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Graduate Student Middle School Mathematics Teachers' Communication Abilities in the Language of Mathematics

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Abstract

It was aimed to investigate graduate student middle school mathematics teachers' communication abilities in the language of mathematics. The study was designed qualitatively with fourteen middle school mathematics teachers who were graduate students in mathematics education program. Data of the study was collected through an open-ended test including ten questions with several items. Responses of the test were analyzed by coding qualitatively.

Keywords: The language of mathematics, communication in mathematics, mathematical literacy

1. Introduction

Mathematics can be defined that it is a language; in fact it is a universal language. However, the language of mathematics differs from other natural languages since there are no native speakers of mathematics language and it requires a natural language to communicate. Additionally, mathematics has its own vocabulary. Therefore, it has a syntactical and rhetorical structure. According to Jamison (2000), the language of mathematics has a nearly universally accepted logical and rhetorical structure, and it has been presented following a format of definition-theorem-proof. Moreover, each of three main elements which are definition, theorem, and proof has a definite rhetorical structure.

The importance of the mathematics language in learning is indisputable. Students should gain the mathematics language skills and mathematics concepts simultaneously to communicate in this language. Thus, mathematics teachers are responsible to provide the mathematics language skills apart from the mathematics concepts. Jamison aimed to show how making the syntactical and rhetorical structure of

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mathematical language clear to students can increase their understanding of mathematical concepts. Then, he concluded if the rules of the language of mathematics are made explicit, students will learn them and use them as tools to understand abstract mathematical concepts. Owens (2006) questioned the reason why students become reluctant once they reached higher grades while they were eager to learn mathematics at the beginning of the school. Owens emphasized that teachers need to know where in the curriculum to introduce the vocabulary words and how to make connections with the students' knowledge to support the use of appropriate mathematical vocabulary in the classroom. Owens considered that every classroom needs a mathematical dictionary, and he constructed a mathematical dictionary that can be used in the primary and middle grade levels.

It is a fact that mathematics teachers generally focus on only mathematical concepts because they expect students to learn mathematics language skills through exposure, they rarely focus on mathematical language knowledge and skills. Gray (2004) worked on why teachers rarely focus on the language while teaching mathematics. According to Gray, the reason is that teachers are either unaware of how to teach the language of mathematics or they may not believe that they are capable of successfully implementing language instruction in their mathematics classrooms. According to Gray, Bandura's self-efficacy theory is a possible explanation for why teachers rarely focus on mathematical language (Bandura, 1997). Bandura's theory emphasizes that teacher's self-appraisal may shape her/ his choices to teach. Gray explained this idea that if teacher does not believe that she can teach a mathematics topic successfully, she will not choose this topic to teach and she will probably not be able to. Gray designed a measurement instrument called Language of Mathematics Teacher Efficacy Scale (LoMTES), to measure teacher' self-efficacy about this issue.

Increasing the importance given mathematical literacy has emerged as one of the results of international examinations like PISA or TIMMS, it is seen that most countries give special importance to communication in the language of mathematics, which is one of the elements of mathematical literacy. In this regard, National Council of Teachers of Mathematics (NCTM) gives importance to students' communicating in mathematical language beginning from the early ages. Communication standards for school mathematics are as follows:

Instructional programs from prekindergarten through grade 12 should enable all students to-

- organize and consolidate their mathematical thinking through communication;
- communicate their mathematical thinking coherently and clearly to peers, teachers, and others;
- analyze and evaluate the mathematical thinking and strategies of others;
- use the language of mathematics to express mathematical ideas precisely.

(2000, p.60)

1.1. Structure of mathematics

As it is mentioned above, mathematics is a language like any other language, so it can be got in semiotic perspective. That is mathematics has own words (symbols), semantic and syntactical rules (Cathcart, 2003). Cathcart states that symbols convey meaning and associated rules. Kenney (2005) emphasized distinction between mathematical content and process. As cited in Kenney (2005), first model of the language of mathematics proposed by the Balanced Assessment Program at the Harvard Graduate School of Education. Kenney states that this model suggests thinking about mathematical nouns, or objects and arrangements and mathematical verbs may be regarded as following four actions:

- Modelling and formulating,
- Transforming and manipulating
- Inferring
- Communicating.

When we consider these actions as elements of mathematical literacy, we have seen that generating a model for mathematical literacy has received a great deal of attention in literature. Pugalee (1999) indicated necessity of definition and a model of mathematical literacy and he studied on a model. According to Pugalee, a model of mathematical literacy must first embody the NCTM's processes through which students obtain and use their mathematical knowledge. The five processes through which students obtain and use their mathematical knowledge were valuing mathematics, becoming confident in one's ability to do math, becoming problem solvers, communicating mathematically, and reasoning mathematically in NCTM standards in 1989. Furthermore, Pugalee adds that as a second, a model of mathematical literacy should demonstrate the intricate interrelationships between various essential processes in development of mathematical literacy and specify 'enablers' facilitate development of the five processes. Pugalee presented a model for mathematical literacy by considering these three standards he indicated as in Figure 1.

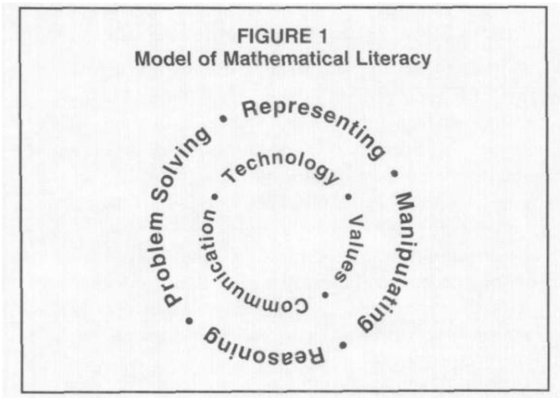

Figure 1. Model of mathematical literacy (Pugalee, 1999, p.20)

On the other hand, as Kenney emphasized, distinction of mathematical processes and content is essential for conceptual knowledge and communication in the language of mathematics. , Tall (2004) claims that symbols that we use for calculation and manipulation are begins with actions as pointing and counting. By using symbols, actions are encapsulated as concepts. Tall states that encapsulation of actions as objects allow us to switch from process to do mathematics to concepts to think about. Gray and Tall (1994) call this switching by the term 'procept'. Tall claims that symbols act dually as process and concept. For instance, a symbol like 1+3 acts as the process of addition or the concept of sum. From Piaget's perspective, when actions on fundamental mental objects are routinized, they are conceptualized as processes. Then, these processes are considered as mental objects at higher levels. We used some aspects of these frameworks in this study.

1.2. Purpose of the study

We aimed to investigate middle school mathematics teachers' communication abilities in the language of mathematics in this study. The research questions that were considered are the following:

1. How are communication abilities of middle school mathematics teachers who were graduate students in mathematics?
2. How are reasoning abilities of middle school mathematics teachers who were graduate students in mathematics?
3. What are conceptual levels of middle school mathematics teachers who were graduate students in number concept?

2. Methodology

The study was designed qualitatively with fourteen middle school mathematics teachers who were graduate students in mathematics education program. We should also note that we can call participants by junior mathematics teachers, since the most experienced of them had seven years experienced. Data of the study was collected through an open-ended test including ten questions with several items.

The test was prepared to asses participants' abilities of reasoning, communication and conceptual level. For instance, some simple sets of numbers were given with common-properties representation in two questions and asked to read with comprehension and the meaning of the given representation to assess participants' reading and conceptual level in number concept. Similarly, a proposition and a number set were given in natural language and required to write in the language of mathematics to assess their writing abilities. Moreover, with some given prepositions in natural language, it is aimed to assess participants' reasoning abilities, structural knowledge of mathematics language that is knowledge of syntactical and semantic rules. For instance, two propositions were given in the language of mathematics and required negations of them. In another question, following mathematical expression was given and asked to evaluate if it was convenient.

$$\text{If } \{x \in \mathbb{R} | 5 < x \leq 8\} \text{ then } x > 7$$

3. Findings

Results of the study indicated that junior middle school mathematics teachers did not have communication abilities in the language of mathematics as expected from a mathematics teacher. It was seen that eight participants out of fourteen had ability of reading with comprehension. In the aspect of writing a mathematical sentence, given in natural language, in the language of mathematics, only seven of fourteen participants were completely successful. Almost all of the participants, except only one, able to write the given set of numbers in the common property notation, while only seven of them could write the preposition given in the natural language in the language of mathematics. Moreover, these seven participants successful about writing were also able to read with comprehension. Furthermore, we saw that only five participants demonstrated that their conceptual levels in the number concept were object level. That is, remaining nine participants were not able to take a set of numbers as a mental object. They expressed the elements of the following number set by reading word by word.

$$\{x \in \mathbb{R} | 5 < x \leq 7\}$$

These participants who gave such an expression explained the elements of the set as an answer of the item in which the meaning of this notation was required. Another important thing was that five participants who demonstrated object conceptual level were also successful about reading with comprehension and writing in the language of mathematics.

Another crucial result obtained from the study was that most participants had weak reasoning ability and weak structural knowledge of mathematics. For instance, answers of the item in which the following conditional sentence was given to be asked to evaluate if it was convenient can be taken as examples of this result.

$$\text{If } \{x \in \mathbb{R} | 5 < x \leq 8\} \text{ then } x > 7$$

Except three participants, all of them evaluate this conditional sentence with the value of x that satisfy the conditional sentence. Therefore, they changed one of the equalities in the sentence. We saw that they were not aware that connective of “if…..then” connects propositions, but hypothesis part of the given conditional sentence was not a proposition. Moreover, also four participants reading with comprehension were not able to evaluate this conditional sentence successfully by reading with comprehension.

In the aspect of reasoning ability, it was seen that junior middle school mathematics teachers had weak abilities. Furthermore, participants who had strong reasoning abilities also demonstrated abilities of reading with comprehension and successful writing in the language of mathematics. One of the questions that we assessed participants’ reasoning abilities included two compound propositions that can be seen in the following. Negations of them were asked in this question.

$$\text{“}For\ all\ \varepsilon > 0\ there\ exist\ at\ least\ one\ \delta > 0\ such\ that\ |x-a| < \delta\ implies\ |f(x) - L| < \varepsilon\text{”}$$

$$\text{“}For\ all\ x < 10, f(x) < 0\text{”}$$

It was seen that any participant could obtain negation of the first compound proposition successfully. Most of the participants negated the first proposition word by word by memory. They negated each quantifiers, symbols and words, while some of others exchanged hypothesis for conclusion to negate it. It was obvious that any of them check the meaning of expressions that they obtained. One of wrong negation can be seen as an example in the following:

$$[\forall \varepsilon > 0, there\ exist\ \exists \delta > 0\ \ni\ |x-a| < \delta\ implies\ |f(x) - L| < \varepsilon]'$$
$$\exists \varepsilon \leq 0\ there\ exist\ \forall \delta < 0\ \ni\ |x-a| \geq \delta\ implies\ |f(x) - L| \geq \varepsilon$$

On the other hand, seven participants out of fourteen were able to obtain correct negation of the second proposition. It was seen that these seven participants expressed the proposition in the language of mathematics first and then negated it. But, this result did not mean that they were able to negate this proposition successfully, since the proposition did not have connective and one can obtain correct result by negating word by word by memory. The student whose negation is seen in above gave following correct answer to the second compound proposition. When her responses for negating were considered, it was thought that she might negate by memory without giving meaning as in the first proposition.

$$(\forall x < 10, f(x) < 0)'$$
$$\exists x \geq 10, f(x) \geq 0$$

Following expressions are examples for the participants who exchanged hypothesis for conclusion and negate word by word to negated it.

$$f(x) > 0, \forall x > 10$$
$$f(x)' > 0, \forall x' > 10$$

4. Conclusion

We concluded junior middle school mathematics teachers had weak abilities of reading with comprehension and writing in the language of mathematics. Moreover, their reasoning ability and knowledge of logical structure were not sufficient to support middle school students' mathematical literacy. For instance, mathematics teachers who were responsible for developing middle school students' mathematical reasoning ability did not evaluate the meaning of the mathematical sentence that they wrote. Consequently our findings indicated necessity of some developments in mathematics teacher training programs in the context of mathematical literacy.

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