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A Counterfactual Analysis of Infinite Regress Arguments

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Abstract I propose a counterfactual theory of infinite regress arguments. Most theories of infinite regress arguments present infinite regresses in terms of indicative conditionals. These theories direct us to seek conditions under which an infinite regress generates an infinite inadmissible set. Since in ordinary language infinite regresses are usually expressed by means of infinite sequences of counterfactuals, it is natural to expect that an analysis of infinite regress arguments should be based on a theory of counterfactuals. The Stalnaker–Lewis theory of counterfactuals, augmented with some fundamental notions from metric-spaces, provides a basis for such an analysis of infinite regress arguments. Since the technique involved in the analysis is easily adaptable to various analyses, it facilitates a rigorous comparison among conflicting philosophical analyses of any given infinite regress.

Keywords Infinite regress · Vicious · Benign · Counterfactual · Similarity · Limit

1 Introduction

Infinite regress arguments are widely used as a method of refutation in philosophy and mathematics. Ancient dialecticians were the first masters of this method, and we owe them some of the most challenging infinite regress arguments, such as Zeno's arguments against the possibility of motion, the Third Man argument directed against Plato's theory of ideas, and Aristotle's argument for the existence of a First Mover (the Unmoved Mover). Recent examples include Russell's argument against resemblance nominalism and Bradley's argument against the thesis that a particular is a bundle of properties.

Despite the plenitude of examples, there is no overarching and widely accepted philosophical theory of infinite regress arguments. This paper aims to be a step forward in this direction. As such, it aims at a clarification of the role of infinite regress arguments in the assessment of theories and the identification of the conditions under which this role is successfully played. My view is strongly based on the

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assumption that infinite regresses are most naturally expressed as infinite sequences of counterfactual propositions: if it were the case that Achilles moves, then it would be the case that he covers a distance *d* during a time interval *t*; if it were the case that Achilles covers a distance *d* during *t*, then it would be the case that he covers two distances consisting of the two halves of the distance *d* during *t*; if it were the case that Achilles covers the two halves of the distance *d* during *t*, then it would be the case that he covers four distances consisting of the four quarters of *d* during *t*; and so forth.

Based on the claim that counterfactual propositions provide a natural way of expressing infinite regresses, the present study promotes a theory of infinite regress arguments within the framework of a theory of counterfactuals. In the next section, a few general notions and distinctions related to the subject are introduced; specific notions needed for the present theory will be introduced in the relevant sections. In the third section, I outline and briefly discuss what I will call the indicative theories of infinite regress. I will be content with emphasizing the points that will present themselves as the main points of difference. For an extensive study of this mainstream view of infinite regresses, see for example Black (1996) and Gratton (2010). In the fourth section, I develop my view while one of the key concepts, namely "approaching to a possible world" is left unexplained. The fifth section is devoted to fill this gap by means of the mathematical notion of limit. In the sixth section, the theory is applied to yield analyses of a few infinite regresses. On the basis of these examples, I want to claim that the present theory meets our intuitions: if an infinite regress is obviously benign (vicious), then the given theory allows only analyses of this regress that establish it to be benign (vicious). If an infinite regress is known to be problematic, then it allows analyses in both directions. That I can make this claim implies that what I present here is really a *theory* of infinite regress arguments that can be strengthened or undermined by examining the cases.

2 Infinite Regresses and Infinite Regress Arguments

To start with a rough description, an *infinite regress* can be said to be an infinite sequence of propositions $A_1, A_2, A_3, \ldots$ such that the truth of any proposition A_i in the sequence "requires" the truth of the next proposition A_{i+1}. A circle consisting of n propositions A_1, A_2, A_3, …, A_n where A_i requires A_{i+1} for $1 \leq i < n$ and A_n in turn requires A_1 will be considered as a special type of infinite regress where the same subsequence recurs consecutively: $A_1, A_2, A_3, \ldots, A_n, A_1, A_2, A_3, \ldots, A_n, A_1, \ldots$. Note that infinite regresses are not just infinite sequences of propositions: the requirement condition is what makes an infinite *sequence* of propositions an infinite *regress*.

In so defining the notion of infinite regress, we do not leave out infinite regresses of entities (e.g., beliefs, justifications, forms, causes) e_1, e_2, e_3, … such that the existence of any entity e_i in the sequence requires the existence of the next one. For whenever we are given an infinite sequence of entities forming an infinite regress, we may form the corresponding infinite regress of propositions: e_1 exists, e_2 exists, ….

An infinite regress raises a philosophical problem when it leads to a disagreement concerning its consequences. Some regresses are said to be pathological, or *vicious*, in that they amount to an unacceptable result, if not an outright impossibility of some kind (e.g., physical, metaphysical or logical). On the other hand, some infinite regresses, for example, the truth regress in the Tarskian hierarchy of languages, are widely accepted to be innocuous, or *benign*. Therefore, one of the main issues in the philosophical theory of infinite regress arguments is to invent tools to distinguish between the vicious infinite regresses and the benign ones.

What makes the study of infinite regresses an important metaphilosophical field is their role in the assessment of philosophical theories. Let us say that an infinite regress of propositions A_1, A_2, A_3,… is yielded by a theory T when it follows from T that for all i, the proposition A_i requires the next one A_{i+1}. It follows from the previous paragraph that when a theory yields an infinite regress, this cannot be categorically identified as a *reductio ad absurdum* against that theory. On the face of the fact that some infinite regresses are benign, to develop an infinite regress *argument* against a theory, an infinite regress *yielded by the theory* should be shown to be vicious; that is, the regress should be shown to produce a result that is *intolerable by the theory.*

Some, such as Black (1996), argue for the view that vicious infinite regresses amount to refutation. Passmore, whose work (1961, Ch. 2) made infinite regresses a metaphilosophical subject, thinks otherwise. For in every case, an infinite regress works only within a context consisting of a body of assumptions, and so it is always possible to give up some of these assumptions and save the theory from that particular regress. For Passmore, infinite regresses strong enough to refute a theory are not philosophical (cf. his discussion of the Waismann's regress argument concerning the irrationality of √2). I accept Passmore's claim that philosophical infinite regress arguments operate only within the context of a body of assumptions, but I take this to imply that the conclusion of an infinite regress argument should be "either the theory or at least one of the assumptions should be dismissed", and in some cases these assumptions are strong enough so that the vicious infinite regress succeeds in weakening the theory. Moreover, *pace* Passmore, I do not think that mathematical infinite regress arguments form an exceptional type with a different kind of relation between the assumptions and the conclusion. The difference, if any, lies in the degree of our belief in the assumptions.

It should be noted that vicious circles should be distinguished from circular arguments (*petitio principii* arguments). A theory should be rejected if it can be shown to yield a *vicious* circle, since such a circle amounts to impossibility. On the other hand, a theory cannot be rejected just by pointing at a circular reasoning offered in its favour. The "God and the Bible argument" is a case in point:

> Ella: God exists.
> Brad: How do you know?
> Ella: The Bible says so.
> Brad: How do I know what the Bible says is true?
> Ella: Because the Bible is the word of God. (Walton 2005, 97)

The circular argument presented by Ella to Brad may be said to fail to support her belief in God. However, this particular failure in argumentation does not prove that God does not exist. Therefore, we should keep the distinction between circular arguments and theories that yield a vicious circle.

Circular definitions are on a par with *petitio principii* arguments. Modulo logical equivalence, we may say that, circular definitions are those such that at least one term that is a part of the *definiendum* is—explicitly or implicitly—also included in the *definiens*. Suppose we have no prior knowledge of the notions "man" and "rational" and let us be given the definition of man as the rational animal where rationality, in turn, is defined to be an essential feature of man. Obviously, we have made no progress in improving our knowledge on either of these notions. Analogous to the case of circular arguments, identifying a definition to be circular may *only* give a direct support to a negative claim about the definition: Circularity in the above definition of "man" does not prove directly that this notion cannot be grasped. Rather, it directs us to seek a non-circular definition, if any.

3 Indicative Theories of Infinite Regress Arguments

Most views on infinite regress arguments treat infinite regresses as infinite sequences of indicative conditionals: if A_1, then A_2; if A_2, then A_3…. To reach a conclusion, the impossibility of an infinite set generated by the infinite regress is exploited. Therefore, these views focus on the study of the structural properties of the sets generated by infinite regresses: Given an infinite regress, if the generated set *S* is inadmissible, then the regress is vicious (cf. Sanford 1975; Gratton 2010; Wieland 2012). Below I rewrite—in linear form—a general schema for infinite regress arguments as given by Gratton (1997, 205):

1. Regress formula
2. Triggering statement
3. Infinite regress (1, 2)
4. Premise(s)
5. Result (3, 4)
6. Premise(s)
7. Result is unacceptable (6)
8. The regress formula is unacceptable (5, 7)

What is usually missing in most examples is a reason to believe (7); that is, the condition that states that the result (5) is inadmissible. Sometimes it is just stated or even assumed (see also Maurin 2007, 2). However, in a philosophical debate on an infinite regress, that is usually the heart of the matter. Consider, for example, the schema of failure regresses given by Wieland (2012). Omitting his reference to a fixed type of objects, his schema of failure regress arguments runs as follows:

1. (Problem) You have to φ at least one thing *x*.
2. (Solution) For any *x*, if you have to φ *x*, you ψ *x*.
3. (Extra premise) For any *x*, if you ψ *x*, then there is a different thing *y*, and you φ *x* only if you φ *y* first.
4. (Infinite regress) For any *x*, you always have to φ a further item first (i.e. before φ-ing *x*). (1–3)
5. (Conclusion) If you ψ anything that you have to φ, then you never φ anything. (1–4)

Consider the following instance of the proposed schema:

1*. (Problem) You have to cover at least one distance *d*.
2*. (Solution) For any distance *d*, if you have to cover *d*, you cover the first half of *d*.

3*. (Extra premise) For any distance d, if you cover the first half of d, then there is a different distance d', and you cover d only if you cover d' first.
4*. (Infinite regress) For any distance d, you always have to cover a further distance first (i.e. before covering d). (1–3)
5*. (Conclusion) If you cover the first half of every distance that you have to cover, then you never cover any distance. (1–4)

As an analysis of this infinite regress argument shows, (see a related example in the sixth section) there is an interpretation under which 1*–4* are all true, while the conclusion 5* is false. This case is obtained by the possibility that tasks forming an infinite sequence may get easier. Therefore, one needs to add one more premise, such as "If you always have to φ a new item if you have to φ any item, then you never φ any item." Augmented with this premise, the argument schema becomes valid. However, as witnessed by the arguably false proposition "If you always have to cover a new distance before covering any distance, then you never cover any distance," the added premise does not hold at all times. Therefore, in each case one should check whether this premise holds or not.

In the case of mathematical theories, such as a formal theory of arithmetic or set theory, establishing inadmissibility is usually reduced to a formal proof-search issue. Consider the widely accepted set theory of the working mathematician, **ZF** (Zermelo–Fraenkel set theory). One of its axioms, the axiom of foundation, guarantees that every set S has a ∈-minimum element; that is, an element s of S such that $s \cap S=\emptyset$. Accordingly, one can easily prove that there cannot be an n-cycle $a_1 \in a_2 \in \ldots \in a_n \in a_1$ or an infinite descending sequence $\ldots \in a_3 \in a_2 \in a_1$ of sets. That is, every such collection of sets is inadmissible. Thus, any theory consisting of a group of set-theoretical propositions leading to the existence of such an infinite or circular ∈-sequence is rejected by **ZF**.

Though the axiom of foundation provides a technical solution to this problem of infinite regress inside this theory **ZF**, why should one believe the axiom of foundation? Indeed, one of the alternative set theories, **ZFA** (anti-well-founded set theory) obtained from **ZF** by replacing the axiom of foundation with the axiom of anti-foundation turns many vicious collections of **ZF** into benign. In **ZFA,** any system of set-theoretic equations e_x has a unique solution. As such, **ZFA** implies the existence of a unique set $a=\{a\}$, which is the unique solution of the equation $x=\{x\}$. In this way, we obtain a set x such that $x \in x$. Or there are exactly two sets $a=\{b\}$, $b=\{a\}$ determined by the system of equations $x=\{y\}$, $y=\{x\}$. In this way, we obtain two sets x and y such that $x \in y$ and $y \in x$. In general, we may produce an n-circle $x_1 \in x_2 \in \ldots \in x_n \in x_1$ for n as big as we want.

The set theory **ZFA** is assumed to be well adapted to the study of circularity (including the phenomenon of self-reference). Barwise and Etchemendy (1987) and later Barwise and Moss (1996) advocated this view. It should be pointed out that none of these major works managed to explain why some circles are vicious while others are not. In this direction, Kühnberger's attempt to explicate vicious circularity as non-eliminable non-well-foundedness is noteworthy: "An entity is circular if and only if there are aspects of this entity that are non-well-founded." (Kühnberger 2001, 40) "A circular entity is pathological if it does not allow an appropriate representation that is well-founded in the mathematical sense of well-foundedness" (2001, 41). It is not clear how the work by Barwise, Etchemendy, Moss, Kühnberger and others on circular phenomena can be generalized to infinite regress arguments. In the case of circular entities, if the only way to demarcate vicious circular entities is to see if it allows a well-founded representation; then the liberation we get after replacing the foundation with the axiom of anti-foundation fades. To avoid begging the question, one should admit the possibility that "the mathematical sense of well-foundedness" could well be the one determined by the axiom of foundation. Indeed, while the recursive definition is seemingly a circular phenomenon, it seamlessly takes its place in **ZF**.

Moreover, the point raised by McLarty against the view that **ZFA** is a plausible framework for circularity is quite strong: whether we choose **ZFA** or **ZF** is irrelevant to the semantics of circularity. A part of his criticism is based on the observation that membership relation is not similar to the relation that holds between two propositions if the first is an immediate constituent of the second (cf. McLarty, 1993). Barwise and Etchemendy (1987) exploited this similarity to support their claim that anti-foundation axiom is required to their semantics of vicious circularity. If McLarty is right, then the anti-foundation approach cannot give support to a successful theory of infinite regress arguments, and in this framework we may develop at best a theory of circular definitions and circular arguments.

In philosophical enterprise, usually there is no such fixed ground theory like **ZF** to determine vicious and benign regresses. For this reason, in the case of philosophical theories yielding an infinite regress or circle, inadmissibility conditions lie at the heart of the controversy. Therefore, we do not make much progress by simply saying that an infinite regress or circle produced by the theory is vicious if the generated set is inadmissible. To produce a successful philosophical infinite regress argument, a justification for an appropriate inadmissibility result should be incorporated in the proposed analysis of the infinite regress under discussion.

4 A Counterfactual Theory of Infinite Regress Arguments

In section 2, an infinite regress was said to be an infinite sequence of propositions $A_1, A_2, A_3, \ldots$ such that the truth of any proposition A_i in the sequence "requires" the truth of the next proposition A_{i+1}. There I did not mention which specific notion of requirement I will be using. In the previous section, we have seen that one possibility is to make use of indicative conditionals. As the first step in the direction of developing a counterfactual theory of infinite regress arguments, I suggest that infinite regresses should be elaborated by means of counterfactual propositions (or counterfactuals, for short). A typical counterfactual is of the form "If *A* were the case, *C* would be the case" and it is symbolized as ($A \mathrel{\Box\!\!\rightarrow} C$). The proposition *A* is the antecedent, and the proposition *C* is the consequent of the counterfactual ($A \mathrel{\Box\!\!\rightarrow} C$). I now redefine an infinite regress to be an infinite sequence of counterfactuals $A_1 \mathrel{\Box\!\!\rightarrow} A_2, A_2 \mathrel{\Box\!\!\rightarrow} A_3, \ldots$ and thus explicate the notion of requirement by means of counterfactual implication.

If counterfactuals are the building blocks of infinite regresses, one should start with a working theory of counterfactuals in order to develop a theory of infinite regress arguments. Lewis (1973a; 1973b) and, earlier, Stalnaker (1968) developed a possible world semantics for counterfactuals. It is fair to say that despite many philosophical and technical criticisms, the variations of the Stalnaker–Lewis semantics still dominate the field. Below I will define a semantics of counterfactuals in this variety, and I assume a sound and complete deductive system for this semantics for the propositional logic of counterfactuals. Thus, when I claim that some proposition is derivable, it means that it is derivable within such a system. Therefore, if successful, the results of this paper will also prove to be an application of an extension of the Stalnaker–Lewis theory.

A model for the propositional logic of counterfactuals is a quadruple M=(W_M, R_M, S_M, V_M), where W_M is the set of worlds, R_M is the accessibility relation, S_M is the comparative similarity relation and V_M is the valuation function of the model M. One may omit the indexes if the model is understood or intentionally left ambiguous.

For any model M, its set of worlds W_M must include an initial world w_0 that represents the actual world. With an abuse of language, w_0 is said to be the actual world.

The accessibility relation is a binary relation defined on the set of possible worlds such that $R(w, w')$ if and only if the world w' is considered as a possible state of affairs from the point of view of the world w. If $R(w, w')$, then it is said that w' is accessible from w. I assume only that R is reflexive as a minimal condition on the relation of accessibility. This means that if a proposition is true, then possibly, it is true.

The comparative similarity relation is a ternary relation defined on the set of possible worlds such that $S(w, w', w'')$ if and only if the world w' is more similar to w than w'' is. The comparative similarity relation may also be interpreted informally as a comparative closeness relation. On this reading, $S(w, w', w'')$ means that w' is closer to w than w'' is. Given a model M, we may fragment its ternary comparative similarity relation S_M into binary relations $S_{M,w}$ (or S_w if the model is understood) so that $S_{M,w}(w', w'')$ if and only if $S_M(w, w', w'')$. Obviously, this fragmentation is reversible. The comparative similarity relation need not be total: it may be the case that for some worlds w, w', w'' neither $S_{M,w}(w', w'')$ nor $S_{M,w}(w'', w')$. Moreover, for any world w, the only most similar world to w is w itself. That is, for any world w and for every different world w', it must be the case that $S_{M,w}(w, w')$ and it is not the case that $S_{M,w}(w', w)$. Since a comparative similarity relation S is strict, it is asymmetric: if $S_{M,w}(w', w'')$, then it is not the case that $S_{M,w}(w'', w')$.

The valuation function V_M of the model M assigns truth-values to propositional variables for each world. Therefore, it is a function that associates each pair (p, w) consisting of an atomic proposition and a world with one of the truth values ***T***(rue) and ***F***(alse). $V_M(p, w)$=***T*** when p denotes an atomic proposition true in w while $V_M(p, w)$=***F*** when p denotes an atomic proposition false in w. For a quantificational logic of counterfactuals, we need more refined structures as models, such as quintuples $M=(W_M, \{D_w : w \in W_M\}, R_M, S_M, V_M)$, where each world $w \in W_M$ is assigned a domain of individuals D_w and the valuation function interprets all individual constants, relation symbols and function symbols of the language. For the purposes of the present paper, I will be content with models for the propositional logic of counterfactuals.

Truth conditions for propositions other than counterfactuals are as given by the usual Kripke semantics (cf. Creswell and Hughes: 1996). Let us be given a propositional language with negation, conditional and possibility symbols as primitive logical constants. As usual, '~' stands for the negation, '→' stands for the conditional, '◊' stands for the possibility connective. The remaining propositional connectives can be introduced with abbreviative definitions and their truth conditions can be easily derived on the basis of those of the primitive ones. Let $M=(W_M, R_M, S_M, V_M)$ be a model and $w \in W_M$. Truth condition for atomic propositions were already defined. Moreover,

1. $V_M(\sim A, w)=\boldsymbol{T}$ if $V_M(A, w)=\boldsymbol{F}$. Otherwise, $V_M(\sim A, w)=\boldsymbol{F}$.
2. $V_M(A\rightarrow C, w)=\boldsymbol{F}$ if $V_M(A, w)=\boldsymbol{T}$ and $V_M(C, w)=\boldsymbol{F}$. Otherwise, $V_M(A\rightarrow C, w)=\boldsymbol{T}$.
3. $V_M(\Diamond A, w)=\boldsymbol{T}$ if in some world w' accessible from w, $V_M(A, w')=\boldsymbol{T}$.

   To state the truth condition for counterfactual propositions briefly, let a world be an A-world if in this world the proposition A is true. Then, $(A \,\Box\!\!\rightarrow C)$ is true in M in a possible world $w \in W_M$ if and only if C is true in all A-worlds $w' \in W_M$ accessible to w such that there are no accessible A-worlds closer to the world w than w':
4. $V_M(A \,\Box\!\!\rightarrow C, w)=\boldsymbol{T}$ if $V_M(C, w')=\boldsymbol{T}$ for all $w' \in W_M$ such that

   a. $R(w, w')$
   b. $V_M(A, w')=\boldsymbol{T}$
   c. $\forall w''\ [w'' \in W_M \ \&\ R(w, w'') \ \&\ V_M(A, w'')=\boldsymbol{T} \rightarrow \sim S_M(w, w'', w')]$

   Otherwise, $V_M(A \,\Box\!\!\rightarrow C, w)=\boldsymbol{F}$.

Although we normally assert a counterfactual $(A \,\Box\!\!\rightarrow C)$ when we believe that the antecedent A is false, the possibility that A is true in the world w does not pose a difficulty when we evaluate the counterfactual $(A \,\Box\!\!\rightarrow C)$. Since I assumed reflexivity of the accessibility relation, and since a world is one of the closest worlds to itself and there is no closer one, if the world $w \in W_M$ is already an A-world, then C must be true in w so that $(A \,\Box\!\!\rightarrow C)$ is true in w.

We must also consider the possibility that there can be more than one closest A-world accessible to w. This is because the comparative possibility relation need not be total: it is possible that for some worlds w', w'', neither $S_M(w, w', w'')$ nor $S_M(w, w'', w')$.

If we allow the possibility that there are no accessible closest A-worlds to w, then for any proposition C, the counterfactual proposition $(A \,\Box\!\!\rightarrow C)$ becomes trivially true in w. I will assume that the following condition holds:

> *Closest world assumption*: If some A-world is accessible to w (that is A is possible with respect to w), then there is at least one accessible closest A-world to w.

This condition is normally named as the limit world assumption, but since I need the notion of limit for some other purpose, I used a different label.

To give a general schema of infinite regress arguments, I introduce an auxiliary formula to the effect that an infinite sequence of accessible possible worlds approaches to another accessible possible world: For any sequence of propositions A_1, A_2, …, let $\Diamond(A_i)_{i\in\mathbf{N}}$ denote the formula such that given any model $M=(W, R, S, V)$,

> $\Diamond(A_i)_{i\in\mathbf{N}}$ is true in w, if and only if there is a sequence of possible worlds w^i starting with w approaching to a possible world w' such that each w^i is an A_i-world accessible to w.

By means of the above notions, I may now propose the following schema as a general representation of infinite regress arguments:

1. T (Assumption)
2. $\Diamond A_1$ (Assumption)
3. For all i, $A_i \,\Box\!\!\rightarrow A_{i+1}$ (1)
4. For all i, $\Diamond A_i$ (2,3)
5. $\Diamond(A_i)_{i\in\mathbf{N}}$ (1,4)

6. ~5 (Premise)
7. ~1 ∨ ~2 Conclusion (5,6)

T, the regress theory, is the target of the whole argument. The third line means that *T* must be shown to yield an infinite regress. This infinite regress together with an assumption such as the one on the second line implies the possible truth of all A_i by the semantics of counterfactuals. The fifth and sixth lines are crucial to have a successful infinite regress argument against the regress theory. The claim that 5 is derivable from *T* with the help of the additional assumption 2 means that from the point of view of *T*, given that A_1 is possible, not only all A_i are possible, but also there is a way to exploit all these possibilities to reach a final possible state of affairs. The sixth line points at the harder task of establishing that there is no way to reach such a state of affairs. Jointly leading to a contradiction, either *T* or the assumption 2 should be dropped.

Consider the Zeno's regress and let us be given the possibility (with respect to the actual world) that Achilles moves; that is, he possibly covers a distance in a finite period of time. Then it follows from a theory of motion (see the related example in the sixth section for some details) that for every n=1, 2, 3… it is also possible that Achilles were able to cover 2^n many distances. Not only that: since the regress is infinite, to cover the whole distance, Achilles should be able to cover infinitely many divisions of a distance. This last assertion corresponds to the fifth line of the argument schema. Now the infinite regress arguer should show that Achilles *cannot* be able to cover infinitely many divisions of a distance no matter which strategy Achilles choses (for example, Achilles may try running at a constant speed or accelerating his steps). In other words, the infinite regress arguer should show that, for every strategy Achilles choses, he cannot avoid stepping in worlds less and less similar to the actual world. The infinite regress arguer may, for example, try to support this claim on the basis of the prima facie plausible claim that the number of distances that Achilles covers constantly increases and it is less likely that one achieves covering more distances in the same amount of time. Therefore, he would conclude that Achilles *cannot* end up with a possible world that is accessible to the actual world, and this corresponds to the sixth line of the above schema.

This infinite regress is claimed to be vicious for it leads to the existence of an accessible possible world including an infinite collection of tasks, while *in this case* there cannot be such a world. This is in fact a special case: in more general terms, an infinite regress would be vicious when it requires the existence of an accessible possible world including an entity with an unacceptable degree of complexity.

The semantics of counterfactuals together with the truth definition for $\Diamond(A_i)_{i\in\mathbf{N}}$ allow us to generalize this idea underlying analyses of infinite regress arguments: Let us be given an infinite regress $I=A_1 \;\Box\!\!\rightarrow A_2;\; A_2 \;\Box\!\!\rightarrow A_3;\ldots$ yielded by a logically consistent theory *T*. Moreover, let us be given a model $M=(W_M, R_M, S_M, V_M)$ such that the initial world of the model w_0 is a structure that satisfies $T \cup \{\Diamond A_1\}$. Since *T* yields *I*, it must be the case that for all i, the counterfactual proposition $A_i \;\Box\!\!\rightarrow A_{i+1}$ is true in w_0.

(1) Let $W_0=\{w_0\}$
(2) Since w_0 satisfies $\Diamond A_1$, there is an A_1-world accessible to w_0. Therefore—by the closest world assumption—there is at least one closest A_1-world accessible to w_0. Let W_1 be the non-empty set of all accessible closest A_1-

worlds to w_0. Since $A_1 \Box\!\!\rightarrow A_2$ is derivable from T and w_0 satisfies T, the counterfactual $A_1 \Box\!\!\rightarrow A_2$ is true in w_0. Therefore, its consequent A_2 is true in all worlds in W_1. Since W_1 is non-empty, it follows that $\Diamond A_2$ is true with respect to w_0. We may then let W_2 be the non-empty set of all accessible closest A_2-worlds to w_0.

(3) By similar reasoning, for all n we obtain non-empty sets W_n of accessible closest A_n-worlds to w_0. Moreover, each world in W_n witnesses the proposition $\Diamond A_{n+1}$ with respect to w_0.

(4) Therefore, given an infinite regress I and a model as described, we may form infinite sequences of worlds $w^0, w^1, w^2, w^3, \ldots$ where w^0 is w_0 and the world w^n is chosen from the set W_n. Let us say that any such sequence of worlds is a sequence of worlds induced by the infinite regress I, or an I-sequence. To satisfy $\Diamond(A_i)_{i\in\mathbf{N}}$ there should be an I-sequence approaching to a possible world accessible to w_0. If this is the case, we say that with respect to the model M the infinite regress I is *benign*. Otherwise, the infinite regress I is *vicious* with respect to the model.

A given infinite regress argument succeeds if it produces only vicious infinite regresses (that is, only non-approaching I-sequences) with respect to every appropriate model. If one succeeds in forming such a model with an I-sequence approaching to an accessible world, then the proponent of T may justifiably claim that the infinite regress is benign, so that the infinite regress argument directed against T fails.

Of course, in most cases when I is believed to be benign, one may construct arbitrary models where no I-sequence witnesses the benignity of I. However, note that the strength of an analysis of an infinite regress by means of a model M depends on the plausibility of the chosen model. Though the notion of plausibility does not allow a clear-cut description, it seems safe to say that richness and relevance of its stock of possible worlds; relevance of its accessibility and comparative possibility relations adds to the plausibility of a given model as a model of analysis of a given infinite regress argument.

5 From Similarity to Distance Functions

The analysis of infinite regress arguments in the previous section requires a study of the overall behaviour of infinite sequences of counterfactuals. Therefore, the Lewis–Stalnaker analysis of single counterfactuals or a finite reiteration of the analysis does not support by itself a complete analysis of infinite regress arguments. For this reason, I exploited the notion "approaching to a possible world". In this section, I present the mathematical counterpart of this and related informal notions involved in the analysis and thereby reformulate it in a more rigorous fashion.

The definition of the truth condition for counterfactual propositions in the previous section employs the notion of relative closeness between worlds. This notion is based in turn on the notion of comparative similarity. It is possible to develop Lewis–Stalnaker analysis further by means of the mathematical notion of distance function, thereby quantizing the relation of comparative similarity among worlds. This facilitates a precise formulation of the overall behaviour of infinite sequences of counterfactuals in terms of the mathematical properties of I-sequences of worlds. Lewis rejects the quantization of the degrees of similarity on the basis of the claim that this would require the dubious assumption "that the degree of symmetry of i to j equals the degree of similarity of j to i." (1973b, 51) The "counterexample" he proposes shows, however, that he has some subjective notion of similarity in mind, while I stick to an objective notion of similarity (one independent of our interests), though I do not deny that some notions of similarity may be epistemologically more interesting or more useful than others.

To quantize the comparative similarity relation between worlds, we interpret the dissimilarity between two worlds as the distance between them. In general, a *distance function* on a set of points X is a function d from $X \times X$ into the set of non-negative real numbers such that the following two axioms hold:

$$\begin{array}{ll} \text{D1.} & d(x,y) = 0 \text{ if and only if } x = y, \\ \text{D2.} & d(x,y) = d(y,x). \end{array}$$

The first axiom is the axiom of indistinguishability and the second is the axiom of symmetry. A set X accompanied with a distance function d is a *distance space*. A distance space is denoted with an ordered pair (X, d). Where a set X is considered as the domain of a distance space, each element of X is said to be a *point* in the space.

That D1 and D2 state intuitively valid conditions on the notion of distance can be seen easily. Moreover, they naturally correspond to properties of any given comparative similarity relation. In terms of comparative similarity, the indistinguishability axiom states that x is the most similar thing to y if and only if x and y are the same; the symmetry axiom states that the degree of similarity of x to y is the same as the degree of similarity of y to x. This parallel suggests that in many cases where we are given a similarity relation, we may find a corresponding distance function. However, recall that comparative similarity need not be total. On the face of this possibility, one must content with a weaker condition than strict correspondence. Let us say that the pair (X, R) consisting of a set X and a comparative similarity relation R on this set is a *comparative similarity space*. Obviously, we will be interested in comparative similarity spaces whose underlying set is the set of worlds in a model and whose comparative similarity relation is the one belonging to this model. Let us be given a model (W_M, R_M, S_M, V_M). Consider its underlying comparative similarity space (W_M, S_M) and let d be a distance function on the set W_M now considered as a set of points. We say that S_M is compatible with the distance function d on W_M (or S_M is given by the distance function d) if for every w, w', w'':

$$S_M(w, w', w'') \text{ if and only if } d(w, w') < d(w, w'').$$

Or, in terms of the binary comparative similarity relations,

$$S_{M,w}(w', w'') \text{ if and only if } d(w, w') < d(w, w'').$$

Once we have a distance space, we can define the notion of limit for infinite sequences of points in this space: Let (X, d) be a distance space and (p_n) a sequence of points in this space. We say that a point p in X is *a limit* of the infinite sequence (p_n), or the sequence (p_n) *converges* to p (in symbols $(p_n) \to p$), if no matter how small real number $\varepsilon > 0$ is chosen, we can find a natural number N_ε such that for all $n > N_\varepsilon$ it is the case that $d(p_n, p) < \varepsilon$. Note that N_ε depends on ε: if we are given a smaller ε, then we may need to choose a larger number N_ε. A sequence is *convergent* if it converges to a point. Otherwise, it is *divergent*.

The notion of limit now replaces the informal notion of "approaching" to a possible world: Let $M=(W_M, R_M, S_M, V_M)$ be a model and $w', w'', w''', \ldots$ an infinite sequence of possible worlds in W_M. Assume also that S_M is given by a distance function d on W_M. If we know that there is a limit of the sequence $w', w'', w''', \ldots$ in the distance space (W_M, d), then we know that there is a possible world $w \in W_M$ such that our infinite sequence of possible worlds becomes more and more similar to w. Moreover, since $w \in W_M$ and d is a total function, this limit world w is of some definite real distance from w.

Let us be given an infinite regress argument directed against a theory T yielding an infinite regress I. Assume further that we are given a model $M=(W, R, S, V)$ such that

(a) S is given by a distance function d and
(b) The initial world satisfies the set of propositions $T \cup \{\Diamond A_1\}$.

These assumptions let us obtain an infinite sequence of sets of possible worlds W_0, W_1, W_2, … Then, by choosing $w^n \in W_n$, one may form I-sequences. The infinite regress I is vicious with respect to M if there is no I-sequence $w_0=w^0, w^1, w^2, \ldots$ in M such that $(w^n) \rightarrow w$ where w is a possible world accessible to w_0. Otherwise, I is benign with respect to M.

To save T from an infinite regress argument based on the regress I yielded by T, one should find an appropriate model M satisfying (a) and (b) such that I is benign with respect to M. The infinite regress arguer wins, if it can be shown that no such model can be constructed.

I finish this section with a technical remark that might prove to be useful for some applications of the present theory of infinite regress arguments. Though one may be content with distance functions, a stronger notion of distance may be desirable especially when analysing complex examples of infinite regresses. These stronger notions of distance are usually obtained from the notion of distance function by additional conditions. The most common one is the notion of metric:

A distance function satisfying the following *triangle inequality* condition is a *metric*:

$$\text{D3.} \quad d(x,y) + d(y,z) \geq d(x,z)$$

A set with a metric on this set is a *metric space*. The notion of metric is also plausible as a notion of distance, for the *triangularity* condition D3 captures the geographical intuition that we cannot shorten the distance between two points x and y by passing through an intermediary point y.

The most important property of metrics that one may need is the uniqueness of limits: The axioms D1, D2 and D3 *jointly* imply that: if $(p_n) \rightarrow p$ and $(p_n) \rightarrow q$, then $p=q$. In a general distance space, a sequence may have more than one limit that is, it could be the case that $(p_n) \rightarrow p$ and $(p_n) \rightarrow q$ with $p \neq q$. In this case, by the axiom of indistinguishability, $d(p, q) \neq 0$. In general, existence of a limit is all that matters, but if one needs to have more control over the limit worlds, one may search for a metric to guarantee uniqueness. (Though there are weaker conditions than triangularity that also implies unique limits.) In fact, the main result of the paper by Schlechta and Makinson (1994) shows that in fairly many cases, the comparative similarity relation is given by a metric. To simplify matters, I refer to a corollary (namely, Theorem 8 in their paper) as the "Schlechta–Makinson theorem" and adapt it to the terminology and notation used here. The most important notational difference is that they would write, for example, $w' <_w w''$ instead of $S_w(w', w'')$.

Let us say that a model $M = (W_M, R_M, S_M, V_M)$ is *metrizable* if its comparative similarity relation S_M can be given by a metric d on W_M. That is, there is a common metric d on W such that for every $w, w', w'' \in W$,

$$S(w, w', w'') \text{ if and only if } d(w, w') < d(w, w'').$$

A relation R is said to be *modular* if there is a function f from the field of the relation R (i.e. the union of the domain and the range of R) into a set totally ordered by a relation R^* such that $R(x, y)$ if and only if $R^*(f(x), f(y))$. Schlechta–Makinson theorem states that M is metrizable given that M satisfies the following conditions:

1. For all $w \in W_M$, the comparative similarity relation $S_{M,w}$ is modular and $S_{M,w}$ is given by a metric d_w,
2. The size of W_M is at most the size of the set of natural numbers,
3. For all $w \in W_M$, if $w \neq w'$, then $0 = d_w(w, w) < d_w(w, w')$.

6 Examples

The technique involved in the theory developed in the last two sections will now be exemplified by analyses of some infinite regresses. I claim that if a regress is commonly accepted to be benign, in any model it should yield only convergent I-sequences starting with the actual world. If an infinite regress is disputable, it should be possible to produce some models with only divergent I-sequences and some others with at least one convergent I-sequence. In this case, the value of the technique lies not in providing a final resolution but in facilitating a rigorous formulation of conflicting analyses. It is a curious fact that in philosophy—as opposed to mathematics—no nontrivial regress is commonly accepted to be vicious.

1-proposition circle I start with a formal regress. This example also makes it clear that in some cases the technique is able to produce a final result.

It immediately follows from the semantics of counterfactuals that the counterfactual proposition $p \mathbin{\Box\!\!\rightarrow} p$ is obviously valid. Therefore, every theory T yields the infinite regress $p, p, p, \ldots$

Let $M = (W, R, S, V)$ be a model with a compatible distance function d such that w_0 satisfies $T \cup \{\Diamond p\}$. There are two cases to consider:

Case 1. The proposition p is actually true (including the case where p is necessarily true). Then, whatever d is, for every $0 < i$, W_i is the singleton set $\{w_0\}$ since the actual world is the only *closest* accessible world that makes an actually true proposition true; by the axiom D1 of indistinguishability, for all other worlds w_i, the distance $d(w_0, w_i) > 0$. Therefore, the only I-sequence is the constant sequence $w_0, w_0, w_0, \ldots$ Since this sequence is convergent, with respect to M, the infinite regress is benign in M.

Case 2. Assume now that p is actually false. By assumption, $\Diamond p$ holds with respect to the actual world. Then there are some p-worlds accessible to w_0. Then,

> by the closest world assumption, there is at least one closest *p*-world. Let W_1 be the set of all such worlds. In fact, since the infinite sequence is formed by the same counterfactual *p* □→;*p*, for every *i*>0, W_i consists of the closest *p*-worlds accessible to w_0. To obtain a convergent *I*-sequence, choose one of these *p*-worlds *w* and let $w^i=w$ for every $0<i$. Being an eventually constant sequence, the resulting *I*-sequence w_0, *w*, *w*, … is convergent and the infinite regress is analysed to be benign with respect to *M* in this case too.

Therefore, we may conclude that in every model there are only convergent *I*-sequences. Therefore, the 1-proposition regress fails to be a refutation of *T.*

Tarski's truth regress Let us be given a language *L* with a Tarskian truth definition such that for each *L*-sentence, writing that sentence inside single quotes yields a name of the sentence in *L*. Tarski's theory of truth for *L* implies all Tarski sentences, that is, sentences of the following form:

T_S : '*S*' is true if and only if *S*.

It is well-known that Tarski's theory leads to an infinite regress: Let *L* be a language rich enough and such that Tarski and us would accept that at least one *L*-sentence is possibly true. Let *S* be an arbitrary but fixed such *L*-sentence. Then, Tarski biconditionals lead to the sequence of sentences

'*S*' is true if and only if *S*
"*S*' is true ' is true if and only if '*S* ' is true
"'*S*' is true' is true' is true if and only if " *S*' is true' is true

.
.
.

Obviously, Tarski sentences hold not only in the actual world, but also in every possible world *w*. Now consider the following sequence of counterfactuals formed from the right-to-left parts of the above biconditionals:

S □→ '*S*' is true
'*S* ' is true □→ "*S*' is true' is true '
"*S*' is true' is true'□→ "'*S*' is true' is true' is true

.
.
.

It follows from the Tarski's definition that each of these counterfactuals is valid (true in every possible world in every model). Consider the first counterfactual, *S* □→'*S*' is true.

1. If *S* is already true in *w*, then *w* is the only closest *S*-world to itself. Since 'If *S*, then '*S*' is true' holds in *w*, "*S*' is true' must hold in *w*. This makes the counterfactual "*S* □→'*S*' is true" hold in *w*.
2. If *S* is contingently false in *w*, then consider any of the closest *S*-worlds *w*′. Since the Tarski conditional 'If *S*, then '*S*' is true' holds in *w*′, by modus ponens "*S*' is true' must hold in *w*′. This makes the counterfactual "*S* □→'*S*' is true" hold in *w*.

3. If S is impossible in w, then the counterfactual becomes trivially true since in this case there is no accessible S-world to check for the truth of the consequent "'S' is true".

By similar reasoning one can see that all of the counterfactuals in the sequence are valid. Therefore, we obtain an infinite regress yielded by the Tarski's theory of truth:

$$S, \text{'}S\text{' is true, "}S\text{' is true' is true, } \ldots$$

Let us now determine whether we can develop a successful infinite regress argument based on this infinite regress. So let M be a model with *any* distance function d compatible with the comparative similarity relation of M. Further assume that S is possibly true in the actual world. There are two cases to consider:

Case 1. If S is actually true, then Tarski's definition implies that "S' is true' is also actually true. Therefore, the actual world w_0 already satisfies the consequent of the counterfactual 'S □→'S' is true'. That is, $w_0 \in W_1$. By the axiom D1 of indistinguishability, w_0 is the only world w such that $d(w_0, w)=0$. Therefore $W_1=\{w_0\}$. Again, given that S is actually true, "S' is true' is actually true, and Tarski's definition implies that "'S' is true' is true' is also actually true. Therefore, for the same reason as above, the actual world is the only closest ('S' is true)-world that is also an ("S' is true' is true)-world. It follows that $W_2=\{w_0\}$. Continuing this way, one may see that $W_i=\{w_0\}$ for every $i>0$. Therefore, one may form only one I-sequence of possible worlds $w_0, w_0, w_0,\ldots$ and this sequence is trivially convergent. Therefore, if S is actually true, the Tarski regress is benign with respect to the model M.

Case 2. S is contingently false but possibly true in the actual world w_0. Then there is an S-world accessible to w_0. By the closest world assumption, there is at least one closest S-world accessible to w_0. W_1 is the set of all such worlds. By the Tarski biconditional,

$$\text{'}S\text{' is true if and only if } S,$$

"S' is true' is satisfied in every W_1-world. In fact, the above biconditional establishes the fact that the set of S-worlds and the set of ('S' is true)-worlds coincide. It follows that, W_2, the set of all closest ('S' is true)-worlds accessible to w_0 is the set W_1. In a similar vein (by using the appropriate Tarski biconditional), we see that $W_i= W_1$ for every i>0. Therefore, by choosing the same world w each time, we obtain an eventually constant sequence of possible worlds $w_0, w, w,\ldots$. By definition, this sequence is convergent independently of the distance function chosen.

Since M is arbitrary, we may conclude that in any model there is no way to make the inadmissibility premise $\sim\Diamond(A_i)_{i\in\mathbf{N}}$ true where $A_1=S$ and $A_2=$'S' is true; $A_3=$"S' is true' is true; and so forth. Therefore, the Tarskian truth regress is benign on the proposed analysis.

Zeno's regress In most of the cases, philosophical infinite regress arguments are challenging but not decisive. For example, while Zeno's infinite regress arguments concerning motion are admirable, nobody sincerely believes that they deprive of all value the usual concept of motion, let alone establish the impossibility of motion. An ordinary language representation of Zeno's regress argument along the lines of the proposed schema of infinite regress arguments would be:

1. *T*: Motion is covering a distance. Covering a distance *d* is to be at the initial point of *d* at time *t* and to be at the terminal point of *d* at time $t'>t$ and passing through every point of *d*. Distance is infinitely divisible. If one was able to cover a distance *d*, and *d′* is a division of *d*, then one would be able to cover *d′* too. Any division of a division of a distance *d* is a division of *d*.
2. Achilles is able to move (or; possibly, Achilles moves.)
3. If it were the case that Achilles moves, then it would be the case that he covers a distance *d* during a time interval *t*; if it were the case that Achilles covers a distance *d* during *t*, then it would be the case that he covers the two halves of the distance *d* during *t*; if it were the case that Achilles covers the two halves of the distance *d* during *t*, then it would be the case that he covers the four quarters of the distance *d* during *t*; and so forth.
   (1)
4. For every *n*, Achilles is able to cover 2^n distances consisting of divisions of *d* during *t*.
   (2,3)
5. It is possibly the case that Achilles covers infinitely many distances during *t*.
   (1,4)
6. It is impossible that Achilles covers infinitely many distances during *t*. (Either there is an *n* such that even Achilles is not able to cover 2^n distances consisting of divisions of *d* during *t* or, even if there is no such definite boundary, Achilles is not able to cover infinitely many distances.)
   (Premise)
7. Either *T* is incorrect or Achilles is not able to move. (Conclusion)

Those who offer the above infinite regress argument as a refutation of *T* may rely on the idea that in all relevant models the similarity relation should measure the number of distances. Let for any possible world *w*, #(*w*) be the number of distances Achilles covers in *w*. Then for such a model *M*,

$$(*)\ S_w(w', w'') \text{ if and only if } |\#(w) - \#(w')| < |\#(w) - \#(w'')|$$

Therefore let $M=(W, R, S, V)$ be such that w_0 satisfies *T* and the proposition that it is possible that Achilles moves. Moreover, assume that the similarity relation *S* of *M* satisfies (*). In this case, we may immediately let $d(w,w')=|\#(w)-\#(w')|$ to obtain a distance function compatible with *S*.

Case 1. Achilles actually moves. In this case, since the counterfactuals are all true in the actual world w_0, the truth definition of counterfactual propositions implies that $W_n=\{w_0\}$ for every *n* and the only *I*-sequence is the trivially convergent constant sequence $w_0, w_0, w_0, \ldots$

Case 2. It is contingently false that Achilles moves. We let $W_0=\{w_0\}$ and W_1 be the set of all closest accessible worlds to w_0 where Achilles moves (such worlds exist by the assumption 2 and the closest world assumption.) Moreover, the counterfactual "If it were the case that Achilles moves, then it would be the case that he covers a distance d during a time interval t." holds with respect to w_0. Since W_1 consists of the closest accessible worlds to w_0 such that Achilles moves, in every W_1-world he covers a distance d during a time interval t." Therefore it is possible with respect to w_0 that Achilles covers a distance d during a time interval t. We let W_2 be the set of all closest such accessible worlds to w_0. If we continue, we see that for every n>1, W_n consist of all closest alternatives of w_0 such that Achilles covers 2^{n-2} distances during the time interval t. Accordingly, however we choose our I-sequence, the distance $d(w^0, w^n)$ increases without limit as the number of distances that Achilles covers in w^n does. It means that on this interpretation, Zeno's regress supports a successful infinite regress argument.

On the other hand, one may try an Aristotelian strategy to save T from the infinite regress argument. This strategy depends on the distinction between the actual and potential infinities. If this course is taken, it is claimed to be misleading to say that the number of distances increases at each step (cf. *Physics*. 263b3–9). There is in fact no definite distance that increases in number. Instead, there is an infinite divisibility of one definite distance. Therefore, to obtain an Aristotelian model, we *conjoin* these divisions together, rather than *count* them as independent entities. Then in any of the models above, we let $d'(w_0, w_0)=0$ and for each $w_i \in W_i$, $d'(w_0,w_i)=L(\sum_{n=0}^{i}1/2^n)$ where L is the length of the distance that Achilles must cover. Since the sequence consisting of the partial sums $(\sum_{n=0}^{i}1/2^n)$ approaches to 1, $L(\sum_{n=0}^{\infty}1/2^n)=L$. Therefore, every I-sequence $w^0, w^1, w^2, \ldots$ approaches to a world w where, *mutatis mutandis*, Achilles is L units away from the starting point. Therefore, T is saved from the Zeno's regress argument on the Aristotelian analysis.

7 Concluding Remarks

I proposed a theory of infinite regress arguments where infinite regresses are represented as sequences of counterfactuals rather than indicative conditionals. In an indicative theory, an infinite regress leads to an infinite set of *entities in the actual world*, or rather to a proposition "There *are* infinitely many …things." Therefore, the line labelled as "the result" on line 5 of the Gratton schema is such a proposition as, "There is an infinite chain of causes" or "There are infinitely many distances to be covered in a finite period." On the other hand, the contradicting proposition labelled as "Result is unacceptable" asserts that "It is *impossible* that there are infinitely many … things" such as "An infinite chain of causes is impossible", or "It is impossible to cover infinitely many distances in a finite period." Thus one tries to contradict an actually true proposition with an impossible proposition, whereas an actual falsity would suffice to contradict an actual truth. The present theory may be considered as an attempt to remove this discrepancy by suggesting a unified presentation and evaluation of infinite regress arguments. "The result" which is now a special kind of possibility proposition is now contradicted with its proper negation.

Using counterfactuals to form infinite regresses and a triggering proposition $\Diamond A$ remarkably changes the semantic analysis of infinite regress arguments. Since an infinite regress is now considered as a sequence of counterfactuals, we may extract a sequence of sets of possible worlds W_i where, intuitively, each set W_i corresponds to all possible ways of carrying on the construction at the i-th stage as required by the regress. In this case, the sequence W_0, W_1, W_2, … of sets of possible worlds can be considered as an infinite decision tree. Now, choosing a sequence of possible worlds w^i one tries to complete a process dictated by the infinite regress. At each stage, one performs an operation and this brings in some changes in the world (unless the world is already designed that way) together with some side effects. Therefore, a proponent of the regress theory T should be able to claim that it is possible to choose an infinite sequence of accessible possible worlds w^i from W_i in such a way that though the sequence of operations on the initial world is infinite, one may end up with a possible world w at the limit which is still accessible to the actual world.

The possibility of completing an infinitary task does not automatically follow from the fact that at each stage we have at least one possible way of proceeding: an intuitionist would accept the existence of natural numbers 0; the successor of 0; the successor of the successor of 0; and so forth, whereas rejecting the existence of the infinite set of natural numbers as a closed whole. Therefore, a justification is required for the completability of the required infinitary task, or the infinitary collections of entities; in our semantic analysis, this corresponds to the existence of a limit in an appropriate model with respect to a compatible distance function. Note that when one works with a distance function that is not known to satisfy some additional conditions, existence of several limits is possible. This possibility is welcome: in our case, it means that there is more than one way to complete a prima facie impossible infinitary task.

Accordingly, to complete a successful infinite regress argument against a theory T yielding an infinite regress, one must now show that there is no way of constructing such an infinite sequence of possible worlds in which case either T or the assumption $\Diamond A$ should be dismissed. Note that in the present analysis, the validation of the inadmissibility condition goes side by side with the analysis of the infinite regress I; in the same class of models, we decide what result I produces (that is, I-sequences) and—on the basis of the distance function embedded in M—whether that result is acceptable (that is, whether all I-sequences converge or not).

Another virtue of the present theory is its explanatory power. Consider the truth regress: I believe it is important that the analysis of this regress in the counterfactual theory not only complies with our belief that this regress is benign; it also provides a non-trivial justification for this belief. One may expect even more in terms of explanatory power if, for example, one uses a richer logic for counterfactuals (such as a quantificational logic for counterfactuals), but one should then face the fact that there is no well-established quantificational logic of counterfactuals, and a great deal of formal issues grounded in metaphysical issues (such as those related to trans-world identity) needs to be resolved in this field.

As the Zeno regress argument shows, the key to develop a rigorous analysis of an infinite regress argument is to find an appropriate distance function that correctly measures the effects of the changes dictated by the counterfactuals forming the underlying infinite regress. At this stage, some technical apparatus from the field where the theory yielding the regress belongs to comes into play. The resulting complexity at the level of semantics makes our analyses sensitive to the subject matter of the argument. This should be a considerable merit on the face of the diversity of areas where infinite regress arguments play some role.

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References

Aristotle (384-22 BCE). (1961). *Physics*. Transl. J. Hope. Lincoln: University of Nebraska Press.
Barwise, J., & Etchemendy, J. (1987). *The liar: an essay on truth and circularity.* New York: Oxford University Press.
Barwise, J., & Moss, L. (1996). *Vicious circles: on the mathematics of non-wellfounded phenomena*. Stanford: CSLI Publications.
Black, O. (1996). Infinite regress arguments and infinite regresses. *Acta Analytica, 16*, 95–124.
Creswell, M. J., & Hughes, E. M. (1996). *A new introduction to modal logic*. London: Routledge.
Gratton, C. (1997). What is an infinite regress argument? *Informal Logic, 18*, 203–224.
Gratton, C. (2010). *Infinite regress arguments*. Dordrecht: Springer.
Jackson, F. (Ed.). (1991). *Conditionals*. Oxford: Oxford University Press.
Kühnberger, K-U. (2001). *Formal frameworks for circular phenomena*. PhD dissertation. Univ. of Tubingen.
Lewis, D. (1973a). Counterfactuals and comparative possibility. *Journal of Philosophical Logic, 2*, 418–46. Reprinted in Lewis 1986.
Lewis, D. (1973b). *Counterfactuals*. Blackwell Publishers.
Lewis, D. (1986). *Philosophical papers (Vol. 2)*. New York: Oxford University Press.
Maurin, A.-S. (2007). Infinite regress: Virtue or vice? In T. Rønnow-Rasmussen et al. (Eds.), *Hommage à Wlodek* (pp. 1–26). Lund University.
McLarty, C. (1993). Anti-foundation and self-reference. *Journal of Philosophical Logic, 22*, 19–28.
Passmore, J. (1961). *Philosophical reasoning (2nd ed. 1970)*. New York: Scribner's Sons.
Sanford, D. (1984). Infinite regress arguments. In J. H. Fetzer (Ed.), *Principles of philosophical reasoning* (pp. 93–117). Totowa, New Jersey: Rowman and Allanheld Publishers.
Schlechta, K., & Makinson, D. (1994). Local and global metrics for the semantics of counterfactual conditionals. *Journal of Applied Non-Classical Logics, 4*(2), 129–140.
Stalnaker, R. (1968). A theory of conditionals. In N. Rescher (Ed.), *Studies in logical theory* (pp. 98–112). Oxford: Basil Blackwell. (Reprinted in Jackson 1991.)
Walton, D. (2005). Begging the question in arguments based on testimony. *Argumentation, 19*, 85–113.
Wieland, J. W. (2013). Infinite regress arguments. *Acta Analytica, 28*, 95–109.