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# A comparison of curricula related to the teaching of addition and subtraction concepts

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## Abstract

In this paper, we are interested in curricula contents related to the addition and subtraction concepts. A comparative study between Turkey, France and United Kingdom is carried out in order to identify different learning methods adopted and to evaluate these methods according to the educational system of each country. The results show that main differences between three countries curricula are due to the structures of pre-school systems and to the use of technology.

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## 1. Introduction

Arithmetic does not only consist in operations on numbers. It is one of the main branches of mathematics. It is always used in daily life and it is a tool which contributes to the development of mathematical thinking. Arithmetic is also the first curriculum content and has an important place in the curriculum of any country. Because, after experiencing with the natural numbers during the youngest ages (3-4 years old), children's first step in the mathematics starts with the addition and subtraction problems. Teaching of these concepts is always an important research topic in mathematics education. Some studies on the teaching of addition and subtraction concepts show that it is necessary to take into account addition and subtraction problem types, strategies followed and challenges faced by pupils in problem solving (Nunes & Bryant, 1996).

Furthermore, the technology has been an educational tool as well as a part of our daily life. For example, calculators have been used starting the first classes. According to a report (Kahane, Jacob, 2002), with the use of the technology, the teaching of addition and subtraction concepts and pupils' abilities related to them have become questionable.

From the literature on these concepts, it is possible to identify two different processes to take into account for their teaching. The first process is when the addition and subtraction problems are introduced. This process can be also defined as pupils' initiation to these concepts with problems containing small numbers. The second process is the stage of problem solving containing multi-digit numbers (Erdoğan, Özdemir-Erdoğan, 2009).

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In this paper, we are interested in the curricula contents related to the addition and subtraction concepts. In particular, we want to know how these two processes are taken into account in the curricula of different countries. Although mathematics is an universal language, teaching mathematics may differ and two countries, even similar culture countries, may apply different learning methods. In this paper, a comparative study between Turkey, France and United Kingdom is carried out. France and UK seem to be countries where research in mathematics education has developed and current educational theories and methods have been adopted. We think that this comparison could allow us to identify some possibilities and constraints concerning the teaching of addition and subtraction concepts.

**2. Comparison of the educational systems**

Before comparing curricula, we present here the grades of primary schools in three countries. Compulsory education in France and Turkey starts at 6 years old while it starts at 5 years old in UK. Primary education in UK and France goes until 10 years old. In Turkey, this education also includes 11 years old pupils. Primary education consists of 6 classes in UK and Turkey and 5 classes in France. Primary education is also divided in two different cycles in each country. This division also varies between the three countries.

Table 1. Compulsory education in France, UK and Turkey

age	5	6	7	8	9	10	11
France		CP	CE1	CE2	CM1	CM2	
		Cycle 2		Cycle 3			
UK	Year 1	Year 2	Year 3	Year 4	Year 5	Year 6	
	Key Stage 1- Inf		Key Stage 2- Junior School				
Turkey		nursery class	1.grade	2.grade	3.grade	4.grade	5.grade
		Pre school	First level of primary education				

As for the structures of the curricula, in UK, content is defined separately for each class. In France, content is given according to two cycles above. Supplementary titles are added for the second cycles. In Turkey, the structure of the curriculum is the same with UK's except for nursery class. For this class, content is given in the pre-school curriculum. This content is defined as pupils' behavior development.

Table 2. Structures of the curricula in UK, France and Turkey

UK	France	Turkey
<ul style="list-style-type: none"> <li>- Block A: Counting, partitioning and calculating</li> <li>- Block B: Securing number facts, understanding shape</li> <li>- Block C: Handling data and measures</li> <li>- Block D: Calculating, measuring and understanding shape</li> <li>- Block E: Securing number facts, relationships and calculating</li> </ul>	<p><b>Cycle 2</b></p> <ul style="list-style-type: none"> <li>- Work with numerical data</li> <li>-Natural numbers</li> <li>-Calculation</li> <li>-Space and geometry</li> </ul> <p><b>Cycle 3</b></p> <ul style="list-style-type: none"> <li>- Work with numerical data</li> <li>- Natural numbers</li> <li>- Simple fractions and decimal numbers</li> <li>- Calculation</li> <li>- Space and geometry</li> <li>- Sizes and measuring</li> </ul>	<p><b>Nursery class</b></p> <p>Psychomotor, social-emotional, language, cognitive, and self-care skills</p> <p><b>First level of primary</b></p> <ul style="list-style-type: none"> <li>-Nombres</li> <li>-Geometry</li> <li>-Measuring</li> <li>-Data</li> </ul>

The curricula structures show that in UK, the content is presented in a process with connections between subjects while the curriculum in France and Turkey only describes the subjects to study.

### 3. Findings and discussion

We notice that the introduction of addition and subtraction is similar in three curricula. They initially aim the acquisition of the number concept. This point is particularly emphasized in French curriculum. For the concept, providing to pupils work to carry out with materials is advised. It is expected that pupils express collection and separation of objects by using numbers as an increase or decrease in quantity. For the next step, a symbolic expression is expected. In this stage, French and Turkish curricula expect different approaches from a teacher. In French curriculum, before starting symbolic expressions, repetition of the operations containing expressions such as “and” or “more” is expected. As for Turkish curriculum, as it is possible in Turkish language to express the four operations by using the expressions “and”, “with” and “more”, it is advised to teachers not to use these expressions only for addition. Some studies underline that it is necessary to be careful in the use of these expressions and stress that an inappropriate use of them could cause some misconceptions (Iskenderoğlu and al, 2004; Erdoğan, Özdemir Erdoğan, 2009).

In the English and Turkish curricula, it is advised to introduce addition as a gathering of two groups of objects (conting on), and subtraction as a separation from a group of objects (take away). These conceptual explanations does not exist in French curriculum. In this curriculum, it is only noted that when operations are expressed symbolically, it is advisable to introduce the expressions  $a+b$  and  $a-b$  at the same time in order to prevent pupils automatical use of the expression  $a + b$ .

With the introduction of the addition, the three curricula give place to the addition table consisting of numbers between 1 and 10 and to addition and subtraction operations using numbers obtained from this table (addition and subtraction facts). When addition and subtraction concepts are introduced in second year (6 years old) English curriculum aims to the acquisition of addition and subtraction as inverse operations one with other. In this level, it is expected that pupils find the subtraction corresponding to a given addition and vice versa. In French and Turkish curricula, this point is underlined among expected acquisitions. In the introduction part of the Turkish curriculum, it is noted that carrying out addition and subtraction with a sequence of numbers like 3,6,9 ( $6+3=9$ ,  $9-6=3$ ,  $9-3=6$ ) could ensure the acquisition of this relation.

Each following year, numbers studied are enlarged with a supplementary digit and techniques or properties taught regarding addition and subtraction are applied with new multi digit numbers. On the other hand, operations with numbers studied in previous years are expected as mental operations in following years. The extension of digit in French curriculum is similar to Turkish curriculum. In both curricula natural numbers are written and read with 9 digit numbers at the end of primary school. However, in Turkish curriculum, addition and subtraction operations are studied with 5 digit numbers when they are studied with 4 digit numbers in English curriculum. This point is not explained in French curriculum.

Another difference between curricula relates to the techniques of addition and subtraction operations. In English and Turkish curricula, these techniques are clearly explained while the French curriculum simply notes that it is possible to provide the technique of addition at the end of the second cycle. However, the introduction of the operations in the three curricula is carried out by using natural numbers and the importance of this step is underlined. Before introducing standard techniques, English curriculum establishes and defines various levels for techniques and work in class. As for Turkish curriculum, standard techniques are given without delaying. As we underlined above, the French curriculum does not give any precision relating techniques and their introduction time. In the table below we present techniques and strategies outlined in English and Turkish curricula.

Table 3. Techniques and strategies outlined in curricula

UK	Turkey	France
- addition and subtraction facts	- the addition and subtraction facts	Non description is included
- a 100-square or jottings on an empty number line/ a number line	- expanded method / Column method	
- patterns ( $90 - 20 = 70$ and $9 - 7 = 2$ )		
- partitioning or rounding and adjusting		
- Expanded method :		
*partition one of the numbers and add or subtract the units, tens and hundreds		

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separately

- \* Subtraction, illustrating 'difference', is complementary addition or counting up
  - \* decomposition method, illustrating the 'take away' model of subtraction
  - Column method
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The use of technological tools has an increasing place in curricula. These tools could be also used for the teaching of addition and subtraction. The first potential tools for this teaching are calculators. However, the use of calculators differs between the three curricula. In French and English curricula, it is noted that pupils may need to use a calculator for arithmetic operations when solving a problem and that an appropriate use of calculator could be taught. The French curriculum contains information about this use from the first grade while English curriculum advises it from the fourth year. In the second cycle, French curriculum focuses on the pupils' ability to identify the process of an operation and its various stages. The use of a calculator is considered as a way to focus pupils on the problem and to avoid the fact that pupils gives up the solution of a problem only for the difficulties of an operation. This use of calculators is not one of the objectives for Turkish curriculum. It is only noted that teachers may use it when necessary.

### 3. Conclusion

According to the results presented above, we can say that when pre-school education is well developed, i.e. English educational system, introduction of mathematical concepts under different aspects starts very early, learning is more experimental and teaching is organized on a long time. Otherwise, when there is a constraint of time, as shown with the case of Turkish pre-school system, introduction of mathematical concepts is often immediate, quite abstract and techniques are taught since at the beginning. This result shows the importance of early pre-school education for learning and teaching mathematics. Furthermore, we noticed that the use of the technology in mathematics education differ between three countries and this difference is kept in the part of the curricula related to the addition and subtraction. Particularly, when arithmetic operations are taught at the primary education, approaches adopted by the three curricula concerning the use of calculators considerably vary. As the aim of this study is limited to a comparison of curricula, it is not possible for us to know if calculators are really used in class and if this use has a positive effect on pupils' understanding of addition and subtraction concepts. This question could be treated in futur studies in order to identify effective approaches to teach addition and subtraction concepts.

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