca öğrenciler tarafından geliştirilen dik üçgen modelinin gelişimi üç aşamada olmuştur: (i) duruma bağlı olarak yorumlamaya yarayan bir araç, (ii) eğimin hesaplanmasında kullanılan henüz fiziksel olarak ortaya konulma gereği duyulan bir araç (iii) fiziksel olarak ortaya konulma gereği duyulmayan bilişsel bir araç.
Anahtar sözcükler: Eğim, Matematikleştirme, Gerçekçi Matematik Eğitimi

1. INTRODUCTION

Students have informal knowledge of certain mathematical terms in their daily lives before having been introduced to terms formally at school. Slope could be one of such. Reflections of slope such as "steep", "elevation", "descent" and "inclined" are common in daily life. The concept of slope exists in various science fields like art, architecture, engineering, and physics. Slope is a pre-requisite concept for a lot of advanced mathematics concepts, especially for

* The present study which is differentiated by master thesis of first author is based on a part of a scientific research project supported by Anadolu University, Scientific Research Projects (BAP). Project number: 1401E005.

**Mathematics Teacher, İnegöl Fenerbahçeliler Derneği Hamamlı Secondary School, Bursa-Turkey, e-mail: omeraga86@gmail.com

*** Doç. Dr., Anadolu University, Faculty of Education, Department of Elementary Education, Eskişehir-Turkey. e-mail: tuygur@anadolu.edu.tr

derivative (Nagle, Moore-Russo, Viglietti & Martin, 2013). Slope is expected to conceptualize as a constant rate of change in middle school, as average rate of change in high school. With this progress in middle and high school, students could be ready for instantaneous rate of change so for derivative (Nagle & Moore-Russo, 2014). As Stanton and Moore-Russo (2012) indicated it is important to consider former conceptualizations of slope in teaching of this concept in eight grade in which it is expected to make meaningful transition from informal knowledge to formal concept. In this study, a learning environment aiming to relate various conceptualizations of the slope was designed and eight grade students' mathematization process of this concept in this learning environment was described.

1.1. The Concept of Slope

People have been faced with environmental slope case like ramp, incline, mountainside and pay regard to steepness. Then they needed to calculate the measurement of the steepness. For instance, architects had to calculate the steepness mathematically while constructing building, road or ramp (Sandoval, 2013). Therefore, as Lobato and Thanheiser (2002) indicate, slope provides the measurement of steepness. Simon and Blume (1994) state that the concept of steepness is a ratio and this ratio should be seen as a measurement. In this regard, the slope is rate of change of vertical distance relative to horizontal distance, while moving on a linear visual from daily life. This ratio can be calculated as "rise over run". On the other hand, literature shows that students calculate the slope by "rise over run" formula without understanding why this formula is used and what it means (Crawfort & Scott, 2000). Lots of students' conceive of slope only as a number, not as a measurement of steepness or rate of change of the vertical distance relative to the horizontal distance (Lobato & Thanheiser, 2002). Barr (1981) claims students' difficulties with this notion result from only focusing on the memorization of certain rules. To give "rise over run" to students as a formula hinder them to construct the slope as a ratio (Walter & Gerson, 2007). Cheng (2010), emphasizes on the relationship between proportional reasoning skills and the constructing the concept of slope and reached the conclusion that students, not being able to form the concept of ratio meaningfully, were able to learn concept only practically. Simon and Blume (1994) conducted a teaching experiment with prospective elementary teachers and asserted that using various ramps, having same slope, can be important in order students conceptualize the slope as a ratio. Lobato and Thanheiser (2002) conducted a teaching experiment with high school students and concluded that there are four components in teaching process of the concept of slope: isolating the attribute that is being measured, determining which quantities affect the attribute and how understanding the characteristics of a measure, constructing a ratio. Rene Descartes (1595-1650) who reflected the slope to the analytic plane gained this concept an algebraic meaning. In this case, given two points (x1,y1) and (x2,y2), the change in x from one to the other is x2 – x1 (run), while the change in y is y2 – y1 (rise). Substituting both quantities into the ratio generates the formula: y2-y1/x2-x1. When the slope was developed as algebraic ratio, it was invited that the slope is a parametric coefficient in the line equation (y=mx+n) and then slope was started to show steepness and direction of the line (Lobato & Thanheiser, 2002). If the slope is negative then the line will go down to the right and if the slope is positive then the line will go up to the right. According to Sandoval (2013), who emphasizes the direction of slope, architects need the slope to build anything straight. Such as roads, houses, buildings, and ramps. Considering the history of mathematics, even the transition from geometric ratio to algebraic transition is apparent, literature indicates that students are often taught slope as a fraction, with the change in y over the change in x not as a ratio (Walter& Gerson, 2007). Moreover, students develop algorithmic knowledge about the slope (Nagle & Moore-Russo, 2013). It was seen in literature that teaching process, not giving opportunity to relate various representations of slope, stimulate this unfavorableness (Stump, 1999, 2001). Stump (1999, 2001) emphasizes on the importance of associating varied representations of slope in terms of conceptual learning and collected these representations under eight topics; geometrical ratio (rise over run), algebraic ratio (y2-y1/x2-x1), physical property (daily mean), real world situation (static, physical situation or dynamic functional situation), functional characteristics (rate of change between variables), parametric coefficient (y=mx+n), trigonometric concept (tangent) and calculus concept (relationship between derivatives). Moore-Russo, Conner and Rugg (2011) extended the eight conceptualizations of slope by adding three conceptualizations: determining property (property that determines whether lines are parallel or perpendicular), behavior indicator (property that indicates the increasing, decreasing, or horizontal trends of a line) and linear constant ("straight" absence of curvature of a line that is not impacted by translation). Stanton and Moore-Russo (2012) investigated how different conceptualizations of slope were considered in U.S. schools. They concluded that the concept of slope should gain both algebraic and geometric formal meaning in eighth grade. To regard students' initial conceptualizations in teaching process provide opportunities students to develop a concept schema including relations among representations. Students should be given opportunities to relate different representations in learning environment of the slope (Moore-Russo, Conner & Rugg, 2011). On the other hand, Stanton and Moore-Russo (2012) indicate that there was a need to study how students move among different conceptualizations of slope. In this study, it is aimed to design a teaching environment such that eighth grade students can construct the slope as geometric and algebraic ratio by reflecting their informal knowledge about slope. The most appropriate approach for this aim was thought as the Realistic Mathematics Education (RME) due to giving students opportunity to reflect their informal knowledge to the learning process in which reinvention of the concept with interconnecting its different conceptualizations is provided. Moreover, it was aimed to investigate students' mathematizing processes of the concept of slope throughout the teaching process designed on basis of RME approach. As mentioned in above, most of the studies related to the slope concept are on high school or college degree in literature. Therefore this study has an original value. Moreover, it also provides an alternative teaching aiming conceptual learning. The study mainly aimed to answer the following research questions.

- How does mathematizing process of slope concept by eighth grade students in this designed teaching environment?
- What are the relationships between mathematization process of the slope concept and the prerequisite knowledge?

1.2. Theoretical Framework

Literature asserts when the knowledge is directly given to the students and various conceptualizations are handled without relating, students are not able to gain the slope concept. Based on the studies, first students should be aware of slope in daily life and then the ratio should be reinvented with the need to measure the slope in the teaching process as in the history of this concept. To construct the slope as algebraic ratio by reflecting the geometric ratio to coordinate plane can overcome algorithmic learning of this concept. In this study, RME, which gives students opportunities to reinvent the concept by relating their informal knowledge in a daily life context.

1.2.1. Realistic Mathematics Education (RME)

Freudenthal (1968) emphasized that mathematics is a human activity and should be taught in the light of RME through mathematization. Gravemeijer (1994) defines activities in a learning process starting with context problems such as composing a problem and organizing it, generating mathematical basis, relating new information with previous. Treffers (1987) distinguishes what he calls progressive mathematization concept into *horizontal* and *vertical mathematization* claiming that the individual should mathematize the target concept or the strategy discovered step by step at each instant based on formal and informal knowledge formerly acquired. In horizontal mathematization student starts from one context to find present relationships in order to model the present situation and then creates a meaning out of it by using designators such as tables, graphs and formulas (Nkambule, 2009). Activities composed of reasoning; generalization and formulation on abstract structures built on horizontal mathematization activities are called vertical mathematization (Rasmussen, Zandieh, King & Teppo, 2005).

There are three heuristics in a teaching process designed in RME framework: *didactical phenomenology, guided reinvention*, and *emergent models* (Gravemeijer, Bowers & Stephan, 2003) (For detailed information see Gravemeijer, Bowers and Stephan (2003)). One of fundamental heuristics of RME is that students should be encountered to contexts, which they see as real. Didactical analysis is needed for reinventing the concept. *Didactical phenomenology* forms the basis for the designing of the teaching process, in which students discover mathematics by presenting the relationship of a concept with others, strategies and notions. The other heuristic, *guided reinvention*, reflects the process of teaching providing opportunities to students to discover the concept themselves and make progressive mathematization starting from context problems. It is important to image a possible teaching process so that students can reinvent the concept (Gravemeijer, 1999). According to Gravemeijer (1999), development in history of mathematics can be a source of inspiration for a possible teaching process. Pursuant to *emergent model* with a critical role in terms of filling the blanks and interrelating formal and informal knowledge, opportunities should be created for students in order to develop their own models and use them during the problem solving process (Fauzan, 2002). Gravemeijer (1999) defines four levels of activity to help students to develop their own models: *task setting (situational), referential, general* and *formal.* During a learning process based on RME first models developed particular to context situation are ones foreknown by students (Fauzan, 2002). This level, in which interpretation and computations are based on how students understand the contextual situation, is called by *situational level.* When there is a need to use these models and strategies, the activities in *referential level* were carried out (Wijaya, Doorman & Keijze, 2011). In *general level*, the model is used in various computations and interpretations (Gravemeijer, 1999). Finally presence of models in terms of cognitive entity, takes place in the process of vertical mathematization and these models can be used as a model for mathematical reasoning (Gravemeijer & Doorman, 1999). Streefland (1985) uses the expression “model of” for modeling developed particular to the context whereas he uses “model for” for modeling generalized free from the problem situation (quoted in Gravemeijer & Doorman, 1999: 117).

2. METHOD

Design of this qualitative study is Design Based Research (DBR). The aim of DBR studies is to develop an innovative and iterative learning environment (Gravemeijer & Cobb, 2006). In DBR studies aiming to develop a local instruction theory three phases are followed: *preparation and design phase, teaching experiment* and *retrospective analyses* (Bakker & Van Eerde, 2015). These phases are explained in 2.1.2, 2.1.3 and 2.1.4 sections. For more detailed information, Bakker & Van Eerde (2015) should be appealed.

2.1. Data Collection and Learning Environment

Data for this qualitatively designed study were collected through open-ended test evaluating the prerequisite knowledge, researcher logs, individual and group papers of students comprised of teaching process and clinical interviews.

2.1.1. Participants

Participants of the study were selected through purposeful sampling method (Yıldırım & Şimşek, 2005) among eighth grade students instructed by one of the researchers of this study. Before the teaching process, it was concluded that line equation, ratio-proportion, dependent-independent variable were the basic prerequisite concepts attained from literature and from experiences of the researchers an open-ended test was prepared. Open-ended test was applied to 16 eighth graders. Students were then separated into five groups according to their performances on test (Figure 1).

1st Group: Three students in this group were able extend the pattern by using the recursive strategy, verbally express the general rule but were not able to see the dependency among two variables. They made mistakes in comparing the quantities and were not able to make proportions. These students without recognition of linearity were not able to show presence in terms of linear equation concept.

2nd Group: Four students in this group were able to extend the patterns only by using recursive strategy being able to verbally express the general rule as well as seeing the dependency relationship among the variables; however they weren't aware of the variable concept. They made mistakes in determining the points on coordinate plane and depicted that they had no recognition of linearity. They were also not able to show presence on linear equation.

3rd Group: Three students in this group who were able to generalize the patterns by functional thinking had problems in symbolization. They were able to recognize the dependency relationship among variables and showed recognition of variable concept. While making mistakes in determining the points on coordinate plane, they stated opinions on linearity. They were able to recall only some of the info practically acquired for the linear equation.

4th Group: Three students in this group were able to think of patterns functionally and had no problems in symbolization process. Having recognized the dependency relationship among variables, they were aware of the concept of variable as well. They made mistakes in determining the points on coordinate plane but were able to state ideas on linearity. They were able to recall only some of the information they acquired practically for the linear equation.

5th Group: Three students in this group who were able to generalize the patterns by functional thinking and who had no problems in symbolization were also able to recognize the dependency relationship among the variables being aware of the variable concept. These students with the highest performance were able to depict the points on coordinate plane without any mistakes. They showed recognition of linear concept and were able to make a presence in terms of practical and cognitive ways.

Figure 1: The groups emerged based on the students' performances on test

One student from each group, who were thought to have no trouble in communication skills by the researcher, was selected as representatives. These five are the participants for this study. Participants are coded respectively as S1, S2, S3, S4 and S5 in accordance with the group they represent.

2.1.2. Preparation for the Teaching Experiment

In this phase starting points, relevant to students' pre-knowledge, learning goals, mathematical problems and assumptions are determined. Mathematical problems and assumptions are constructed based on students' potential learning processes. In the present study, in order to determine participants' starting points, an open–ended test was developed. Open-ended questions were on the concepts that were determined as pre-requisite concepts for slope. For instance, independent-dependent variable concepts were assessed with the problem as seen in Figure 2.

A daily construction worker earns 5 liras in a day, 10 liras in 2 days , 15 liras in 3 days, ... Accordingly,

a) Fill in the blanks on table below. Find the rule that tells the money to be made in "n" days.

b) Demonstrate the rate of the number of days to the money earned for each day (for the 1st, for the 2nd day,...). What does attract your attention in the relationship between these rates? What is the name given to this relationship between rates in mathematics?

Number of Days	1	2	3	4	5	6	7	...	n
Money Earned (TL)	5	10	15					...	

Figure 2: An Example of open-ended test related to independent-dependent variable

Stanton and Moore-Russo (2012) emphasize the significance of taking initial conceptualizations into account while students are introduced to the concept of slope. In this study, students are expected to call the physical properties, informal conceptualizations of slope in real world situations and its linear constant property in various contextual situations. Due to the need to create a mathematical basis for the informal linear constant conceptualizations and to measure steepness, it was foreseen that the students will reinvent the formation of geometric ratio. In order for the slope to be readjusted as an algebraic ratio, we aimed that students would be able to reflect the geometric ratio to the coordinate plane. We expected that the conceptualization of the linear constant would support this process. Accordingly our learning goals and assumptions about learning process are seen in Figure 3.

1st Learning Goal: Recall the notion of slope from their daily life (Conceptualizations: Real-world situation and physical property).

Conjectured learning process:

- Students can reflect their daily life knowledge in the context of riders' difficulties with the ways having various slopes to learning process.

2nd Learning Goal: Realize the variables, which the slope is dependent on (Conceptualizations: Real-world situation and physical property).

Conjectured learning process:

- Students can notice that vertical and horizontal distance affect the slope in the contexts in which they inquiry the changing of the slope.
- Students can construct relationship among the slope, horizontal and vertical distance by constructing situation specific models (situational level).

3rd Learning Goal: Realize the slope is not change on different points on the same linear visual (Conceptualization: From real-world situation and physical property to linear constant property).

Conjectured learning process:

- Students assumed to have need a foundation for the informal knowledge that the slope is not change on various points on same visual. Then, students can inquiry if the slope is change when the horizontal and vertical distances change.
- Students assumed that they could reason by using situation specific models (situational to referential level).

4th Learning Goal: Invent the slope is a ratio (Conceptualizations: From real-world situation and linear constant property to geometrical ratio).

Conjectured learning process:

- Students can invent that vertical distance over horizontal distance on various points of a linear visual is constant and this ratio is slope.
- When students need to find the slope, students can calculate the slope as a ratio.
- It was not expected that the model could be independent from situation in this level. Students can use the model for calculation, interpretation and justification as a tool in this level (referential level).

5th Learning Goal: Interpret the ratio of slope algebraically (Conceptualizations: Geometrical ratio and linear constant property to algebraic ratio).

Conjectured learning process:

- Students can extend their understanding of the slope as a ratio to coordinate plane for a line slope
- Through progressive mathematization activities, students can invent "y_2-y_1/x_2-x_1" formula for the slope.
- The model can be used by interpreting without drawing in this level (referential to general level).

Figure 3: Learning goals and assumptions about learning process of the slope

2.1.3. Teaching Experiment

DBR studies required teaching experiments aiming improve the course of action in order to reach the learning goals (Bakker & Van Eerde, 2015). In preparation and conduction processes of teaching experiment, design researchers conduct thought experiments on how students can progress practically and theoretically and how teachers support their progression (Freudenthal, 1991). In this study, researchers conduct thought experiments on how students can construct slope concept as a ratio based on the literature and then the teaching process designed on basis of RME approach. This teaching process is detailed in 2.1.3.2.

2.1.3.1. Learning environment

At present study, five heterogeneous groups were made in accordance first with the prerequisite knowledge students have, difficulties faced and mistakes made in the preliminary test. While great care was taken for the five participants to stay in different groups, they were also assigned to take notes to group work sheet during intergroup sessions. Individual and group work sheets composed of notes taken during the teaching process of intergroup and in-group sessions, individual performances of participants, generalizations made and symbolizations acquired were also used as a data collection tool. In order to minimize the risk of data loss on essential and important points such as the interaction of participants with their mates along with their individual performances, class order, in-group and intergroup session processes were recorded to researcher log.

On a teaching process based on RME starting a lesson with a context that is experientially real to the students helps them and creates opportunities for them to recall their informal knowledge and solution strategies for the process. For the teaching process planned out for this study, a total of seven real life contexts were prepared for the notion of slope. These contexts composed of three phased periods with two hours of class were planned out as a total of six hours of class for the teaching process with four during first two periods (1st and 2nd learning goals), two during following two periods (3rd and 4th learning goals) one during last two periods (5th learning goal). Individual and group work sheets used during the process were collected by the researcher at the end of each class. One on one clinical interviews following each two class periods were made for the purpose of acquiring in-depth information on mathematization processes of the participants. Over the course of the study, total of 15 clinical interviews were made to track the mathematization process cognitively.

2.1.3.2. Teaching process designed based on RME

During the first two hours period of class, a common scene from daily life context of a cyclist going up a slope was handled (Figure 4). It was estimated that each participant would have an opinion regarding this context situation, which aimed to create a need for the interpretation of slope. It was thought that this context would arouse attention in class and would be a driving force supporting their mathematical skills.

Figure 4: Cyclist on road with different slope

Following the previous one, two more contexts respectively with same horizontal distances and different heights and with same heights and different horizontal distances with a theme of climbing to the peak point of a mountain foot on foot for the students to discover variables related to slope (Figure 5) were presented. It was assumed that students could develop a situation specific model and notice the horizontal and vertical distances by relating the slope. The instructor then addressed the following questions for students to discover the relationship between the slope of a linear visual with variables such as height, horizontal distance, angle, through inquiry between in-group and intergroup sessions: *"Who has the most difficulty?, Why does he have the most difficulty?, Why do you think this road is steeper?, If the heights are the same what about the slope?, Explain why if you think heights are same but the slopes are different?, Are there any* *other factors affecting the slope other than height?"* Likewise, students were inquired for the purpose of guiding towards the discovery with pre-prepared questions such as *"Which lengths change with the change of that angle?, How does the change in angle affect the slope?"* against the possibility of interrelating slope with angle.

Figure 5: Contexts enabling the discovery of dependency relationship of slope with height and horizontal distance.

On class covering the third and fourth teaching periods, whole class was exposed to a context situation for the purpose of realizing that slope does not change on the same linear visual according to the point taken or recalling of the information from their informal experiences (Figure 6). Students faced the most critical step of their discovery in this context where they felt that slope shouldn't change in line with their daily life experiences but faced difficulties in supporting their claims with mathematical basis. Since students faced a cognitive imbalance due to a change both in height and the horizontal distance, the aim was for them to discover the ratio between height and horizontal distance. In this level it was thought that the model, which was expected the students develop, can gain dynamic structure. Moreover it was assumed that students can see that changing of horizontal and vertical distances were proportional by using this dynamic model. Instructor addressed the following questions for them to discover that the proportional relationship between the height and the horizontal distance did not change the slope: "*Why does/doesn't the slope change?, What does and doesn't it change when people walk along the mountain foot?, What causes the slope not to change?, If the height and the horizontal distance change and the slope remains the same how does this happen?, Are there any unchanged factors leaving the slope stable?, (After the students use correct, point tags), The slope doesn't change while the person or points on the line change, what else remains the same?, Height and horizontal distance change but what doesn't?, (After the discovery that the ratio between the two remain same) Slope along with something else remains the same then what does the slope equals to?".*

Figure 6: Context depicting the unchanged of slope to the point taken on the same linear visual, mathematization of slope as a ratio between height and horizontal distance

During the process of discovering that the proportional relationship between height and horizontal distance equals to slope, discussions on leading questions were held and after solving questions requiring slope calculations with students they were provided with an opportunity to enter the vertical mathematization process. Likewise, to prevent a disconnection in their cognitive processes homework were given from textbooks and workbooks with questions related to physical, real world situation and geometric interpretation of slope.

At last two periods, lesson started with a context of climbing a highland and students were expected to discover how to calculate the slope of a line on coordinate plane within this context (Figure 7).

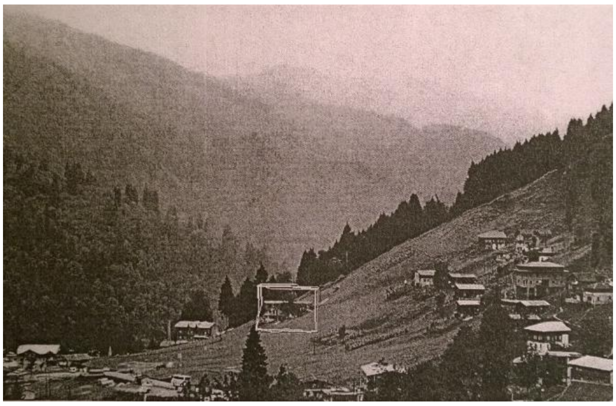

Figure 7: Graphic display of the relationship between height and horizontal distance and transition context to a slope of a line on coordinate plane

On highland context selected for attracting students' attention for the required mathematization, they were asked to question how the slope will change for a person on highland for the purpose of recalling the knowledge that the slope doesn't change and the ratio between height and horizontal distance remains the same. With the help of table and graphic depicting the distance travelled horizontally and vertically on highland, students were pushed into in-group sessions where they inquired how to show whether the slope changed or not and with intergroup sessions an opportunity was given for them to see that even though everyone came up with different height and horizontal distances the slope remained constant. Students in process of calculating the slope of the linear graphic started to get the hint that height and horizontal distances could be found by making use of axes. Context then progressed with activities providing opportunity for a step by step mathematization offering transitions from geometrical ratio to algebraic ratio such as the slope of a line respectively visualized on coordinate plane. In this level, it was expected that referential model can be revealed and then it can be a mental entity that can be used as y2-y1/x2-x1 without reflect on coordinate plane. Following questions were asked for progression to discovery: "Can you calculate the slope of this line segment on the given coordinate plane?, If two points are given how would you find it?, Can you calculate the line without drawing?, Think what you need for the slope and how you can find it without drawing, If the numerical values are large and such large coordinate plane cannot be drawn then how would you calculate the slope of a line passing from these two points?, Can you make an algebraic generalization?, If more than two points of a line were given then what would you do?". Homework including questions related to the slope of a line on coordinate plane were given at the end of last two periods.

2.1.4. Retrospective Analyses

The fundamental principles of DBR studies consist of analyzing the research data in a holistic approach and refining the developed local instruction theory (Gravemeijer & Cobb, 2006). Researchers' observations during the teaching experiments may be inconsistent with their initial assumptions. Based on this inconsistency, the local instruction theory has to be revised and the observed trajectory should guide the subsequent design research (Bakker & Van Eerde, 2015). During the process of retrospective analysis, all data should be analyzed together with the intention of finding patterns that can explain the progress of the students (Gravemeijer & Cobb, 2006). In this study, after each teaching session clinical interviews were conducted with each participant. Clinical interviews, researcher logs, individual and group papers of students were analysed based on participants' mathematization process and if the assumptions were met.

In this study, two types of analyses were used. First, in order to direct the process, identify the necessary revisions and to closely examine the mathematization process we conducted ongoing preliminary analysis of data collected from the clinical interviews. In order to describe the mathematization process in a coherent manner, and demonstrate the consistency of learning goals and assumptions we also conducted retrospective analysis at the end of the teaching experiment. We analyzed the data obtained from in- group and inter-group discussion sessions and the researcher logs by using the method of constant comparison analysis (Bakker, 2004). Based on the research questions and learning goals, two researchers independently coded the collected data by identifying the noteworthy words, phrases, and expressions. Then, the researchers came together to compare the coding, to discuss the consistency of the data and to interpret results collaboratively. These interpretations were corroborated by insights gained from data collected through clinical interviews.

3. FINDINGS

In the teaching process designed for the purpose of mathematization of slope phases such as the "recognition of dependent variables of slope of a linear visual", "discovery of slope being the ratio between height and horizontal distance" and "readjusting of the concept of slope for a line on coordinate plane" were approached respectively. Findings will be approached under these headings.

3.1. Recognition Process of Dependent Variables of Slope of a Linear Visual

At the beginning of the first two periods of teaching process students were presented with the visuals of a cyclist on road with different slopes and were asked on which one he will have the hardest time. Following the short termed in-group sessions, they were easily able to decide that he will sweat more on the steeper one. Their explanations were based on their daily lives such as "*more steep, ramp, the descent is higher*". It was clearly seen that students weren't able to assert a sound mathematical defense with explanations "*Steepest road is d. Because he spends more energy when climbing the road in d.", "one in d has a harder time because the descent seems steeper*". S2 was able to recognize the dependency relationship among variables in preliminary test but where he couldn't make a presence on variable concept participant recognized the height variable with the help of a mate from group 5: "*d is steeper because the height between the starting and ending points are larger".* S5 who recognized the dependency relationship in preliminary test was also aware of the concept of variable in mathematical concept and lead the group to discovery and interpretation process by being able to state that the factor affecting the slope was height. Where students were expected to interpret the slope of a visual with equal heights and different slopes, S5 stood out in recognizing the formation of a right triangle model as well as discovering horizontal distance both in individual and group worksheets. This participant emphasized on individual worksheet that even though the heights were the same the reason why the slope was different was due to the horizontality of the road and with the interpretation that "*slope changed and steepness increased since the horizontal road is shorter*" S5 started making proportional relationship as well. When S5 conveyed similar interpretation on group worksheet by making a right triangle model to enlighten his mates, the other students started to use right triangle model for the following class discussions (Figure 8). Students accepting the affect of height on slope had a hard time in understanding that horizontal distance affected the slope as well.

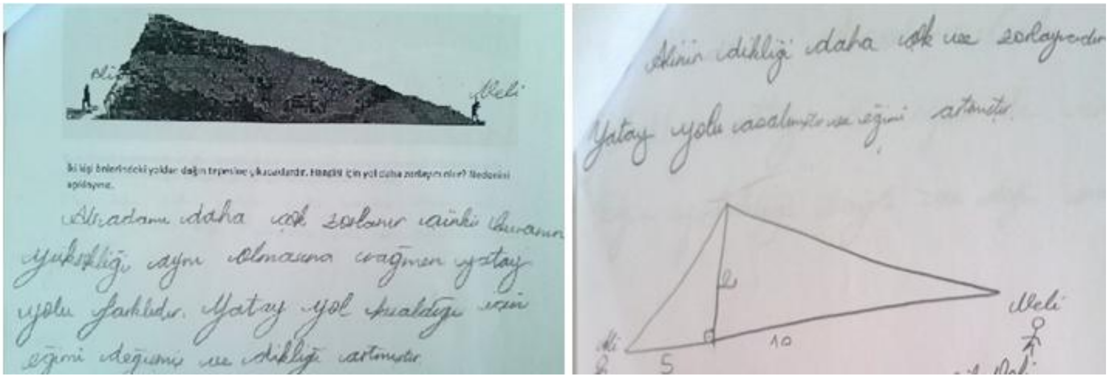

Figure 8: Images from S5’s individual and group worksheets

Following the unanticipated statement that road length is also a factor affecting the slope, all groups were addressed with question if the track length would affect the slope directly or not and a new thinking and session process was started. Most of the students faced a dilemma and during the intergroup sessions a student from S2’s group, who recognized variable concept with no difficulty in ratio-proportion concept as well, set the example of a stick against a wall and showed that the slope might change according to the leaning point of a stick with same length (Figure 9). This student depicting statement that height and horizontal distance change even if the length of a line remains the same was supported with his group mate S5 with assertion that “*the slope of horizontal one of two roads with same distance is zero, slope changes as the angle between the road and the horizontal distance increases”.*

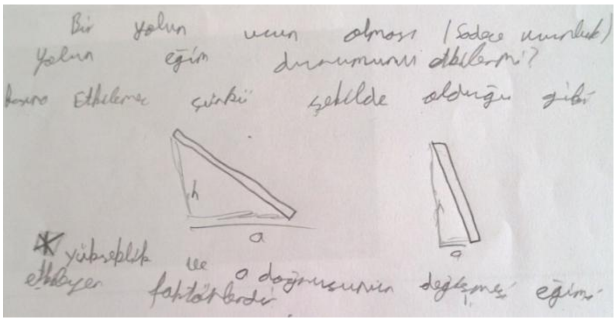

Figure 9: An image of an individual’s worksheet from S2’s group

During the interviews made after first two classes of period it was observed that all participants were able to express the height and horizontal distance as variables related to slope. Where all participants during the teaching process were observed to have used emerging right triangle model, all participants other than S1 were successful in interpreting the slope on model according to the variables affecting it. For instance, S3 was able to present models with same heights and different slopes or with same horizontal distances and different slopes taking the angle variable into consideration (Figure 10).

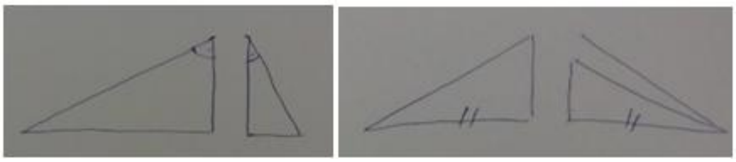

Figure 10: Images of models presented by S3 at 1st interview

It was observed during the first two periods of process that the right triangle model developed by the students had emerged specific to the situation. While this model still remained static for S1, others started to dynamically play with it.

3.2. Discovery That Slope is the Ratio between Height and Horizontal Distance

It was observed that nearly all students were able to recognize that the slope will not change on the same linear visual and that they made informal descriptions such as *"the road being straight, not sweeping, not weaving"* during the process starting with a context situation which provided them an opportunity to inquire the slope of the linear visual for people standing on different points of a mountain foot visualized linearly. However, the notion that slopes should change as a result of mathematical change in height and horizontal distance was also presented. This instability in the information students acquired from informal experiences and the information formally acquired during the teaching process prompted them to reflect upon the subject, discuss their reflections and try to create stability. When students were observed to create a cognitive balance once again, leading questions like why the slope remains the same, what does and doesn't it change while the person moves along the mountain foot pushed them towards the discovery of proportional relationship. At the end of this long and enduring waiting period two students came very close to the discovery and were given opportunities to exchange info within the group for the whole class. S4 claimed that "*the ratio of height to horizontal distance is always the same on road*" whereas S5 had reached the desired point with the discovery that "*slope and the ratio of height to horizontal distance remain the same… thus the slope equals to the ratio of height to horizontal distance*". Couples of problems and exercises were given to students for slope calculations and it was seen that they were able to calculate the slope by dividing height to horizontal distance easily (Figure 11).

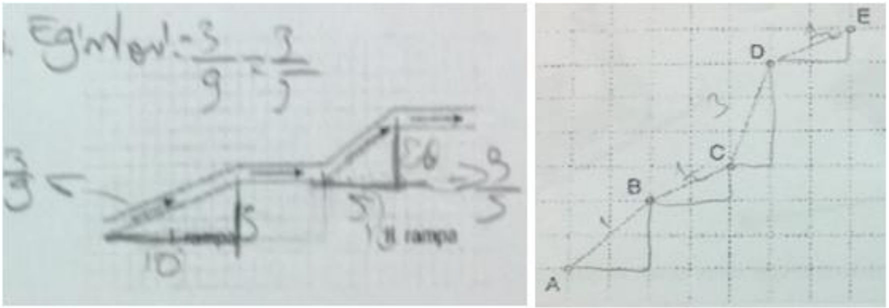

Figure 11: Images of S1's (left) and S5's (right) slope calculations

During the second interview S1 interpreted the slope of the same line or the linear visual according to a point taken on that line by saying that "*Slope changes because height changes, horizontal distance changes so does the slope*". This participant calculating the slope from rise over run was observed not being able to recognize this concept as the ratio between height and horizontal distance and recognized it only as an algorithm where the height will be divided by horizontal distance.

Where S4 wasn't able to interpret that there is a fixed ratio among directly proportional quantities in preliminary test, she didn't relate that horizontal distance and height increase and decrease in same ratio in his argument and as a result couldn't recognize the fixed ratio between two (Figure 12). Making mistakes in slope calculation by dividing the horizontal distance constantly to height supports the result that participant couldn't make sense the slope as a ratio.

S4: *If reduced to half, 1 m, meaning 1/4…*

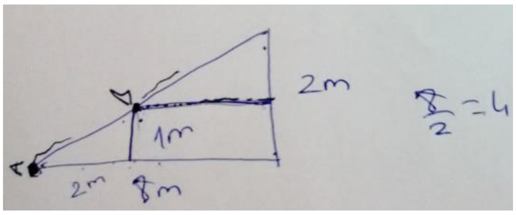

Figure 12: S4's model showing the fact that she could not make sense of the fact that rise and run increase and decrease in the same ratio

S2 stood out in terms of taking positive steps towards proportional reasoning even though making mistakes in proportioning quantities in preliminary test and having difficulties in formally making proportions in problem situations, but still had difficulty in interpreting group mate's *"same increase and decrease ratio in height and horizontal distance"* discovery. Even though the concept of ratio was an issue, participant still adopted the slope based on proportional relationship. He emphasized on the subject that the slope will not change on the same linear visual throughout the interview by suggesting that horizontal distance and track length decrease at the same ratio. On the last interview where S2 said that the stationary state of slope on the same line according to the claim that "*Slope doesn't change. Height decreased proportionally with horizontal distance*" supports the notion that S2 was in a phase in making sense slope as a ratio.

During the second interview S3 suggested that "*slope changes since height and horizontal distance change*" according to the point taken on the same linear visual at start. When asked for an example to explain, it was seen that S3 recognised the directly proportional change of height and horizontal distance on the right triangle model and by saying "*slope doesn't change because the ratio between height and horizontal distance is always constant*" S3 provided a mathematical basis to informal knowledge for the stability of slope. With the advantage of having prerequisite knowledge, S5's ability to construct slope as the ratio between height and horizontal distance was clearly seen. This participant argued for the stability of slope on linear visual both visually and with proportional relationship on right triangle model and was even able to envision this model for argument.

> ***S5:*** *With a small triangle I'll calculate slope. Then take half of its height which equals to taking half of its horizontal distance. Or I'll take a quarter, 10%... whatever… horizontal road will be calculated based on it.*
>
> ***I:*** *Won't the slope of that triangle change if it's small?*
>
> ***S5:*** *No, because it will be proportional.*

During this process it was observed that all participants used the right triangle model as a tool for calculating or arguing their interpretations whereas during the process of interpreting slope as a ratio S2, S3 and S5 were able to dynamically move the right triangle model directly in height and horizontal model whereas S4 moved it without taking heed of the directly proportional relationship. S1 was observed to have a more static model. When the development process of right triangle model is taken into consideration, it can be said that the process turns into a cognitive tool for slope becoming independent from the modelling of the situation.

3.3. Readjusting the Term Slope for a Line on the Coordinate Plane

In a context climbing up a linearly visualised highland by bicycle, it was observed that all students recognised slope as a critical factor and calculated height by proportioning it to horizontal distance by recalling that factor. The right triangle model was observed to be made by taking the extreme points (starting and ending points of road) and an opportunity was given for them to make a diagram depicting the change in height and horizontal distance during the path. All groups were able to show the directly proportional increase of height and horizontal distance over the slope during the diagram phase which is a positive step in adoption of slope with the directly proportional increase of height and horizontal distance (Figure 13).

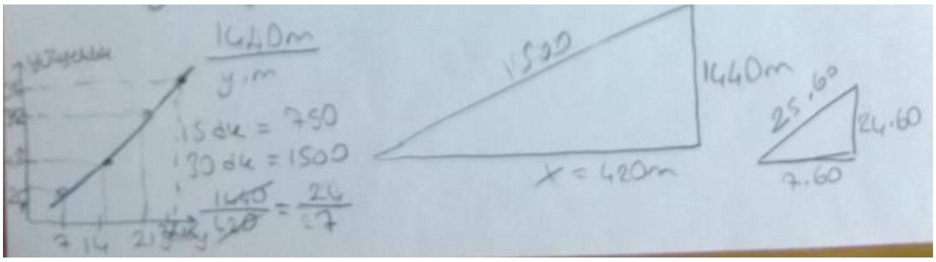

Figure 13: S3's diagram depicting the change in height-horizontal distance and slope calculation from individual worksheet.

The question how the slope of a line given on a coordinate plane can be calculated was addressed to whole class and at the end of in-group sessions all groups made similar statements saying that they will find the height and horizontal distance and proportion them to each other. In order for them to readjust slope as an algebraic ratio following assignments were given respectively: (i) slope of a line segment visualized on coordinate plane, (ii) slope of a line with coordinates from only two points and (iii) two points with high numerical values of coordinates and finding the slope of a line passing these two points. During this process enabling step by step mathematization it was observed that when finding the slope of a line visualized on coordinate plane all groups used the right triangle model accepting the line as hypotenuse to calculate and proportion the height and horizontal distance with the help of axes (Figure 14).

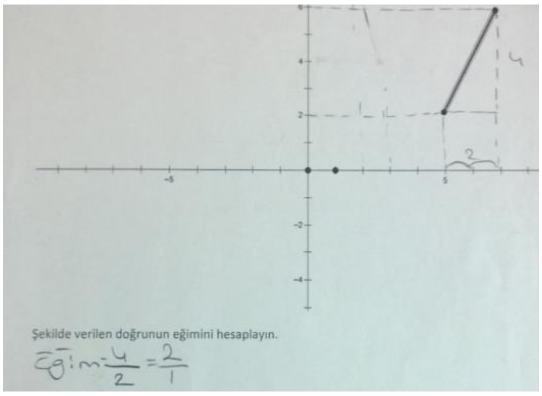

Figure 14: S1's calculation of slope of a line visualized on coordinate plane

Afterwards coordinates of only two points were given for the calculation of slope and in-group discussions were sparked by asking if they can find a more different or shorter way to solve the problem. While some group mates of S1 and S2 who interpreted and drew the line graph at preliminary test and had difficulties on determining of points on coordinate plane were able to reach the point of discovering the difference between coordinates, they still found the result through height/horizontal distance by visualizing the line segment on coordinate plane (Figure 15). Whereas they made mistakes in determining the points on coordinate plane during preliminary test, S3 and S4 were able to state opinions on linear relationship and for the line equation concept they were only able to perform in terms of recalling knowledge acquired through practice. It was observed that as a group they didn't think of reaching the result through difference between the coordinates and calculated the slope by rise over run by visualizing the line segment.

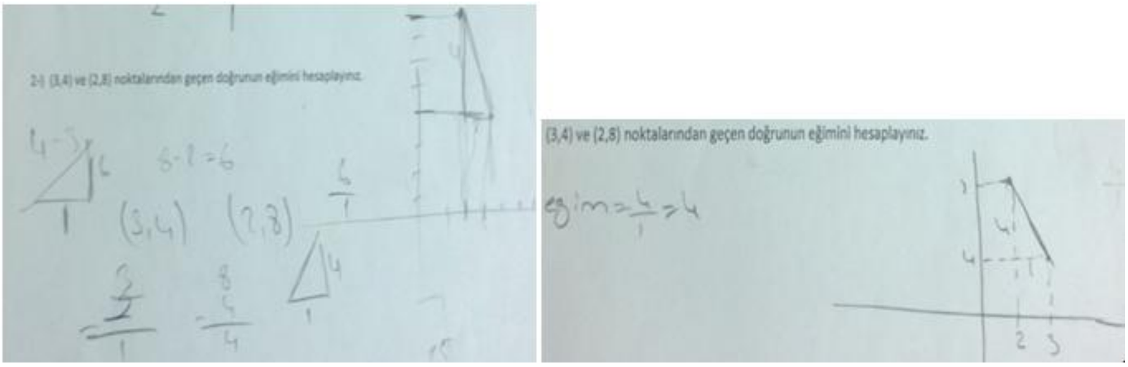

Figure 15: S1's group mate's discovery of slope with inter-coordinate difference (left) and S1's calculation of slope with rise over run by visualizing the line (right)

The participant showing that she had no mistakes on the coordinate plane at preliminary test, interpreted the linear relationship and showed conceptual as well as practical knowledge on linear equation concept addressed the question if there would be a difference in slope when the linear visual was downgrade or upgrade and the instructor then addressed the latter to the whole class. During this process where a negative slope appeared students were able to make discoveries with more visuals such as "*downgrade means negative*", "(*with hand gestures*) *when like this (line inclined to right) positive, when like this (line inclined to left) negative*" only when the instructor's leading question "*you only discovered positive slope until now*" was addressed.

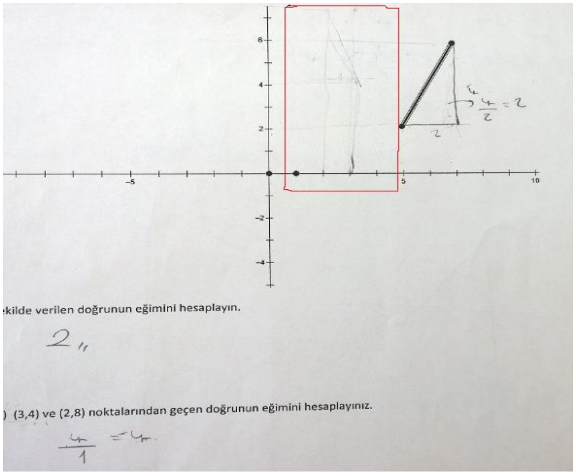

Figure 16: S5's discovery that difference between the coordinates gives the height and horizontal distance

On the assignment where the slope of a line passing through two points which seem difficult to be visualized on coordinate plane due to very high numerical values for coordinates for the purpose of benefiting from difference between coordinates, S1, S2 and S3 benefited from the difference between coordinates since not being able to visualize the process and they showed great effort on making sense this discovery. On previous activity where S5 took the lead on making the discovery here the participant's directly finding the difference between coordinates and proportioning it was very significant in terms of acting independent from the right triangle model and coordinate plane.

At last interviews S1 calculated the slope of a line visualized on coordinate plane, by calculating lengths with the help of axis, with rise over run and found the slope of a line with only two given coordinates with the algorithm *"take y coordinates out find height, x coordinates out and find horizontal distance and divide them to each other"*. Response of S1 to questions why bothered to find the difference between coordinates or why use such method were as follows: *"I* *find height through height and horizontal distances out as well and divide them to each other. This is what we did in class".* This and the generalization "*this is a formula I'll use in high numerical coordinate values"* depicts the difficulties she has on making sense slope as an algebraic ratio and seeing he had difficulty in relating geometrical ratio representation. However, participant's recognition that y coordinates apply to height and x coordinates to horizontal distance is regarded as positive steps for the interpretation of algebraic generalization. S2 and S4, who respectively had no recognition of linearity to show presence on line equation and who were able to state opinions on linearity but recalled practical knowledge for the graph of line, calculated the slope with rise over run by always visualizing the desired line rather than memorizing formula or using a step such as algorithm.

S3 who was able to interpret linear relationship on preliminary test and was only able to perform in recalling knowledge practically acquired for drawing the line and S5 who was able to answer questions such as why linear or line equation in linearity and line equation concepts were participants who both calculated the intended slope with rise over run by visualizing the line on coordinate plane and by calculating through y_2-y_1/x_2-x_1 generalization without visualizing. Between the two participants of who were both able to make transitions between the algebraic and geometrical ratio interpretations of slope S3's explanation "*by taking x out of x and y out of y I find the horizontal distance and height. Actually it is just like in here (rise over run from right triangle model on coordinate plane)"* proves that participant established a relationship between algebraic generalization and geometrical ratio.

Another striking point is where S5 interpreted the slope on right triangle model without drawing and mostly visualizing activities on her mind. For instance, the process of finding the slope of a line with given equation it was striking how the participant explained the right triangle model by visualizing it in mind. S5's explanation on how she can calculate the slope without drawing the line on coordinate plane by using the right triangle model formed by a line intercepting the axis without physically drawing the triangle supports the idea that S5 integrated the right triangle model to slope scheme as an integral part. In the meanwhile, S5 was able to explain negative slope both with the inclination of a line to left-right and with the angle between line and horizontal distance being wide and narrow.

> ***I:**. Can you calculate the slope of a line with equation 5x+4y-40=0? How?*
> ***S5:** I'll assign zero. Find x and y coordinates. I draw it then divide height to horizontal distance.*
> ***I:** What if I say "without drawing on coordinate plane", what do you do?*
>
> ***S5:** I'll find points to assign zero but in another way. Now here I assigned zero to x and y becomes 10. This time y will be zero and x, 8. Here y is height and x is the horizontal distance. It is 10/8 here and this is 5/4.*

4. DISCUSSION and CONCLUSION

At the end of the study, it was concluded that some revisions should be on some assumptions. Related with the second learning goal, it was seen that students can take the length of way as a variable while interpreting the slope. It was seen that the class discussions about if linear visuals, which have same length but different slope, are possible enable that students develop right triangle model to see the changing of vertical and horizontal distances. Moreover, in relation to the fifth learning goal, it was seen that even students could regulate the slope notion as "y_2-y_1/x_2-x_1", they prefer to use right triangle by means of *"rise over run"*. It was realized that the important thing for conceptual learning in this level is that students can switch to various conceptualizations of slope meaningfully in different problem situations. Furthermore, it was seen that the students can make sense of the fact that the slope can be negative by using visual of line.

In this respect, it is observed that while determining whether the slope is negative or zero, students mainly rely on the conceptualization of behavior indicator.

Simon and Blume's (1994) suggestion that for the purpose of understanding of slope as a measurement, ramps with the same slopes but different sizes can be used for the purpose of recognizing that slope is a ratio between height and horizontal distance shows consistency with the results of this study. In fact, during the interviews where S1 and S4 had difficulties in making sense of the slope as a ratio when the height and horizontal distance changed, they emphasized that slope changes as well and they weren't able to interiorize to the concept of slope with the ratio between height and horizontal distance which remained constant. The constant ratio between height and horizontal distance on a point taken randomly at the same linear visual ensures the stability of slope resulting in argument that slope is the ratio between height and horizontal distance which is a very critical phase for the adaptation of slope as a geometrical ratio.

Stump (1999; 2001) and Stanton and Moore-Russo (2012) emphasized the importance of establishment of a relationship between representations on conceptualizing of slope. In this study, it was clearly depicted that the mathematization process of slope starting yet from eighth grade can prevent the development of useless knowledge provided through transition between representations. Following the transition from physical characteristics and real world situation conceptualization to geometrical interpretation via linear constant property, the discovery that slope is an algebraic ratio as well through activities requiring mostly vertical mathematization after the geometrical ratio representation of slope is reflected on coordinate plane provided establishing relationships between representations. Making transitions between representations with mathematization activities prevented representations from being seen as rules disconnected from each other.

The right triangle model that emerged since the beginning of mathematization process of slope concept was at first presented as the modeling of the situation, but then it evolved as a tool on which assertions, interpretations, descriptions and calculations were made on slope. It turned into a cognitive tool required to be physically presented by becoming independent from the situation since vertical mathematization activities were made following the discovery of slope being a ratio. In this study where transition of the emergent right triangle model emerges as a model of concept then turns into a model for concept clearly depicts the bouncing process from model-of to model-for phase as asserted by Streefland.

The mathematization processes of slope for the purpose of first level formal introduction of slope to eighth grade students were observed in this study. Students will extend the slope scheme by making readjustments for different interpretations during high school years. For that reason there is a need to study the mathematization processes supporting meaningful transitions between different representations of slope on high school level as well. Finally enlightening steps for making sense and conceptualization of this concept setting the basis for various concepts and derivative in particular will be taken.

5. REFERENCES

Bakker, A. (2004). Design research in statistics education: On symbolizing and computer tools. Doctoral dissertation. Utrecht University, Utrecht.

Bakker, A., & Van Eerde, D. (2015). An introduction to design-based research with an example from statistics education. In A. Bikner-Ahsbahs, C. Knipping, & N.C. Presmeg (Eds.), *Approaches to Qualitative Research in Mathematics Education* (pp. 429-466). Netherlands: Springer.

Barr, G. (1981). Some student ideas on the concept of gradient. *Mathematics in School*, *10*(1), 14-17.

Cheng, D. S. (2010). *Connecting proportionality and slope: Middle school students' reasoning about steepness*. Doctoral dissertation. Boston University, Boston.

Crawford, A. R., & Scott, W. E. (2000). Making sense of slope. *The Mathematics Teacher*, *93*(2), 114-118.

Fauzan, A. (2002). *Applying Realistic Mathematics Education (RME) in teaching geometry in Indonesian primary schools.* Unpublished doctoral dissertation. University of Twente, Enschede.

Freudenthal, H. (1968), Why To Teach Mathematics So As To Be Useful. *Educational Studies İn Mathematics, 1,* p. 3-8.

Freudenthal, H.(1991). *Revisiting Mathematics Education: China Lecturers*. Dordrecht: Kluwer,

Gravemeijer, K. (1994). Educational development and developmental research in mathematics education. *Journal for Research in Mathematics Education*, *25*(5) 443-471.

Gravemeijer, K. (1999). How emergent models may foster the constitution of formal mathematics?. *Mathematical Thinking and Learning* 1(2), 155–177.

Gravemeijer, K. & Cobb, P. (2006). Design research from the learning design perpective. In J. van den Akker, K. Gravemeijer, S. McKenney, & N. Nieveen (Eds.) *Educational Design research* (pp. 45-85). London: Routledge.

Gravemeijer, K., & Doorman, M. (1999). Context problems in realistic mathematics education: A calculus course as an example. *Educational studies in Mathematics*, *39*(1-3), 111-129.

Gravemeijer, K., Bower, J. & Stephan, M. (2003). A Hypothetical learning trajectory on measurement and flexible arithmetic. In Stephan, M., Bowers, J., Cobb, P., & Gravemeijer, K. (Eds.), *Supporting students' development of measuring conceptions: Analyzing students' learning in social context* (pp. 51-66). JRME Monograph 12. Reston, VA: National Council of Teachers of Mathematics.

Lobato, J., & Thanheiser, E. (2002). Developing understanding of ratio-as measure as a foundation for slope. In B. Litwiller, (Ed.), *Making sense of fractions, ratios, and proportions: 2002 yearbook* (pp. 162–175). Reston, VA: National Council of Teachers of Mathematics.

Moore-Russo, D., Conner, A. & Rugg, K. (2011). Can slope be negative in 3-space? Studying concept image of slope through collective definition construction. *Educational studies in Mathematics*, *76*(1), 3-21.

Nagle, C., Moore-Russo, D., Viglietti, J., & Martin, K. (2013). Calculus Students' and Instructors' Conceptualizations of Slope: A Comparison Across Academic Levels. *International Journal of Science and Mathematics Education*, *11*(6), 1491-1515.

Nagle, C., & Moore-Russo, D. (2014). The concept of slope: Comparing teachers' concept images and instructional content. *Investigations in Mathematics Learning*, *6*(2), 1-18.

Nkambule, T. (2009). Teaching and learning linear programming in a grade 11 multilingual mathematics class of English language learners: exploring the deliberate use of learners' home language. *Pytagoras*, *71*, 3-12.

Rasmussen, C. , Zandieh, M., King, K. & Teppo, A. (2005). Advancing Mathematical Activity: A Practice-Oriented View of Advanced Mathematical Thinking. *Mathematical Thinking and Learning*, *7*(1), 51-73.

Sandoval, C. (2013). Transcript of History of Mathematics: Slope of a Line. Retrieved from https://prezi.com/8q5czopcujx9/history-of-mathematics-slope-of-a-line/

Simon, M. A. & Blume, G. W. (1994). Mathematical modeling as a component of understanding ratio-as-measure: A study of prospective elementary teachers. *The Journal of Mathematical Behavior*, *13*(2), 183-197.

Stanton, M., & Moore-Russo, D. (2012). Conceptualizations of slope: A review of state standards. *School Science and Mathematics*, *112*(5), 270-277

Stump, S. (1999). Secondary mathematics teachers' knowledge of slope. *Mathematics Education Research Journal*, *11*(2), 124-144.

Stump, S. (2001). Developing preservice teachers' pedagogical content knowledge of slope. *Journal of Mathematical Behavior*, *20*, 207-227.

Treffers, A. (1987). *Three dimensions. A model of goal and theory description in mathematics education: The Wiskobas Project.* Dordrecht, The Netherlands: Reidel.

Walter, J. G., & Gerson, H. (2007). Teachers' personal agency: Making sense of slope through additive structures. *Educational Studies in Mathematics*, *65*(2), 203-233.

Wijaya, A., Doorman, L. M. &, Keijze, R. (2011). Emergent Modelling: From Traditional Indonesian Games to a Standard Unit of Measurement. *Journal of Science and Mathematics Education in Southeast Asia*, *34*(2), 149-173.

Yıldırım, A. ve Şimşek, H. (2005). *Sosyal Bilimlerde Nitel Araştırma Yöntemleri*. (5. Baskı) Ankara: Seçkin Yayıncılık.

Uzun Özet

Günlük yaşamda bayır, dik, yokuş gibi yansımaları sıklıkla görülen eğim kavramı, türev başta olmak üzere birçok matematiksel kavrama temel oluşturmaktadır (Nagle, Moore-Russo, Viglietti ve Martin, 2013). Eğim en genel anlamıyla bir dikliğin ölçümü olarak tanımlansa da farklı alanlarda daha detaylı tanımları da mevcuttur. Eğime formal bir anlam yüklenmeden önce, ona ilişkin informal deneyimlerden elde edilen bir kavram imajına sahip olunduğu bilinmektedir. Öğrencilere oldukça tanıdık gelen kavramın matematiksel anlamda yapılandırılmasında zorluk çekildiği ve genellikle anlamlandırılmaksızın sadece işlemsel olarak hesaplandığı (Crawfort ve Scott, 2000; Barr, 1981; Lobato ve Thanheiser, 2002) dikkat çekmektedir. Eğimin öğretim sürecinde öğrencilere hazır formüller sunulması, onların eğimi kavramsal öğrenmesini olumsuz etkilemektedir. Stump (1999, 2001) eğimin farklı temsillerini ilişkilendirmenin kavramsal öğrenme açısından önemine dikkat çekmiş ve bu temsilleri; geometrik oran, cebirsel oran, fiziksel özellik, gerçek yaşam durumları, fonksiyonel özellik, parametrik katsayı, trigonometrik kavram ve kalkulüs kavramı olmak üzere sekiz başlık altında toplamıştır. Moore-Russo, Conner ve Rugg (2011) ise belirleyici özellik, davranış gösterici özelliği ve doğrusal sabitlik özelliği ile eğimin temsillerini, onların deyişiyle öğrencilerin eğim kavramsallaştırmalarını genişletmiştir. Stanton ve Moore-Russo (2012) Amerika Birleşik Devletleri okullarında eğimin farklı kavramsallaştırmalarının ele alınışını araştırmış ve ilkokul yıllarından başlayan eğim anlayışlarının, sekizinci sınıfta hem cebirsel hem de geometrik oran olarak formal bir anlam kazanmasının beklendiğini ortaya koymuştur. Dolayısıyla sekizinci sınıf eğim öğretim sürecinde öğrencilerin önceki kavramsallaştırmalarının hesaba katılmasının onların temsiller arası bağların kurulu olduğu bir kavram şeması geliştirmesine olanak tanıyacaktır. Bu sebeple eğim öğretiminde farklı temsiller arasında ilişki kurdurup, yansıtma yapılmasına fırsat veren bir öğrenme ortamı sağlanmalıdır (Moore-Russo, Conner ve Rugg, 2011). Bu çalışma sekizinci sınıfta, eğim ile ilgili edinilen informal anlayışların- ki bunlar en başta fiziksel özellik, gerçek yaşam durumları ve doğrusal sabitliktir (Stanton ve Moore-Russo, 2012)- öğrenme sürecine yansıtılması ve bu anlayışlardan yola çıkarak eğimin geometrik ve cebirsel oran olarak yapılandırılmasına olanak veren bir öğretim deseni ortaya koymayı amaçlamaktadır. Bu öğretim desenine en uygun yaklaşım ise, öğrencilerin sahip oldukları informal bilgi ve stratejilerini öğrenme sürecine yansıtmalarına fırsat vererek, formal bilgiyi bizzat kendilerinin oluşturdukları modeller aracılığıyla yeniden keşfetmelerine olanak tanıyan bir yaklaşım olması nedeni ile Gerçekçi Matematik Eğitimi (RME) olarak belirlenmiştir. RME yaklaşımı altında desenlenen öğretim sürecinde, öğrencilerin eğimi matematikleştirme süreçlerinin gelişimi ve öğrencilerin önbilgilerinin matematikleştirme sürecine etkisi incelenmiştir.

RME ilkelerine dayalı bir öğretim sürecinde eğim kavramının matematikleştirilme sürecinin incelenmesini ele alan bu nitel çalışma, bir kavramın öğrenilme sürecini ortaya koymaya yönelik öğrenme yol haritası geliştirmeyi amaçlayan desen tabanlı bir araştırmadır (Design Based Research-DBR). Çalışmanın katılımcıları, araştırmacılardan birisinin aynı zamanda matematik öğretmenliğini yürüttüğü 16 sekizinci sınıf öğrencisi arasından amaçlı örneklem yoluyla seçilmiştir. Öğretim öncesinde yapılan alan yazın taraması ve araştırmacıların deneyimleri doğrultusunda eğim için önkoşul bilgiler olduğu sonucuna varılan doğru denklemi, oran-orantı ve bağımlı-bağımsız değişken kavramlarına yönelik bir açık uçlu test geliştirilerek uygulanmıştır. Elde edilen sonuçlar doğrultusunda performanslarına göre beş gruba ayrılan öğrencilerden her grubu temsilen iletişim kurmakta sıkıntı yaşamadığı düşünülen birer öğrenci katılımcı olarak belirlenmiştir. Araştırmanın verileri bu katılımcılarla öğretim boyunca gerçekleştirilen klinik görüşmeler, araştırmacı günlükleri ve öğrenci çalışma kağıtları ile araştırma öncesinde uygulanan açık-uçlu test yoluyla toplanmıştır. İkişer derslik periyotlar halinde toplam altı ders saati süren öğretim sürecinde her iki derslik periyodun ardından katılımcılarla birebir klinik görüşmeler gerçekleştirilmiş ve bu sayede onların eğim kavramını matematikleştirme süreçlerine yönelik derinlemesine bilgi sahibi olunmuştur. Bu çalışmada eğimin gerçek yaşam durumları ve fiziksel özellik temsilinden yola çıkılarak doğrusal sabitlik anlayışı temelinde geometrik oran (yükseklik/yatay mesafe) oluşunun keşfedilmesi ve geometrik oran olarak koordinat düzleminde bir doğru için yansıtılması yoluyla cebirsel bir oran (y_2-y_1/x_2-x_1) formunda yeniden yapılandırılmasını öngören bir varsayımsal öğrenme süreci ortaya konmuştur. Öğrenme amaçları ve varsayımlar doğrultusunda gerçekleştirilen öğretim deneyinde varsayımların gerçekleşme durumu incelenmiş ve çalışma sonunda öne sürülen olası öğrenme sürecinde revize edilmesi gereken durumlar belirlenmiştir.

Elde edilen bulgular doğrultusunda öğrencilerin eğim ile ilgili ön kavramsallaştırmalarını öğrenme sürecine çağırmakta zorlanmadıkları ve kendilerinin yaptıkları etiketlemelerle eğimin bağlı olduğu yükseklik, yatay mesafe ve açı değişkenlerini keşfedip yorumlayabildikleri görülmüştür. Bunun yanında doğru uzunluğunu da eğimi etkileyen bir değişken olarak ele alabilecekleri ortaya çıkmıştır. Uzunlukları aynı fakat diklikleri farklı olan doğrusal görsel örneklerinin, istenmeyen bu durumu onların yükseklik ve yatay mesafeyi fark etmelerini sağlayacak şekilde olumlu yöne çevirebileceği ortaya konmuştur. Doğrusal bir görsel üzerinde alınan herhangi bir noktada yükseklik ve yatay mesafe arasındaki oranın sabit kalışının keşfedilerek, informal deneyimlerden edinilen eğimin değişmemesi bilgisi ile ilişkilendirilmesinin eğimin bir oran olarak anlamlandırılmasında kritik bir öneme sahip olduğu görülmüştür. Bu süreçte açık uçlu testte özellikle oran-orantı kavramında sıkıntı yaşamadığı görülen öğrenciler keşif sürecine önderlik etmişlerdir. Gerçekleştirilen görüşmelerde açık uçlu testte oran kavramına yönelik bir varlık gösteremeyen katılımcı ile bu kavramı sadece işlemsel olarak ortaya koyabildiği sonucuna varılan diğer katılımcının eğimi yükseklik ile yatay mesafe arasındaki bir oran olarak yapılandıramadığı dikkat çekmiştir. Eğimi işlemsel olarak hesaplayabilen tüm katılımcıların koordinat düzleminde bir doğru için yeniden düzenleme sürecinde *"yükseklik/yatay mesafe"* yi yansıtabildikleri görülmüştür. Ancak gerçekleştirilen görüşmelerde eğimi bir oran olarak yapılandırmakta zorlanan iki katılımcının cebirsel oran temsilini de daha çok işlemsel olarak kullandıkları ya da geometrik oran temsilini kullanarak sonuca gittikleri dikkat çekmiştir. Eğimi bir oran olarak yapılandırabilen diğer katılımcılardan açık uçlu testte önkoşul bilgilere sahiplik konusunda en başarılı gruptan gelen katılımcı, geometrik ve cebirsel oran temsilleri arasında dinamik geçişler yapabildiğini ve önceden karşılaşmadığı, doğrudan eğim ile ilişkili olmayan problem durumlarında da eğimin farklı temsillerini yansıtıp sonuca ulaşabildiğini göstermiştir. Diğer iki katılımcının da eğimi yorumlayabildikleri ve eğim hesaplarken attıkları adımların ne anlama geldiğini açıklayabildikleri dikkat çekmiştir. Oran-orantı bilgisinin eğimin matematikleştirilmesinde kritik oluşunun net bir şekilde görüldüğü, bunun yanında doğru denklemi ile ilgili hazırbulunuşluğun da özellikle eğimin cebirsel bir oran olarak yapılandırılmasında ve daha matematiksel bir anlam kazanmasında önemi ortaya konmuştur. Bir diğer önkoşul bilgi kabul edilen bağımlı-bağımsız değişken kavramı için ise henüz sekizinci sınıf düzeyinde, günlük yaşamdaki bağımlılık ilişkisinin fark edilerek yorumlanmasının yeterli olacağı sonucuna varılmıştır.

Eğim kavramının matematikleştirilmesi sürecinin başlangıcından itibaren ortaya çıkan dik üçgen modelinin, önce durumun modellemesi olarak ortaya konulduğu ancak ilerleyen zamanda eğim ile ilgili savunuları, yorumları, açıklamaları ve hesaplamaları destekleyen bir araç olarak geliştiği görülmüştür. Öğrenciler tarafından geliştirilen dik üçgen modelinin gelişimi kısaca üç aşamada olmuştur: (i) duruma bağlı olarak yorumlamaya yarayan bir araç, (ii) eğimin hesaplanmasında kullanılan henüz fiziksel olarak ortaya konulma gereği duyulan bir araç (iii) fiziksel olarak ortaya konulma gereği duyulmayan bilişsel bir araç.